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Original scientific paper

PLANT HEIGHT IN SOME PRILEP TOBACCO VARIETIES

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ABSTRACT

Height is distinctive morphological feature for each type and variety of tobacco. Investigations on this character were conducted in 2009 and 2010 in the Experimental field of Tobacco Institute Prilep with six oriental varieties of the type Prilep: Prilep P-23 (Ø), P 12-2/1, NS-72, P 66-9 /7, P-79-94 and Prilep Basma 82. The average values for the height of the stalk with inflorescence ranged from 59,3 cm in Prilep P 12-2/1 to 148,1 cm in Prilep Basma 82. Compared to the check, only in variety Prilep P 12-2/1 the stalk height is lower and in all other varieties it is higher.

Key words: Tobacco, type Prilep, P-23, P-12-2/1, NS-72, P 66-9/7, P-79-94, Prilep Basma 82, genotype

ВИСОЧИНА НА РАСТЕНИЈАТА КАЈ НЕКОИ СОРТИ ТУТУН ОД ТИПОТ ПРИЛЕП

Височината како морфолошко својство е карактеристика на секој тип и сорта на тутун. Истражувањата за ова својство се вршени во 2009 и 2010 година на опитното поле од Институтот за тутун Прилеп на шест ориенталски сорти од типот прилеп: прилеп П-23 (Ø), П 12-2/1, НС-72, 66-9/7, П-79-94 и прилеп басма 82. Просечните вредности за височината на стракот со соцветие се движат од 59,3 cm кај сортата прилеп П 12-2/1 до 148,1 cm кај прилеп басма 82. Во однос на контролата само сортата прилеп П 12-2/1 има пониско стебло, а сите останати истражувани сорти имаат повисоко стебло.

Клучни зборови**:** Тутун, тип прилеп, П-23, П-12-2/1, НС-72, П 66-9/7, Р-79-94, прилеп басма 82

INTRODUCTION

Tobacco is a very "plastic" plant, which, depending on the growing conditions, can have large differences in morphological properties. Although these properties are considerably variable and greatly depend on external environmental conditions, it can be stated that plant height is a typical and varietal characteristics. Selectioners make efforts to create a variety with higher number of leaves and stalk height in the limits that correspond to oriental tobaccos, i.e. they are trying to maintain their qualitative and quantitative characteristics which are of major importance both for tobacco growers and for purchasers.

MATERIAL AND METHODS

The following six varieties of the type Prilep served as material for comparative studies of stalk height: P-23 - Ø (check), P12-2/1, NS-72, P-66-9/7, P-79-94 and Basma-82 (Figures 1 - 6).

Field trial was set up in diluvial-colluvial soil in 2009 and 2010 in four replications, with one deep plowing (to 35 cm depth) in autumn and three plowings in spring. Fertilization was applied before the second spring plowing with complex mineral fertilizer NPK 10:30:20 in amount of 300 kg / ha.

All necessary cultural practices were applied during the growing period of tobacco. In both years of investigation, nutrition with 27% KAN in amount of 60 kg/ha was made. Measurements of this morphological trait were performed according to usually applied methods in genetics and tobacco breeding.

Photo 1. Prilep P-23

Photo 2. Prilep P 12-2/1

Photo 3. Prilep NS-72

Photo 4. Prilep P-66-9/7

Photo 5. Prilep-79-9/4

Photo 6. Prilep Basma -82

CLIMATE CHARACTERISTICS

The mean monthly air temperature of 18.9^{0}C in 2009 and 2010 during the growing period satisfies the requirements for obtaining a good quality tobacco raw.

Table 1 Climate characteristics

Meteorological factors	Year	Months					
		V	VI	VII	VIII	IX	/Σ
Mean monthly air temp., ^{0}C	2009	15.8	18.5	21.9	21.4	17.1	18.9
	2010	15.3	18.8	21.3	23.1	15.9	18.9
Precipitations, mm	2009	55.0	75.0	8.0	43.0	15.0	196.0
	2010	64.0	87.0	55.0	45.0	47.0	298.0

Distribution of rain during the growing period in 2009 (196 mm) was non-uniform and had a negative effect on tobacco yield and quality. To reduce this negative effect, irrigation appears as an indispensable practice. According to the data on amount (298 mm) and distribution of precipitation during the growing period in 2010, there were excessive rainfalls, but their distribution was non-uniform and did not meet the requirements of tobacco. Therefore, during draught periods it is necessary to apply irrigation.

RESULTS AND DISCUSSION

Tobacco height is type and varietal trait, which is affected by soil and climate conditions of the environment.

Uzunoski (1985) reported that tobacco height is quantitative trait which highly depends on variety and environmental conditions.

Stalk height was measured with and without inflorescence, because in some varieties it is inverted in the top leaves and in others it is exerted.

Data presented in Table 2 show that in 2009 only the variety P 12-2/1 is lower compared to the check. This is the oldest tobacco variety with lower number of leaves and it is understandable why its height is lower.

Table 2. Height of the stalk with inflorescence in 2009 (cm)

Variety	Replication				Average	Index
	I	II	III	IV		
Prilep P-23 (Ø)	61.4	69.1	67.4	63.9	65.5	100.00
Prilep P 12-2/1	57.1	55.4	59.5	50.4	55.6	84.88
Prilep NS-72	88.0	85.8	83.6	96.4	88.5	135.11
Prilep P 66-9/7	104.0	112.0	110.0	98.2	106.1	161.98
Prilep P-79-94	94.6	92.8	93.6	81.1	90.5	138.16
Prilep Basma 82	146.0	142.6	146.6	134.0	142.3	217.25
Average	91.8	92.9	93.4	87.3	91.4	139.54

LSD 0.05 = 7.51 cm
0.01 = 10.31 cm
0.001 = 14.35 cm

Table 3. Height of the stalk with inflorescence in 2010 (cm)

Variety	Replication				Average	Index
	I	II	III	IV		
Prilep P-23 (Ø)	62.2	81.6	69.0	72.7	71.4	100.00
Prilep P 12-2/1	68.7	57.5	60.4	65.4	63.0	88.24
Prilep NS-72	95.9	103.0	88.4	124.0	102.8	143.98
Prilep P 66-9/7	114.0	123.0	116.0	121.0	118.5	165.97
Prilep P-79-94	101.1	107.0	109.0	96.0	103.3	144.68
Prilep Basma 82	153.0	154.0	157.0	152.0	154.0	215.69
Average	99.15	104.35	99.97	105.18	102.17	143.10

LSD 0.05 = 12.18 cm
0.01 = 16.85 cm
0.001 = 23.28 cm

In the check variety Prilep P-23, the average stalk height with inflorescence is 65.5 cm. Lower values for this trait (55.6 cm) were observed only in P 12-2/1 and all other varieties had higher stalk with inflorescence, ranging from 88.5 cm in NS-72, 90.5 cm in P-79-94, 106.1 cm in P 66-9/7 up to 142.3 cm in Prilep Basma-82.

Expressed in percentages, P 12-2/1 variety had 15.12% lower height compared to the check, while all other investigated varieties were higher. Thus, NS-72 was higher for 35.11% , P-79-94 for 38.16%, P 66-9/7 for 61.98% and Basma 82 for 117.25% higher compared to the check.

Results from investigations carried out in 2010 (Table 3) were similar to those of 2009. Again, P 12-2/1 was the lowest and Basma 82 was the highest. The average height measured in the check variety P-23 was 71.4 cm and only the variety P 12-2/1 with a height of 63.0 cm was lower compared to it. The other tobacco varieties had higher stalk with iflorescence, ranging from 102.8 cm in variety NS-72, 103.3 cm in P-79-94, 118.5 cm in P 66-9/7 to 154.0 cm in Basma 82.

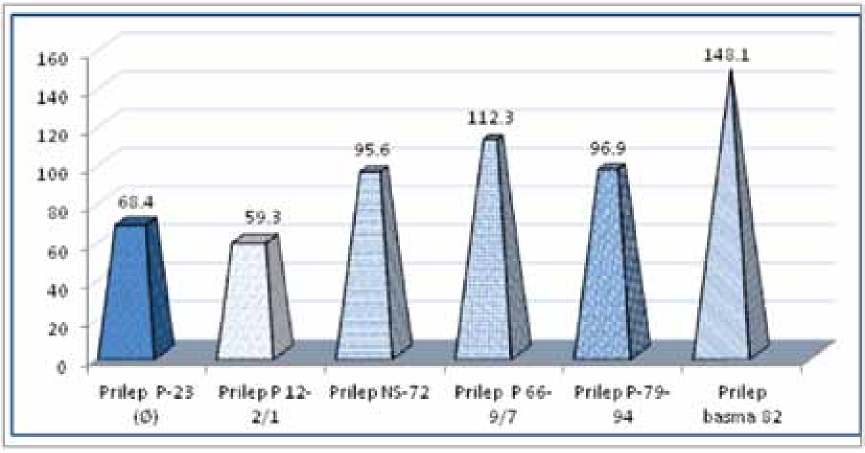

Figure 1 Average height of the stalk with inflorescence, cm (2009 – 2010)

Relative difference between the investigated varieties in relation to the check was negative only in variety P 12-21 (-11.76%) and in all other varieties it was positive for 43.98%, 44.68%, 65.97% and 115.69% in NS-72, P-79-94, P 66-9/7 and Basma 82, respectivaly.

Based on statistical data analysis for both years of investigations, it can be stated that all differences between varieties are highly significant at a level of 0.001, indicating that they are a product of the different genetic constitution.

From the presented data on the average two-year results for stalk height with inflorescence in analysed tobacco varieties (Figure 1), we can only confirm the statement that has been almost the same in 2009 and 2010. Tobacco control variety P-23 has an average height of 68.4 cm. The lowest stalk height of 59.3 cm was measured in P 12-2/1 and it is 13:31% lower compared to the check variety. Greater stalk height was found in variety NS-72 (95.6 cm, i.e. 39.77% higher). Somewhat lower height was observed in P-79-94 (96.9 cm, or 41.67% higher compared to the check). Greater average height of 112.3 cm (or 64.18% more) was observed in P 66-9/7 and the highest was variety Prilep Basma 82 with 148.10 cm, i.e. 116.52% higher than the check.

Beside stalk height with inflorescence, measurements were also made on stalk height without inflorescence, because the shape and size of the inflorescence can greatly differ, which has an impact on this character.

Our investigations on tobacco stalks without inflorescence carried out in 2009 (Table 4) gave similar results as for height of the stalks with inflorescence. The check variety had an average height of 59.0 cm and variety P 12-2/1 had 50 cm, i.e. 15.25% shorter. Higher stalk height was achieved in varieties NS-72 (80.9 cm), P-79-94 (82.7 cm, i.e. 40.17% higher than the check), P 66-9/7 (102.1 cm) and the highest was the stalk of Basma 82 variety with 131.7 cm, i.e. 132,22% more than the check.

Analysis of the results on stalk height without inflorescence obtained in 2010 (Table 5) show small differences compared to the year 2009. The check variety P-23 has an average height of 64.0 cm and only the variety P12-2/1 is lower than it, measuring 54.2 cm. Higher than the check are all other varieties: P-79-94 with 92.7 cm, NS-72 with 93.8 cm, P 66-9/7 with 108.5 cm and the highest was Prilep Basma 82 with 143.5 cm.

Table 4. Stalk height without inflorescence (cm), 2009

Variety	Replication				Average	Index
	I	II	III	IV		
Prilep P-23 (Ø)	55.9	61.0	60.1	58.8	59.0	100.00
Prilep P 12-2/1	50.6	50.5	54.5	44.3	50.0	84.75
Prilep NS-72	81.1	77.6	76.3	88.4	80.9	137.12
Prilep P 66-9/7	95.8	104.4	101.5	91.6	102.1	173.05
Prilep P-79-94	87.6	83.6	86.9	73.2	82.7	140.17
Prilep Basma 82	134.4	132.6	136.0	123.6	131.7	232.22
Average	84.2	84.9	85.9	80.0	84.4	143.05

LSD 0.05 = 7.36 cm
0.01 = 10.17 cm
0.001 = 14.05 cm

Table 5. Stalk height without inflorescence (cm), 2010

Variety	Replication				Average	Index
	I	II	III	IV		
Prilep P-23 (Ø)	56.3	74.9	60.4	64.5	64.0	100.00
Prilep P 12-2/1	60.3	48.1	52.8	55.6	54.2	84.69
Prilep NS-72	87.2	93.6	81.1	113.1	93.8	146.56
Prilep P 66-9/7	104.0	112.9	106.0	111.0	108.5	169.53
Prilep P-79-94	90.6	95.0	99.0	86.0	92.7	144.84
Prilep Basma 82	143.0	143.0	146.0	142.0	143.5	224.22
Average	90.23	94.58	90.88	95.37	92.78	145.00

LSD 0.05 = 11.67 cm
0.01 = 16.13 cm
0.001 = 22.29 cm

Statistical data processing on stalk height without inflorescence in 2009 and 2010 confirms that differences are highly statistically significant, i.e, that each of the investigated tobacco varieties is unique and has its own genetic code.

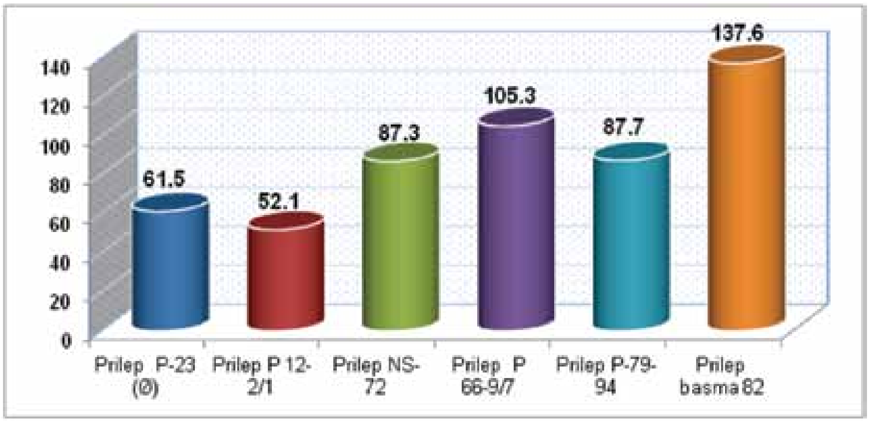

Figure 2. Average stalk height without inflorescence in cm (2009-2010)

According to the average height for the two investigated years (Figure 2), it can be noted that the check variety has an average height of 61.5 cm and that only the variety P 12-2/1 has a shorter stalk - 52.1 cm, which is 15:29% lower. Higher stalks compared to the check were measured in varieties Prilep NS-72 with 87.3 cm (41.95% higher), P-79-94 with 87.7 cm (42.60% higher), P 66-9 / 7 with 105.3 cm (71.22% higher) and the highest was Prilep Basma 82 with 137.6 cm (123.74% higher).

The height of Prilep tobacco varieties was studied by a number of authors. Our data are in correlation with their results.

Odic (1973) reported that the average height of tobacco type Prilep average was 83 cm. Uzunoski (1985) found that Prilep tobacco is the lowest of all other types, with an average height of 40 to 50 cm.

According to Dimitrieski (1985), the height of Prilep tobacco variety P 12-2/1 is 45.5 cm. Dimitrieski (2001), in his investigations of several varieties of the type Prilep in conditions of irrigation, came to the following data: variety P12-2/1 with inflorescence is 46 cm high, variety P-23 reached a height of 57 cm, while Prilep NS-72 had a stalk height of 74 cm.

Korubin-Aleksoska (2004) reports the following average stalk heights: P-23 - 65 cm, P 12-2/1 – 55cm and P-79-94 - 70 cm.

Dimitrieski (2011), in his investigations on variety P-66-9/7 suggested that under normal conditions for growth, plant height ranges from 65 to 75 cm.

From previously presented data on the height of the stalk with and without inflorescence, a general conclusion can be drawn that the differences between investigated tobacco varieties are significant and that they are primarily a result of the genotype.

CONCLUSIONS

Based on the two-year investigations on plant height in some varieties of tobacco type Prilep, the following conclusions can be drawn:

Height of the stalk with inflorescence in the check variety Prilep P-23 was 68.4 cm. The lowest stalk height (59.3 cm) was measured in variety Prilep P 12-2/1. Higher stalks compared to the check waere recorded in varieties Prilep NA-72 with 95.6 cm, Prilep P-79-94 with 96.9 cm, P 66-9/7 with 112.3 cm and the highest was Prilep Basma 82 with 148.1 cm

- The height of the stalk without inflorescence has a similar pattern of differences mong the investigated tobacco varieties.
- Climate conditions in 2010 were better for growth of tobacco plant, because the average height in all investigated varieties was increased by 8.4 cm compared to 2009.
- Differences in stalk height with and without inflorescence in all tobacco varieties is a result of the genotype, i.e . it is a varietal trait.

REFERENCES

1. DIMITRIESKI Miroslav (1995). Efekti od laserskata svetlina vrz prinosot I kvalitetot na tutunot. 30.06.1995 godina. Doktorska disertacija-Zemjodelski fakultet – Skopje.
2. DIMITRIESKI Miroslav (2001). Reoniranje i mikroreoniranje (vo uslovi na navodnuvanje I sušni uslovi) na postojnite priznati orientalski sorti vo tutunoproizvodnite regioni I reoni. Proekt. JNU Institut za tutun - Prilep.
3. DIMITRIESKI Miroslav (2011). Nova poproduktivna sorta na tutun od tipot prilep. Tutun/Tobacco, Vol.61. str. 59-62. Prilep.
4. KORUBIN-ALEKSOSKA Ana (2004). Sorti tutun od Institutot za tutun - Prilep.
5. MITRESKI Milan (2005). Nasleduvanje na považnite morfološki i biološki svojstva i sodržinata na nikotin kaj F_1 hibridite kaj nekoi sorti tutun. Magisterski trud. Fakultet za zemjodelski nauki i hrana-Skopje.
6. MITRESKI Milan (2012). Komparativni proučuvanja na považnite proizvodni, tehnološki i kvalitetni svojstva kaj nekoi sorti tutun od tipot prilep. Doktorska disertacija. Naučen institut za tutun - Prilep.
7. ODIĆ Mirko (1973). Ispituvanje biološkog potencijala nekih makedonskih i hercegovačkih sorata duhana. Tutun/Tobacco. 1-12, str.11-16. JNU Institut za tutun - Prilep.
8. UZUNOSKI Mile (1995). Proizvodstvo na tutun. Stopanski vesnik. Skopje.

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Original scientific paper

COMPARATIVE TESTING OF ORIENTAL TOBACCO VARIETIES OF THE KRUMOVGRAD ECOTYPE IN THE REGION OF NEVROKOP

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ABSTRACT

A comprehensive assessment was made on biological and economic properties of five varieties of oriental tobacco ecotype Krumovgrad in agro-environmental conditions of the experimental field in the town of G. Delchev, the Nevrokop region. According to their morphological and biological properties, all tested varieties fit the typical characteristics of the ecotype Krumovgrad. Varieties Krumovgrad 78C and 68M have a potential to achieve higher yield and quality of dry tobacco and exceed the tested varieties of the ecotype Krumovgrad. From the general assessment it can be predicted that the investigated varieties can be grown not only in the selected regions but also in other regions with similar soil and climate characteristics.

Key words: *tobacco varieties, Krumovgrad, Nevrokop, yield*

КОМПАРАТИВНО ИСПИТУВАЊЕ НА ОРИЕНТАЛСКИ ТУТУНСКИ СОРТИ ОД ЕКОТИПОТ КРУМОВГРАД ВО РЕОНОТ НА НЕВРОКОП

Извршено е детално испитување на биолошките и економските својства на пет сорти тутун од ориенталкскиот екотип Крумовград во агроеколошките услови на експерименталното поле во градот Гоце Делчев (Неврокопски реон). Според нивните морфолошки и биолошки карактеристики, сите испитувани сорти се типични за екотипот Крумовград. Сортите Крумовград 78Ц и 68м имаат потенцијал да постигнат повисок принос и квалитет на сувиот тутун во споредба со останатите испитувани сорти од овој екотип. Од општата проценка може да се предвиди дека испитуваните сорти можат да се одгледуваат не само во посочениот реон туку и во други реони со слични почвени и климатски услови.

Клучни зборови: сорти тутуn. Крумовград, Неврокоп, принос

INTRODUCTION

Variety is one of the factors responsible for good quality of the received material and the level of yields and fluctuations in morphological characters is an objective component in the evaluation of genotype (variety) (Dyulgerski, 2011). The biological factor is a powerful tool for improving yield and quality of tobacco. It is also of particular importance to preserve quality characteristics of the varieties in different soil and climate conditions. At the same time we should find a favorable combination of morphological, technological, and smoking properties specific to the type of tobacco (Bojinova and , Djulgerski , 2006; Dimanov and Vitanova, 2011; Mutafchieva, 2005; Stoeva, 2006).

Market requirements for formation of large batches of high quality tobacco imposed cultivation of varieties developed for other agro-ecological regions in the Nevrokop area. Starting from 2005, oriental tobacco varieties from other ecotypes, mainly the ecotype Krumovgrad , have been distributed and grown widely in the region of Nevrokop.

Oriental ecotype Krumovgrad is one of the most respectable and well accepted tobaccos in international markets (Yancheva and Yordanov, 1997). In recent years it has emerged as one of the most demanded tobaccos, both from Bulgarian and from major international tobacco companies. The demand for aromatic tobacco as a commodity is mainly related to the specific smoking habits of the consumers.

The aim of this study is to make a comparative assessment of morphological and commercial properties and response to diseases and pests of the Krumovgrad tobacco varieties in different conditions of their habitat range.

MATERIALS AND METHODS

The field experiment was carried out during 2008-2010 with five genotypes of the varietal group Basma, ecotype Krumovgrad (varieties Krumovgrad 988, Krumovgrad 58, Krumovgrad 90, Krumovgrad 68M and Krumovgrad 78C). A comparative study was conducted in experimental station - Gotse Delchev using randomized block design in four replications. Nevrokop-1146, which has been the most cultivated variety in the region for decades, was used as a control. The area of the experimental elementary plot was 20 m2

Experience is displayed in rotation with wheat. Technology is growing at a standard farming practices of oriental tobacco, developed and adopted by TTPI- Markovo.

The following characters are reported :

- Plant height / cm /
- Number of leaves
- Leaf size / cm /
- Length of the vegetation period from planting to flowering, in days,
- Economic indicators - yield and quality of dried tobacco

The data were statistically processed (Zapryanov and Dimova, 1995).

METEOROLOGICAL CHARACTERISTICS

The values of meteorological parameters during the study (2008, 2009, 2010) are listed in tables. They affect significantly the levels of expression of the biometric, economic and technological indicators.

Monthly average temperatures and precipitation for the Nevrokop region, 2008

Months	IV	V	VI	VII	VIII	IX
Temperature	11.6	15.3	20.2	21.9	23.0	17.4
Rainfall	39.2	20.0	95.6	45.7	41.9	68.2

Monthly average temperature and precipitation for the region Nevrokop 2009

Monts	IV	V	VI	VII	VIII	IX
Temperature	11.4	17.1	19.9	22.2	22.0	17.3
Rainfall	49.4	24.4	61.4	27.1	10.0	39.4

Monthly average temperatures and precipitation for the region Nevrokop 2010

Monts	IV	V	VI	VII	VIII	IX
Temperature	11.6	16.3	18.8	21.6	24.2	17.8
Rainfall	22.3	33.7	130.0	54.2	21.9	65.5

For the three years of investigation, the maximum rainfall was recorded in June 2010, 130 L/m^2. All other values of the average monthly temperature and rainfall were close to normal for the area.

RESULTS AND DISCUSSION

The results of accompanying indicators are compiled tables.

Table 1 presents data from biometric measurements of height, number of leaves and length of growing season. In terms of plant height measured at the end of the vegetation period, the lowest average height was observed in variety Krumovgrad 988 (92 cm), followed by Krumovgrad 68 (119.4 cm) Krumovgrad 90 (120.5 cm) and Krumovgrad 78S (122.5 cm), and the highest in the standard Nevrokop 1146 (132.8 cm) and in the variety Krumovgrad 58 (134.5 cm).

For the character average number of leaves (technically fit) for the testing period, the lowest number of leaves (22) was observed in variety Krumovgrad 988 and highest in varieties Krumovgrad 78C and 68M (42-48 leaves). In other variants the number of leaves was 24 – 32.

Table 1 Biometric measurements for the period 2008 - 2010

Variants Varieties	Plant height in cm	Number of leaves	Length ofvegetation period in days	Leaf 7 length / width	Leaf 14 length / width
Krumovgrad 988	92.0	22	58	22.2/ 14.2	18.1/ 11.1
Krumpvgrad 90	120.5	24	68	23.8/ 15.4	20.1/ 13.1
Krumovgrad 58	134.5	32	64	27.4/ 19.3	22.8/ 15.0
Krumovgrad 78C	122.5	48	74	21.7 /13.2	19.1/ 11.7
Krumovgrad 68M	119.4	42	72	21.4/ 14.5	19.9/ 12.9
Nevrokop 1146-standard	132.8	32	64	26.4/ 17.8	24.2/ 14.4

According to the data, the longest period of vegetation (from planting to full bloom in days) of was observed in varieties with increased leaf number (Krumovgrad 68M-72 days, Krumovgrad 78C-74 days. Outer leaves remain greener, because they can not fully ripen well in the area conditions. Shorter growing season was recorded for the vatiety Krumovgrad 988 - 58 days. The standard Nevrokop 1146 and variety Krumovgrad 58 have average vegetation period of 64 days.

Table 1 presents data (average values for the test period) from measurements of length and width of leaves from the lower and middle harvesting layer, which is also defined by the form of the leaves. In the lower harvesting belt (7 leaves), the largest leaves are found in Krumovgrad 58 and in standard variety Nevrokop 1146. Smaller leaves are observed in varieties Krumovgrad 68M, Krumovgrad 78C, Krumovgrad 988 and Krumovgrad 90.

The highest length / width ratio in the average harvesting layer (14 leaves) was observed in Krumovgrad 58 and in the standard Nevrokop 1146.

The black shank disease, caused by *Phytophthora parasitica var.nicotianae* is one of the economically most important diseases responsible for obtaining lower yields and quality of the test varieties.

During 2008-2010, natural and artificial infections of tobacco plants were made to study the resistance to black shank.

The data show disease susceptibility of 6.1% in variety Krumovgrad 988, followed by Krumovgrad 90 (3.8%), Krumovgrad 78C (3.5%), Krumovgrad 58 (2.3%) for 1200 observed plants . The trend was confirmed in 2009, with values increased by 2-3% when irrigation water is significantly higher due to dry periods. This favors the development of the fungus far more than rainfall water.

Standard variety Nevrokop 1146 is generally resistant to black shank, but it also accounts for 1.0% diseased plants, which determines its relative stability.

Another important disease is Tobacco Mosaic Virus (TMV) I, Smith. In the work on resistance to TMV with artificial contamination by the method of Ternovskiy, all Krumovgrad varieties were found to be susceptible and react with TMV to make systemic infection.

Table 2. Yield and percentage of classes for the period of investigation

Variants Varieties	Average yield kg / dka	% of classes		
		I	II	III
1. Krumovgrad 988	179.15	41.2	48.1	10.5
2. Krumovgrad 90	201.75	48.0	47.5	4.5
3. Krumovgrad 58	216.15	47.5	39.3	13.2
4. Krumovgrad 78C	231.50	42.4	45.6	52.0
5. Krumovgrad 68M	221.65	49.2	39.3	11.5
6. Nevrokop 1146	250.00	36.0	40.0	24.0

ANOVA table

Cause of variation	SQ	FG	S2	F
Total	758.83	23		
Blocks	6.021	3		
Varieties	752.21	5	150.4	3.76
Errors	0.599	15	0.04	

Error of the difference-0 14kg/dka

The analysis of data on economic characteristics proves that the highest yields were achieved in the standard Nevrokop 1146, variety Krumovgrad 78C and Krumovgrad, 68M, followed by Krumovgrad 58 and lower yields were obtained in Krumovgrad 988 and Krumovgrad 90.

In terms of quality expressed by the percentage of classes, the difference is very small both between varieties and compared to the standard.

CONCLUSIONS

Comprehensive assessment was made of biological and economic properties of five oriental varieties of tobacco ecotype Krumovgrad in agro-environmental conditions of the experimental field in the town of G. Delchev / region of Nevrokop /.

All varieties tested fit the typical ecotype Krumovgrad in their morphological and biological characteristics.

Varieties Krumovgrad 78C and Krumovgrad 68M showed higher productive potential yield and quality of dry tobacco and outperformed the other tested varieties of the ecotype Krumovgrad.

From the integrated assessment it can be predicted that the investigated varieties can be grown not only in regions that were selected, but also in other regions with similar soil and climate characteristics.

REFERENCES

1. Bojinova R., J., Mason, 2006, studies show the growth of domestic and foreign varieties of Burley tobacco type depending on the weather, Proceedings of the First International Symposium - "Ecological approaches to the production of safe food," 221-226
2. Dimanov D., E. Vitanova, 2011. Study the possibility of foreign otgluzhdane oriental varieties in agro-ecological conditions of the region Nevrokop, Office No. 1 from 0.5 to 8.
3. Dyulgerski J., 2011. Selection-genetic studies of Burley tobacco. Thesis.
4. Zapryanov Z., D. Dimova, 1995. Manual for experimental work with biometrics. Zemizdat, Sofia
5. Mutafchieva M., 2005, Characterization of the economic performance of new hybrids selected line. Jubilee International Conference '60 ITTI, 138-141
6. Stoeva, D. 2006. Morphological and biological characteristics of tobacco varieties, forming the origin Municipality. Jubilee International Conference "SMS '20 - Smolyan"
7. Yancheva A. V.Yordanov, 1997, Variety Krumovgrad 68 M. Bulgarian tobacco, № 5, 14-18

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Original scientific paper

CHANGE IN QUANTITATIVE AND QUALITATIVE CHARACTERISTICS OF TOBACCO VARIETY JK-48 DEPENDING ON MINERAL NUTRITION

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ABSTRACT

Field experiments were carried out during 2005 - 2006 in Scientific Tobacco Institute -Prilep to investigate the effects of different rates of mineral fertilizers on yield and quality of oriental tobacco variety JK-48. According to the results, the best effect on the yield of JK-48 with an increase of 26.39 % has the Nutrifert fertilizer applied at 30 kg/ha. The average purchase price of tobacco, expressed in % of quality classes, was increased by only 1.34 % in the variant fertilized with 30 kg/ha NPK (8:22:20). In all other variants only insignificant decrease of tobacco quality was observed. To evaluate the fertilizers effect on investigated characters of tobacco all results were statistically processed by analysis of variance and LSD test.

Key words: mineral fertilizers, oriental tobacco JK-48, yield, qualitative characteristics

ПРОМЕНИ НА КВАНТИТЕТНИТЕ И КВАЛИТЕТНИТЕ КАРАКТЕРИСТИКИ НА ТУТУНОТ, СОРТА ЈК-48, ВО ЗАВИСНОСТ ОД МИНЕРАЛНАТА ИСХРАНА

Целта на двогодишните истражувања беше да се одреди влијанието на различни комбинации и концентрации на минерални ѓубрива врз приносот и квалитетот на ориенталската сорта тутун, ЈК-48.

Испитувањата се извршени на опитното поле од НИТП, во периодот 2005 - 2006 година.

Врз основа на добиените резултати, најдобар ефект врз приносот на тутунот е забележан кај ѓубривото Нутриферт со доза од 30 kg N/ha, при што истиот се зголемил за 26,39%. Просечната откупна цена изразена преку % на квалитетни класи е незначително зголемена за само 1,34% кај варијантата ѓубрена со NPK (8:22:20) со доза од 30 kg N/ha. Кај сите останати варијанти се забележува незначително намалување на квалитетот на тутунот.

За да се оцени ефектот на употребените ѓубрива врз испитуваните карактеристики на тутунот резултатите од испитувањата се статистички обработени со анализа на варијанса и LSD тестот.

Клучни зборови: ориенталски тутун ЈК-48, минерални ѓубрења, принос, квалитетни карактеристики

INTRODUCTION

Fertilization is one of the most important agricultural practices for ensuring a strong agricultural production. Yield and quality of oriental tobacco are closely associated with its availability to absorb nutrient elements from soil. Tobacco is particularly sensitive to the quantities of nitrogen in soil. This very important nutrient has a positive impact on yield and quality of tobacco to a certain limit. Then, the yield can be increased but the quality of produced tobacco substantially declines. The use of larger amounts of fertilizers before planting and frequent nourishments of tobacco, after each irrigation, is a common practice that increases the quantity, but the quality is drastically reduced. In conditions of strong fertilization and higher nitrogen availability tobacco forms larger leaves with prominent nervature, rougher and thicker tissue, difficult to dry and with poor color (K. Naumovski et al. 1977), with higher percentage of nicotine and protein, reduced precentage of sugar and bad smoke properties (Atanasov D., 1965, Dimitrievic R, Tomic K., 1963, Donev H., 1976).

The effects of fertilization can be positive and negative. Controlled use of fertilizers secures safe production of crops. There are frequent cases when enormously high amounts of fertilizers are used in order to achieve higher yields. Uncontrolled use of fertilizers has a negative effect not only on tobacco quality but it also increases the production costs and has negative impact on the environment, especially on underground waters. That is why fertilization is a very complex process which should be paid serious attention.

Lately, the market has offered complex fertilizers with various formulations for a broad range of crops. Knowing that each crop has specific physiology and different needs for nutritious elements in various stages of development, choosing the right fertilizer and formulation requires a very cautious approach. So far, mineral fertilizer NPK 8:22:20 has been recommended for the process of tobacco production.

Taking these facts into consideration, testing was made of several new formulations of mineral fertilizers to study their impact on yield and quality of oriental aromatic tobacco in the producing region of Prilep.

MATERIAL AND METHODS

Studies were performed in 2005 and 2006, at the field of the Scientific Tobacco Institute – Prilep, with oriental tobacco JK - 48 and different combinations and concentrations of mineral fertilizers, as follows:

1. Ø- unfertilized check
2. NPK (8:22:20) + Ammonium nitrate for feed (36,5% N) - 30 kg N/h
3. NPK (8:22:20 + Ammonium nitrate for feed) - 50 kg N / ha
4. Nutrifert 6 (6:12:24 +2 MgO) + fertimon for feed (25% N) - 30 kg N/ha
5. Nutrifert 6 (6:12:24 +2 MgO) + fertimon for feed (25% N) - 50 kg N/ha
6. Magnifert (14:7:14 +5 MgO + microns) + fertimon for feed (25% N) - 30 kg N/ha
7. Magnifert (14:7:14 +5 MgO + microns) + fertimon for feed (25% N) - 50 kg N/ha

The experiment was set up in randomized complete block design in 7 variants, with 3 replications. Meteorological conditions during the experiment were recorded by the Meteorological station located near the experimental field of Tobacco Institute.

Soil tests were done before setting up the experiment in order to determine the agrochemical and physical properties of the soil. 50% of the total amount of fertilizer was applied before transplanting of tobacco, and the remaining 50% two weeks after, on the first digging. Each plot consisted of 5 rows, three of which for harvest and two for protection. Sedlings were planted at a spacing of 40x12 cm. All indispensable agrotechnichal and phyto-protection practices were applied during the vegetation period of tobacco.

Harvesting was carried out in six insertions, after which tobacco was sun-cured under polyethylene. To measure the yield, quality and chemical composition of tobacco 63 plants were picked from the middle rows of the experimental plots. Qualitative assessment of processed tobacco was made according to the Rules for measurement and purchase of raw tobacco. The obtained results on yield, average price and gross income were statistically processed with LSD test.

RESULTS AND DISCUSSION

Climate and soil are the most important factors that affect the yield and quality of aromatic tobaccos (Pasoski, 1980). The Prilep production area is characterized by a warm continental climate (Filiposki 1997). From meteorological factors, precipitation and air temperature play a major role in tobacco production (Georgievski, 1990). The results of our investigations on these two parameters are presented in Table 1.

Table 1. Sum of precipitation and average air temperature during 2005 and 2006

Month	Decade	Sum of precipitation, mm		Average air temperature, °C	
		2005	2006	2005	2006
May	I	16,0	17,5	15,4	11,1
	II	1,3	12,6	17,2	15,9
	III	30,5	---	17,5	20,8
	sum	**47,8**	**30,1**	**16,7**	**16,1**
June	I	27,6	10,5	16,5	14,3
	II	29,1	8,4	18,2	19,4
	III	0,1	7,0	22,1	24,9
	sum	**56,8**	**25,9**	**18,9**	**19,5**
July	I	1,9	40,5	22,5	19,6
	II	16,8	---	21,6	20,9
	III	0,6	11,6	24,3	24,4
	sum	**19,3**	**52,1**	**22,9**	**21,5**
August	I	57,9	30,3	21,3	22,6
	II	0,3	0,5	21,3	23,4
	III	21,8	2,4	20,9	21,5
	sum	**80,0**	**33,2**	**21,1**	**22,4**
September	I	1,4	---	18,8	20,0
	II	1,7	5,1	19,6	17,4
	III	4,7	17,7	15,2	15,8
	sum	**7,8**	**22,8**	**17,9**	**17,7**
Total		**211,7**	**164,1**	**19,5**	**19,4**

In 2005, the total amount of precipitation in May and September was 211.7 mm and in 2006 it reached 164.1 mm. In this period there were 40 rainy days. Sediment quantities were even higher than those required for a good quality oriental tobacco (Atanasov 1965), but the rainfall distribution was very uneven, especially in July and August when water requirements were the highest. The uneven distribution of precipitation had a strong impact on yield and quality of tobacco. Greater amounts of water after transplanting resulted in root development near the soil surface. In later phases longer dry periods appeared, followed by high temperatures which accelerated the evaporation, so that shallow tobacco root had no ability to satisfy the water requirements for normal plant growth. In dry periods, when soil nutrients are unavailable for tobacco plants, irrigation is an indispensable measure.

The temperature is major climatic factor for development of tobacco. The optimum temperature for tobacco growth is 20-30^0 C (Atanasov 1965, Uzunoski 1985), the growing is best when the night temperatures are 18 - 21^0 C (Hawks S., Colins W, 1994), and the best temperature for maturation is when it is not lower than 20 °C (Uzunoski 1985).

According to Georgievski (1990), mean daily temperature of 22-25^{0}C throughout the growing season is accepted as an optimum temperature equivalent, and the limit equivalents are between 18 and 30^{0}C.

During the two-year field investigations, the mean daily and monthly temperatures ranged within the optimum values. The maximum temperature values were observed in July and August, with an average of 21.1 and 22.9^{0}C.

From the analysis of climatic conditions (Fig 1 and Fig 2) it can be concluded that dry periods which appear in the warmest months of the year have a negative impact on tobacco growth. The lack of water should be overcome by irrigation, which will reduce harmful effects on tobacco yield and quality.

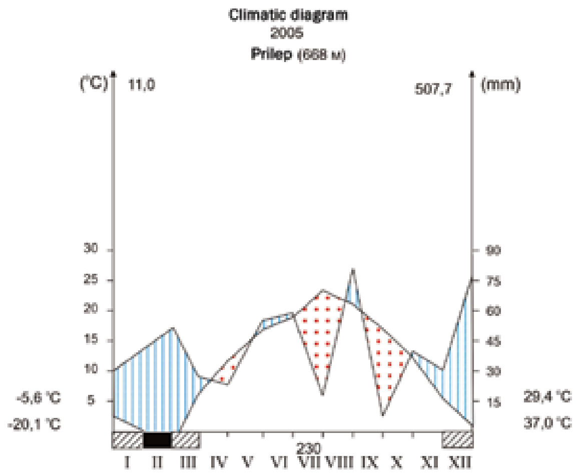

Fig. 1. Climate diagram for the year 2005

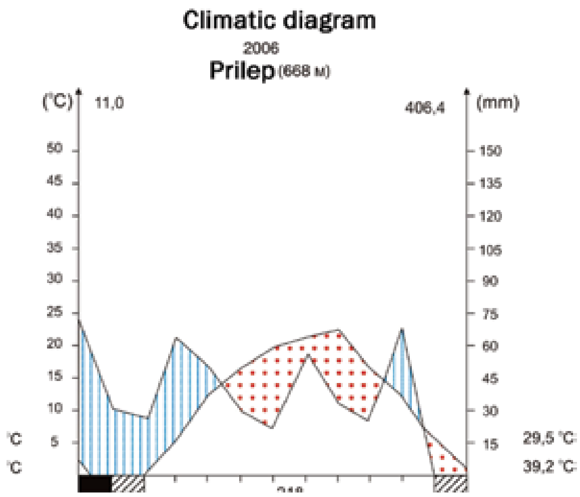

Fig. 2. Climate diagram for the year 2006

Investigations were carried out in colluvial-alluvial soil, quite common for tobacco producing region of Prilep.

According to its mechanical composition (Table 2), the soil of the arable layer is light loam, physical clay fraction is represented by 22.4%, field water capacity is 25.75% and with porosity of 31.84% it belongs to the group of low permeability soils. In terms of chemical properties, the soil is low acidic, with low humus content, low content of easily available phosphorus and medium supply of potassium.

Table 2. Physical and chemical properties of soil

Depth (cm)	Porosity vol. %	Water capacity vol. %	Fisical clay %	Texture	pH in H_2O	Humus %	mg/100 g soil	
							P_2O_5	K_2O
0 - 30	31,84	25,75	22,4	Light loam	6,00	0,53	7,3	13,3

The results on average yield (Table 3) show that all fertilized variants have higher yield compared to the check. It can be also concluded that higher yield was achieved with variants fertilized with 50 kg nitrogen/ha. The highest yield of 3353 kg/ha was obtained in variant 3, with application of Nutrifert, and in relative terms the increase was 26.39 % compared to the check.

In variant 5, fertilized with 50 kg N/ha, the yield was 25.39% higher compared to the check but 1.0% lower compared to variant 3, which was fertilized with significantly lower rate (30 kg N/ha). In variants 6 and 7 the increase was respectively 21,58% and 26,09%, compared to the unfertilized check. It can be stated from the results that the best results were obtained with variants fertilized with 30 kg N/ha. The use of higher rates of nitrogen fertilizers is unjustified and it will only make tobacco production per unit area more expensive.

Statistical analysis of results showed significant statistical difference at 0.001 level between fertilized variants and the unfertilized check. This leads to the conclusion that all investigated formulations and rates of fertilizers have a positive effect on the yield of oriental tobacco variety JK-48. No statistical significance exists among the variants fertilized with higher rates of nitrogen, while in variants with lower rates of nitrogen statistical differences of 5% and 1% were observed. The data indicate that the investigated fertilizers Nutrifert and Magnifert gave better results compared to NPK 8:22:20, which has been commonly recommended fertilizer in tobacco production so far.

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Table 3. Average tobacco yield (kg/ha)

Variants	years			
	2005	2006	average	%
Check	2797	2509	2653	100.00
NPK 30 kg2	3258	2863	3061	115.36
NPK 50 kg3	3301	3005	3153	118.84
Nutrifert 30 kg4	3387	3320	3353	126.39
Nutrifert 50 kg5	3440	3214	3327	125.39
Magnifert 30 kg6	3356	3096	3226	121.58
Magnifert 50 kg7	3577	3114	3345	126.09

		2005 year	2006 year
LSD	0.05 =	62.19 kg/ha	101.97 kg/ha
	0.01 =	85.18 kg/ha	139.69 kg/ha
	0.001 =	116.08 kg/ha	190.36 kg/ha

Tobacco quality is expressed by the average purchase price per 1 kg of tobacco. The data presented in Table 4 show that statistical significance was observed only in variant 2, fertilized with 30 kg N/ha, with 4.43% higher average price compared to the check. In all other variants there was a decrease of tobacco quality. This means that the increase of nitrogen rates to a certain level has a stimulative effect on yield and quality, but if that level is exceeded, it has a negative impact, particularly on quality.

Table 4. Average tobacco price (den/kg)

Variants	years			
	2005	2006	average	%
Check	103.77	102.69	103.23	100.00
NPK 30 kg2	111.55	104.07	107.81	104.43
NPK 50 kg3	99.97	99.84	99.91	96.78
Nutrifert 30 kg4	101.51	102.23	101.87	98.68
Nutrifert 50 kg5	101.35	100.01	100.68	97.53
Magnifert 30 kg6	97.21	96.99	97.10	94.06
Magnifert 50 kg7	94.85	95.98	95.42	92.43

		2005 year	2006 year
LSD	0.05 =	5.70 den/kg	2.31 den/kg
	0.01 =	7.99 den/kg	3.23 den/kg
	0.001 =	11.29 den/kg	4.57 den/kg

The economic effect, expressed through monetary income (Table 5), is a synthesis between the achieved yield and the average tobacco price per unit area. It can be stated from the results that fertilization is an important agrotechnical measure which has a strong impact on the increase of tobacco yield per unit area.

High significance during the research period was observed in the increase of gross income in all variants investigated. The best economic effect was achieved in variant 4, with 24.25% higher income per hectare and in all other variants the increase ranged from 10.16% (variant 6) to 23.89% (variant 5), compared to the check.

Despite the fact that fertilization increases the costs of production, investigations have shown that this agricultural practice is still profitable investment that increases money gains to 40%.

Table 5. Gross income of tobacco for 2005 and 2006 (den/ha)

Variants	years			
	2005	2006	average	%
Check	290.065	257.497	273.781	100.00
NPK 30 kg2	363.197	298.644	330.920	120.87
NPK 50 kg3	330.695	298.508	314.602	114.91
Nutrifert 30 kg4	340.867	339.485	340.176	124.25
Nutrifert 50 kg5	348.320	330.089	339.204	123.89
Magnifert 30 kg6	324.686	278.512	301.599	110.16
Magnifert 50 kg7	336.971	267.600	302.285	110.41

	2005 year	2006 year
LSD 0.05	= 22382.32 den/ha	12717.91 den/ha
0.01	= 31380.45 den/ha	17830.76 den/ha
0.001	= 44353.77 den/ha	25202.36 den/ha

Great influence on tobacco quality has its chemical composition, in particular total alkaloids, protein, soluble sugars, mineral matter and their mutual ratio (Bajlov, Popov, 1964). The content of chemical components depends on variety, climatic conditions and, above all, on applied agricultural practices (Devcic, 1975, Beljo et al. 1994, Dimitrov 1964, Tatarcev 1955).

Table 6 presents the average values of the investigated parameters which determine the chemical composition of tobacco. The lowest content of nicotine was determined in the check (0.62%) and the highest in the variant 3 (1.15%). The content of proteins is in optimal range (Shmuk 1948) between 6.98% and 8.77%. Soluble sugars and mineral matters are also within the limits typical for this type of tobacco (Bogdanceski et co. 1997, Grabuloski 1999). The obtained results show that the applied fertilizers do not violate the harmonious chemical composition of tobacco variety JK-48.

Table 6. Chemical composition of tobacco

Variants	2005-2006			
	Nicotine %	Proteins %	Soluble sugars %	Mineral matter %
Check	0,72	6,31	18,34	9,91
NPK 30 kg2	0,94	6,54	20,27	11,70
NPK 50 kg3	1,01	6,62	20,94	10,18
nutrifert 30 kg4	1,06	6,04	22,54	11,28
nutrifert 50 kg5	1.21	6,50	20,91	11,80
magnifert 30 kg6	1,16	7,06	21,38	11,87
magnifert 50 kg7	1,29	6,50	19,95	12,37

CONCLUSIONS

Two year-investigations were carried out to study the influence of three mineral fertilizers applied in different rates on qualitative and quantitative properties of the oriental tobacco

Variety JK-48.

The results have shown that all fertilized variants have a positive impact on yield and gross income.

No statistical significance was observed among the variants fertilized with nitrogen rates up to 30 kg/ha. By further increase of nitrogen rates, the quality of tobacco decreases insignificantly.

The applied mineral fertilizers showed a positive influence on cost-effectiveness of the oriental tobacco variety JK-48.

REFERENCES

1. Atanasov D., 1965. Tjutjunoproizvodstvo. Plovdiv
2. Bailov D., Popov M., 1965. Proizvodsvo i prvicna obrabotka na tutunot. Zemizdat. Sofija.
3. Beljo J., Bajtek M., Vuletic N., 1994. Utjecaj kolicine i rasporeda oborina na prinos i sadrzaj nikotina kod duvana tipa berlej. Izvesce o znanstvenom i strucnom radu u 1994, Vol.21, Zagreb
4. Bogranceski M., Dimitrieski M., Miceska G., 1997. Proizvodni i kvalitetni svojstva na nekoi sorti tutun od tipot Jaкa vo strumickiot region, Tobacco, Vol. 47, No. 1-6, 3-13, JNU Institut za tutun - Peilep
5. Grabuloski T., 1999. Vlijanie vrz kvalitetot na makedonskite orientalski tipovi tutun vrz zastapenosta na katran i nikotin vo tutunskiot cad. Tobacco.Vol. 49, N 7-12, 119-126, JNU Institut za tutun - Prilep
6. Georgievski K., 1990. Tutun i tutunski prerabotki- so poseben osvrt vrz ekologijata na tutunot, Makedonija Tabak, Skopje 1990.
7. Dimitrievic R., Tomic Lj., 1963. Poznavanje duvanske surovine. Beograd.
8. Devcic J, Kozumplik., 1975. Ujeti i neke karakteristike proizvodnje – flue cured duhana u Podravini. Tutun br. 1-2 17-22. JNU Institut za tutun - Peilep
9. Donev N., Zlatev G., 1974. Tutunot vo Bugarija. Plovdiv.
10. Dimitrov D., 1964. Dejstvie na mineralnite i organicnite torove vrhu tjutjuna – Sofija.
11. Uzunoski M., 1985. Proizvodstvo na tutun, Skopje
12. Hawks S.N., Collins W.K., 1994. Nacela proizvodnje virginiskog duhana, Zagreb
13. Naumoski K., Boceski D., Grozdanoski M., Karajankov S., Ackoski B., 1977. Sovremeno prozivodstvo na tutunot. Skopje.
14. Filiposki K., 1989. Vlijanie na mineralnata ishrana vrz prinosot i kvalitetot na tutunot od sortata P-7. Тутун/Tobacco, Vol.39, N° 1-20, 17-34, Prilep
15. Filiposki K., i sor. 1997. Iznaogane na ekonomski opravdan najpogoden hranidben i poliven rezim na povrsinite pod tutun od hidrosistemot "Prilepsko pole"-Prilep, JNU Institut za tutun - Prilep
16. Pasoski D., 1980. Vlijanie na Mg, B, Cu, Zn vrz prisnosot, hemiskiot sostav i pojavata na basara kaj tutunot od tipot prilep. Doktorska disertacija. Skopje.
17. Pelivanoska V., 2009. Kvalitativni i kvantitativni karakteristiki na sitnolisnata orientalska sorta tutun P-66. Тутун/Tobacco, Vol.59, N° 9-10, 213-219. Naucen institut za tutun -Prilep
18. Tatarcev M., 1955. Pridones za kvalitativnata vrednost na makedonskite izvozni tutuni i nivnata hemiska karakteristika. Tutun br.3, Prilep
19. Shmuk A. A., 1948. Hemija tabaka i mahorki. Moskva.

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Original scientific paper

THE USE OF *TRICHODERMA ASPERELLUM* IN THE CONTROL OF PATHOGENIC FUNGUS *RHIZOCTONIA SOLANI* KÜHN

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ABSTRACT

Fungi of the genus Trichoderma are widely used in production of biological products which show high efficiency in the control of many soilborne pathogens. Laboratory tests with *Trichoderma asperellum* grown together with the pathogenic fungus *Rhizoctonia solani* as dual culture showed high inhibitory effect of this antagonist on the pathogenic fungi. It represses the growth of the pathogen and develops on its colony. The growth of pathogenic fungus grown in the presence of antagonist was reduced by 36.74%, with percentage of inhibition being 63.26%. Biological agent *T. asperellum* was tested for control of *R. solani* on tobacco seedlings in protected conditions. Tobacco seedlings were inoculated both with pure culture of the pathogen and with dual culture, where the pathogen was grown in the presence of the antagonist. Seedlings treated with dual culture were healthier and had a more rapid growth, while those treated with pure culture showed high percentage of infestation.

Key words: tobacco, *Trichoderma asperellum, Rhizoctonia solani*, antagonism

КОРИСТЕЊЕ НА *TRICHODERMA ASPERELLUM* ЗА КОНТРОЛА НА ПАТОГЕНАТА ГАБА *RHIZOCTONIA SOLANI* KÜHN

Габите од родот Trichoderma најдуваат широка примена за изработка на биолошки препарати, кои покажуваат висока ефикасност во контролата на поголем број почвени патогени. Во лабораториските испитувања кои ги направивме со габа *Trichoderma asperellum* одгледувана заедно со патогената габа *Rhizoctonia solani* во двојна култура, се покажа дека овој антагонист има високо инхибиторно дејство врз патогената габа. Таа го инхибира порастот на патогенот, го потиснува и се развива врз неговата колонија. Порастот на патогената габа одгледувана во присуство на антагонистот е намален за 36,74%, додека процентот на инхибирање изнесуваше 63,26%. Биолошкиот агенс *T. asperellum* за контрола на *R. solani* кај тутнскиот расад, беше испитуван во заштитен простор. Тутунскиот расад беше инокулиран со чиста култура од патогенот и со двојна култура, каде патогенот е одгледуван во присуство на антагонистот. Расадот третиран со двојна култура беше здрав и со побрз развој, додека кај расадот третиран со чиста култура од патогенот имаше висок процент на зараза.

Клучни зборови: Тутун, *Trichoderma asperellum, Rhizoctonia solani,* антагонизам.

INTRODUCTION

Phytopathogenic fungus *Rhizoctonia solani* is the major causing agent of root rot disease on tobacco and other crops. Very often, chemical products do not provide full protection of seedlings from this pathogen. Even the most frequently recommended chemicals so far chlorothalonil, thiophanate-methyl and iprodion (12) have shown low effectiveness in the control of root rot in many vegetables. Presently, researchers and growers of agricultural crops, supported by environmentalists, are interested in application of biological means for control of plant pathogenic microorganisms. According to Baker and Cook (1974) (1), biological control in fact reduces the production of inoculum, i.e. the activity of pathogen carried out by one or more organisms.

In phytopathology, the term biological control refers to the use of microorganisms-antagonists of specific pathogens – agents of plant diseases (9) called biological control agents-BCA. In our country, biological control of plants is still recommended as a segment of the integral control.

In literature, a number of mycoparasites are found for biological control of soil pathogens. Some of them have proved to be good antagonists of *R. solani*. The most popular among them are species of the genus Trichoderma. Species belonging to this genus are distributed widely in the nature and can be easily isolated from soil as a pure culture. The rapid growth of the culture on nutrient medium and production of great number of conidia colored with different shades of green are the basic characteristics of fungi of this genus (14). One of the most important features of these species is their ability to parasitize other fungi.

Trichoderma genus includes imperfect fungi which have anamorphic stage of propagation (with no sexual stage) (5, 14). They are optional parasites that parasitize a large number of fungi, but can live as saprophytes, too. Species *Trichoderma spp*. not only parasite the fungal plant pathogens, but can also produce antibiotics, causing systemic or local resistance against the pathogen and improve the development of the plant. The potential of these species to be used as biological agents in the control of plant diseases has been known since the 1930s (16). This activity is due to different mechanisms (14). One of the mechanisms is the competition for food, and the second one, through formation of small groups limits the growth of the pathogen. In both cases the pathogen *R. solani* does not produce sclerotic lesions, which means that Trichoderma species can control the development of pathogen sclerotions in the soil. Thus, Trichoderma species act as mycoparasites, they produce antibiotics and have enzymatic system capable of attacking a wide range of plant pathogens. According to Shalini (13), the mechanism and mode of action of Trichoderma against *R. solani* consists of bending around the hyphae of the pathogen, penetration into their interior and decomposition of cell.

Biopesticides use the beneficial microorganisms or the products of their metabolism in plant protection (6). They act through several mechanisms: direct competition, antibiosis, predation or parasitism and induced resistance in host plant (6). The most common biological agents in the control of soil pathogens are *T. harzianum*, *T. viride* and *T. viriens* (11) and a number of biofungicides based on these agents (*T. harzianum, T. viride*, *T. asperellum*, and *T. polysporum*) can be found in the trade. (1). *T. asperellum* was used as an antagonist to obtain the biofungicide Trifender WP, which showed high efficiency in protection of tobacco seedlings from the pathogen (Taskoski, 15). This biochemical also achieved high efficiency in potato protection from *R. solani* (18).

The use of antagonistic fungus *T. asperellum* and its application in biological control of the pathogen *R. solani* in tobacco seedlings was the main objective of this study.

MATERIAL AND METHODS

Studies were performed *in vitro*, in laboratory conditions, with cultures of the antagonistic and pathogenic fungi grown as dual culture, and *in vivo*, on tobacco seedlings. Antagonistic effect of *Trichoderma asperellum* on pathogenic fungus *Rhizoctonia solani* was investigated. Pure culture of *R. solani* was obtained from infected tobacco seedlings grown on potato-dextrose agar. The culture of the antagonistic fungus *T. asperellum* was isolated from the biofungicide Trifender WP obtained from the fungus, and than grown on nutrient medium potato-dextrose agar.

Antagonistic ability of the fungus *T. asperellum* against the pathogen *R. solani* was investigated using the dual culture technique described by Dennis and Webster (1971) (14).

Fragments sized 3 mm with mycelia of the pathogenic fungus and the antagonist were placed at 3 cm distance from each other in Petri dishes with 10 cm diameter, on potato-dextrose agar medium. Four trials with three replications were made for this study. Pure culture of the pathogenic and antagonistic fungi set up in triplicate was used as a check. Prepared Petri dishes were incubated in a thermostat at 25^0 C for a period of 10 days. Radial growth of mycelial colony of the pathogen grown in the presence of the antagonist as a pure culture which served as a check was regularly measured in a period of 7 days. The percentage of growth of mycelial colony from the pathogenic fungus grown as a pure culture was calculated by the formula of Siameto (11):

$$\% = \frac{\text{radius of growth in the presence of the antagonist}}{\text{radius of growth in the check}} \times 100$$

Percentage of inhibition of the pathogen by *T. asperellu,* was calculated by the formula of Mudri (8) and Siameto (11):

$$\%\ \text{of inhibition} = (a - b/a) \times 100$$

where:
a = radial growth of the pathogen in the check
b = radial growth of the pathogen in the presence of the antagonist

Inhibition of colony growth of the pathogen (Zivkovic, 19) can be presented on 0 - 4 scale, where:

0 = no inhibition,
1 = 1-25% inhibition,
2 = 26-50% inhibition,
3 = 51-75% inhibition,
4 = 76-100% inhibition

Biological control of the pathogen *R. solani*, the causing agent of root rot disease on tobacco seedlings was checked in protected conditions. For this purpose two experiments were set up in three replications. Tobacco seedlings of the oriental variety P66 were planted in pots and grown using standard agrotechniques. In the stage when seedlings were 2-4 cm long, inoculation was made with suspension prepared from fungal mycelium. Investigations were performed in three variants:

- Seedlings treated with pure culture of the pathogenic fungus *R. solani*
- Seedlings treated with dual culture of the pathogen and *T. asperellum*
- Check - untreated seedlings

Fungal culture was grown on potato-dextrose agar, in a thermostat at 25^0 C for a period of 10 days. Culture of the pathogenic fungus was grown separately, and dual culture between the pathogenic fungus and the antagonist was placed in other Petri dishes.

Mycelium from one Petri dish was used to inoculate seedlings grown in a 380 cm^2 pot. Mycelial colony was mixed in 200 ml distilled water and the resulting suspension was used for foliar spraying of the seedlings. Inoculation was made on 22.06.2011 in the first trial and on 25.07.2011 in the second trial. Seedlings in the pots which were used as a check were treated with pure water. Health condition of tobacco seedlings was evaluated according to the number of infected plants, i.e. to the percentage of infected area.

RESULTS AND DISCUSSION

The soilborne phytopathogenic fungus *R. solani* can be easily isolated from infected plants. Grown on potato-dextrose agar it has slower growth, creating dirty white mycelial colony which in older cultures gets brighter shade of brown and the appearance of concentric circles (Fig. 1). *T. asperellum* showed similar development as the pathogenic fungus, forming white colony which turned green after a few days, as a result of conidiophores and conidia development (Fig. 2).

The results of laboratory investigations of mycelial colonies in phytopathogenic fungus *R. solani* and antagonistic fungus *T. asperellum* grown both in pure culture and as dual culture were used to calculate the percentage of growth of pathogen colony and the percentage of its inhibition by the antagonist.

Fig. 1. *R. solani* – pure culture

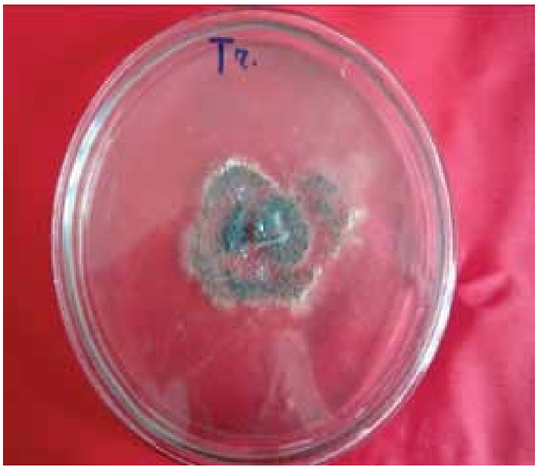

Fig. 2. *T. asperellum* – pure culture

Daily growth of fungi in pure culture and in dual culture is presented in Table 1, through the mean values from the four trials with three replications. .

Table 1 Fungal growth (mm) in the period of incubation by days

Variant	Days of incubation						
	1	2	3	4	5	6	7
Rhizoctonia solani	5.50	15.00	20.00	29.00	38.00	42.50	48.75
R. solani+T. asperellum	4.50	11.20	13.00	14.00	15.00	17.00	17.50
Trichoderma asperellum	3.50	14.00	19.00	26.00	38.00	46.00	52.50

Pathogenic fungus *R. solani* grown in pure culture on the nutrient medium showed somewhat slower growth. 24 hours after planting, radius of the mycelial colony was 5.50 mm and on the seventh day, at the end of observation, it measured 48.75 mm. Antagonistic fungus *T. asperellum* grown in pure culture was similar in size with the pathogenic fungus, measuring a radius of 3.50 mm on the first day and 52.50 mm on the seventh day of observation. When grown in dual culture with the antagonistic fungus, the pathogenic fungus showed high inhibitory effect on its growth. On the first day, radius of the colony was 4.50 mm and on the seventh day it reached 17.50 mm. Pure mycelial growth of the pathogenic fungus appeared as a result of the antagonistic effect of *T. asperellum.*

On the seventh observation day, there were no major differences in colony size of the fungi grown in pure culture. The results obtained in all four trials were almost identical (2). *R. solani* reached its maximum colony growth in the first and fourth trial, where a radius of 55.00 mm was measured. Slower growth was measured in the third and second trial, reaching 40.00 mm and 45.00 mm, respectively. On the seventh day of observation *T. asperellum* reached a radius of 50.00 mm in the first and fourth and 55.00 mm in the second and third trial. Unlike this, the pathogen *R. solani* in dual culture with antagonistic fungus showed a very slow growth, with colony size ranging from 10.00 mm in the fourth to 25.00 mm in the first trial. In the second and third trial the radius measured 15.00 mm and 20.00 mm, respectively.

Table 2. Fungal colony growth on the 7th day of incubation

Variant	Radial growth of the colony in mm by trials				Average, mm
	1	**2**	**3**	**4**	
Rhizoctonia solani	55,00	45.00	40.00	55.00	48.75
R. solani+T. asperellum	25.00	15.00	20.00	10.00	17.50
Trichoderma asperellum	50.00	55.00	55.00	50.00	52.50

In average, *R. solani* grown in pure culture on nutrient medium has 36.74% higher growth of mycelial colony compared to the same pathogen grown in dual culture, in the presence of antagonist (Table 3). Thus, colony growth in the fourth trial was greater for 18.18%, while in the third trial it was 50.00% greater.

Table 3. Percentual growth of the colony of pathogenic fungus *R. solani*

Variant	Radial growth of the colony in the check variant, mm	Radial growth of the colony in the presence of antagonist, mm	Colony growth, %
I Trial	55.00	25.00	45.45
II Trial	45.00	15.00	33.33
III Trial	40.00	20.00	50.00
IV Trial	55.00	10.00	18.18
	Average		**36.74**

Data obtained for radial growth of mycelial colony of the pathogen in the check variant (grown in pure culture) and that grown in dual culture were used to calculate the percentage of inhibition of pathogenic fungus by the antagonist. In our investigation, the percentage of inhibition averaged 63.26% (Table 4). The highest percentage of inhibition of 81.82% was observed in the fourth trial and the lowest of only 50.00% in the third trial. In the first and second trial, the inhibition of the mycelial growth of the pathogen was 54.55% and 66.66%, respectively.

Table 4. Inhibitory effect of *T. asperellum* on *R. solani*

Variant	Radial growth of the colony in the check variant, mm	Radial growth of the colony in the presence of antagonist, mm	Inhibition, %	Index
I Trial	55.00	25.00	54.55	3
II Trial	45.00	15.00	66.66	3
III Trial	40.00	20.00	50.00	2
IV Trial	55.00	10.00	81.82	4
	Average		**63.26**	3

Results of the investigation confirmed the high antagonistic effect of the fungus *T. asperellum* on this pathogen. Its growth on the colony of *R. solani* in dual culture is presented in Fig. 3 and Fig. 4.

Fig. 3. Dual culture of *R. solani* and *T. asperellum*

Fig. 4. Development of *T. asperellum* on the colony of *R. solani*

Results obtained in this research coincide with those of Soares (12), who reported that Trichoderma species inhibited the growth of *R. solani* for over 60%, and T. koningii – species which produces a huge amount of antibiotics, inhibited the growth for 79-82%. Similar results on antagonistic effect of 15 isolates of *T. harzianum* were reported by Siameto (11) and by Shalini (13), who tested 17 species for their effect on soilborne phytopathogenic fungi grown in dual culture. All isolates showed serious antagonistic effect on mycelial growth of pathogenic fungi, and the maximum inhibition of growth of *R. solani* was 61.55%.

Biological control of *R. solani* was tested under protected conditions, on tobacco seedlings grown in pots. Inoculation was made with a pure culture of the pathogenic fungus and with pathogenic and antagonistic fungi grown in dual culture.

The first symptoms of infection in seedlings treated with pure culture of the pathogen in both trials were observed two days after inoculation. Infection spread rapidly and it only took few days for over half of the seedlings to be destroyed. Unlike this, low percentage of natural infection was observed in the check variant (untreated) and a very small percentage of plants were infected in the seedlings inoculated with dual culture. 10-15 days after inoculation, seedlings inoculated with the pure culture of the pathogen were completely destroyed. In seedlings inoculated with dual culture the spreading of infection was not only stopped, but they had better growth and development compared to the check (Fig. 5 and Fig.6).

Fig. 5. Inoculated seedlings (left-*R. solani*, right- *R. solani+T. asperellum*), I trial

Fig. 6. Inoculated seedlings (left-*R. solani*, right-*R. solani+T. asperellum*), II trial

The high effectiveness of Trichoderma species in reduction of diseases caused by the pathogen *R. solani* in gardening crops was confirmed in many studies (16). It was observed that these crops were characterized by faster growth and higher yields compared to the check variants. High effectiveness in protection of the lettuce and tomato seedlings from the pathogens *R. solani* and *P. debaryanum* was achieved with biological product TRI 003 based on *T. harzianum*, compared to the standard fungicides Previcur and Dithane (3, 4, 10). In bean plants, the infection by *R. solani* was reduced to 92% when the seed was treated with *T. lignorum* (2). Effectiveness of 89% was obtained in the protection of cucumber from *R. solani*, using a compost prepared from plant waste enriched with the biological control agent *T. asperellum*, isolate T-34 (17). The biofungicide Trifender WP based on *T. asperellum* showed over 90% effectiveness in tobacco seedlings protection from the pathogen *R. solani* (Taskoski, 15). According to the results obtained *in vitro* and *in vivo*, the fungus *T. asperellum* proved to be a good antagonist and a real mycoparasite which inhibits the growth of the pathogenic fungus *R. solani*. It uses its mechanisms of action - antagonism and mycoparasitism to suppress the pathogenic fungus *R. solani* and to prevent its infection on tobacco seedlings.

CONCLUSION

Phytopathogenic fungus *R. solani* grown as a pure culture on potato-dextrose nutrient agar has 36.74% faster growth compared to the same fungus grown with the antagonistic fungus as a dual culture.

Fungus *T. asperellum* showed high antagonistic effect on mycelial colony growth of the pathogenic fungus. The percentage of inhibition of colony growth ranged from 50.00 % to 81.82 %, or 63.26 % in average.

Tobacco seedlings inoculated with a pure culture of the pathogenic fungus was infected and completely destroyed, while in tobacco seedlings treated with inoculum prepared from culture of the pathogen grown together with the antagonist, the percentage of infected plants was very small. The seedlings grown in the presence of the antagonistic fungus had a more raoid growth and better development compared to the check.. *T. asperellum* fungus appeared as a real antagonist and mycoparasite on the fungus *R. solan* and it can be used for biological control of this pathogen in production of tobacco seedlings and other gardening crops.

REFERENCES

1. Anonymous. 2011. Biopesticidi u svetu. http://www.agroservis.rs/biopesticidi u svetu. Agroservis, informativna agencija, noemvri.
2. Aziz N.H., El-Fouly M.Z., El-Essawy A.A. and Khalaf M. A. 1997. Influence of bean seedling root exudates on the rhizosphere colonization by *Trichoderma lignorum* for the control of *Rhizoctonia solani*. Botanical Bulletin of Academia Sinica. Vol. 38, 33-39.
3. Baličević R., Parađiković N., Ćosić J., Rozman V. 2008. Suzbijanje zemljišnih parazita (*Pythium debaryanum, Rhizoctonia solani*) na salati biološkim pripravkom. 43. Hrvatski i 3. međunarodni simpozij agronoma, Opatija, Hrvatska, Zbornik radova 465-468.
4. Baličević R., Parađiković N., Ćosić J., Vrandečić K., Šamota D. 2009. Utjecaj supstrata na biološku zaštitu presadnica salate protiv patogena *Rhizoctonia solani* i *Pythium debaryanum*. 44. Hrvatski i 4. međunarodni simpozij agronoma, Opatija, Hrvatska, Zbornik radova 407-410.
5. Brozova J. (2004): Mycoparasitic fungi *Trichoderma* spp. in plant protection. Plant Protect. Sci., 40: 63–74.
6. Grahovac M., Inđić D., Lazić S., Vuković S. 2009. Biofungicidi i mogućnosti

primene u savremenoj poljoprivredi. Pestic. Fitomed. (Beograd), 24 (4), 245-258.

7. Mudri S., Sušinjak I. 2000. Prototip pripravka za stimulaciju biljnog rasta na osnovu gljive *Trichoderma harzianum*. Studij Bilinogojstvo, usmjerenje zaštita bilja, agronomski fakultet Sveučilišta u Zagrebu, Svetošimunska 25.
8. Pal K.K., Gardener B. Mc S. 2006. Biological control of plant pathogens. The Plant Health Instructor. DOI:10.1094/PHI-A-1117-02.
9. Pariđiković N., Baličević R., Vinković T., Pariđiković D., Karlić J. 2007. Biološke mjere zaštite u proizvodnji gerbera i presadnica rajčice. Agronomski glasnik 5, 355-364.
10. Siameto E. N., Okoth S., Amugune N. O., Chege N. C. 2010. Antagonism of *Trichoderma harzianum* isolates on soilborne plant pathogenic fungi from Embu District, Kenya. Journal of Yeast and Fungal Research Vol. 1 (3), pp. 47-54.
11. Soares I. M., Faull J. L. 2000. Parasitism of *Rhizoctonia solani* by strains of Trichoderma spp. Sci. agric. Vol. 57, N^{o} 1, Piracicaba jan./mar.
12. Shalini S., Narayan K.P., Lata and Kotasthane A. S. 2006. Genetic relatedness among Trichoderma isolates inhibiting a pathogenic fungi *Rhizoctonia solani*. African Journal of Biotechnology Vol. 5 (8) pp. 580-584.
13. Shalini S. and Kotasthane A. S. 2007. Parasitism of Rhizoctonia solani by strains of Trichoderma spp. Electronic Journal of environmentsl, Agricultural and food chemistry, 6 (8), 2272-2281.
14. Ташкоски П., Чифлигароски З. 2011. Можности за примена на биофунгицидот Trifender WP во заштитата на тутунскиот расад. Заштита на растенија, год. XXII, Vol. XXII, 29-35.
15. Tran N. Ha. 2010. Using Trichoderma species for biological control of plant pathogens in Vietnam. J. ISSAAS Vol. 16, N^{o}. 1:17-21.
16. Trillas M. I., Casanova E., Cotxarrera L., Ordovas J., Borrero C., Aviles M. 2006. Composts from agricultural waste and the *Trichoderma asperellum* strain T-34 suppress *Rhizoctonia solani* in cucumber seedlings. Biological Control 39, 32–38.
17. Zoltan O. 2008. A Trifender alkakmazasa a burgonyaban. Virages Tarsa Bt. 3529 Miskole Szilvas ut 2 II/4.
18. Živković S., Stojanović S., Ivanović Ž., Gavrilović V., Popović T., Balaž J. 2010. *Colletotrichum acutatum* and *Colletotrichum gloeosporioides*. Arch. Biol. Sci., Belgrade, 62 (3), 611-623.

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Original scientific paper

INVESTIGATION ON THE EFFECTIVENESS OF SOME FUNGICIDES IN THE CONTROL OF *RHIZOCTONIA SOLANI* IN CONDITIONS OF ARTIFICIAL INOCULATION

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ABSTRACT

Investigations of biological effect of fungicides in the control of pathogenic fungus *Rhizoctonia solani* - causing agent of the damping off disease on tobacco seedlings were carried out in biological laboratory at Scientific Tobacco Institute-Prilep in 2009 and 2010. The aim was to study the effectiveness of fungicides that have already been used in the production of tobacco seedlings, and some other fungicides, i.e. active ingredients. The results will contribute to possible expanding of the list of products aimed for protection of tobacco seedlings from this disease.

Artificial inoculation was made with suspension of pure culture of the pathogen and products with appropriate concentrations were applied through watering. The poorest results were obtained with the fungicide Previcur (0,15%). Products with active ingredient chlorothalonil showed an average effectiveness of 78.27%. The best results were achieved with Top M (0,1%) and Quadris (0,2%) with 87.85% and 94.36% average effectiveness in both years respectively.

Key words: *Rhizoctonia solani*, fungicide, active ingredient, effectiveness

ИСПИТУВАЊЕ НА ЕФЕКТИТЕ ОД НЕКОИ ФУНГИЦИДИ ЗА КОНТРОЛА НА *RHIZOCTONIA SOLANI* ВО УСЛОВИ НА ВЕШТАЧКА ИНОКОЛУАЦИЈА

Испитувања на биолошкото дејство на фунгицидите за сузбивање на патогената габа *Rhizoctonia solani* – предизвикувач на болеста сечење на тутунскиот расад беа извршени во Биолошката лабораторија при Научниот институт за тутун-Прилеп во 2009 и 2010 год. Тие имаа цел да се испита ефикасноста на фунгицидите кои веќе се употребуваат во производството на тутунски расад, но и некои други фунгициди, односно активни материи. Резултатите би придонеле за можно проширување на листата на препарати за заштита на тутунскиот расад од сечењето.

Истражувањата беа извршени со вештачка инокулација со суспензија од чиста култура од патогенот, а препаратите со соодветната концентрација беа аплицирани со полевање. Најслаби резултати имаше фунгицидот Previcur (0,15%). Препаратите со активна материја chlorothalonil покажаа просечна ефикасност 78,27%. Најдобри резултати покажаа препаратите Top M (0,1%) со 87,85% и Quadris (0,2%) со 94,36% просечна ефикасност во двете години.

Клучни зборови: *Rhizoctonia solani,* фунгицид, активна материја, ефикасност

INTRODUCTION

Tobacco seedlings production suffers from serious damage caused by damping off disease each year. It occurs in all stages of seedling development and can be manifested as pre-emergence damping off, post-emergence damping or as rot pruning (Numez, 2005). Bearing in mind the importance of seedlings for successful tobacco production, the seriousness of the damage becomes more obvious.

The most common cause of this disease in our conditions is the pathogenic fungus *Rhizoctonia solani*. It is a widespread pathogen known for its harmful effects on many important agricultural and horticultural crops (Grosh, 2003) including beans, peas, soybeans, black tomato, cucumber, corn, alfalfa, and many others (Uchida, 2012). Damages that *R. solani* causes to yield are proportional to agricultural land and can reach up to 50% (Wallwork, 2000, loc cit. Hollaway, 2008).

The application of preventive measures is of great importance in plant protection. These measures include crop rotation, optimum seed quantities, proper use of fertilizers in accordance with the growing stage, moderate irrigation, aeration, etc. (Uchida, 2012). Protection programs should be based on prevention of pathogen infection and maintenance of vigorous plants (Pataky, 1988).

Despite this, the fight against this pathogen is difficult. According to Nunez (2005), *R. solani* is a soilborne pathogen which lives in many types of soils. It persists for many years as sclerotia or as mycelium on oganic substances, in various environmental conditions (Pataky, 1988; Grosh, 2003). Sclerorotia have an ability to float and to exist in water and thereby they present a primary inoculum (Ceresini, 1999).

Hollaway (2008) reported that the control of this pathogen is difficult because the fungus has a wide circle of hosts, i.e. limited possibilities for crop rotation, and there are not resistant varieties because the fungus can live and develop even in the absence of living plant organism – it has a "saprophytic ability." Because of all this, it can not be eliminated, but it can be controlled to a level that will not cause economic losses.

Due to previous statements, the application of chemical products in protection of tobacco from this disease is necessary.

A number of active ingredients have been reported for the control of this pathogen. Frisina (1988) made investigations with benomyl, iprodione, chlorothalonil and benodanil. Mueller (1996) recommended the use of pentachloronitrobenzene (PCNB). From there investigations of ED50 of many active ingredients Csinos and Stephenson (1998) recommended flutolanil, iprodione, fluazinam and tebuconazole to prevent the spreading of infection in tobacco seedbeds.

According to Mocioni et al. (2003), azoxystrobin and trifoxystrobin, as well as tebuconazole applied in three different formulations showed significant effectiveness in the control of *Rhizoctonia*.

Fungicides that contain PCNB (Terrachlor), Iprodione (Rovral) or Azoxystrobin (Quadris) are effective against *Rhizoctonia solani* (Koenning, 2007). Wong (2008) gives an extensive list of active ingredients and preparations for control of *R. solani* in lawns. Fludioxinil, maneb, penthiopyrad, thiophanate-methyl, PCNB and azoxystrobin, under different trade names, are recommended by Schwartz and Gent (2012) in potato.

The number of products for protection of tobacco seedlings is limited and in certain cases unsuitable fungicides are applied which are not effective in the control of specific causing agent. Therefore, the purpose of this research was to study the effectiveness of some fungicides in the control of causing agent of damping off disease - the pathogenic fungus *R.solani* in conditions of artificial inoculation.

MATERIAL AND METHODS

Investigations were carried out in Biological laboratory of Tobacco Institute-Prilep, in 2009 and 2010. For this purpose, seeds of P 23 tobacco variety were sown in 8 pots for each variant in both years. The seedlings were grown in traditional way.

Pathogenic fungus *R. solani* was isolated from infected tobacco seedlings and grown on

potato dextrose agar (PDA). 15-day pure culture of the fungus was used for inoculation.
Artificial inoculation of the seedling was carried out prior to the stage of intensive growth and fungicide treatment was applied the following day. The check variant was only inoculated.

Fungicides were applied by watering with 3 l/m2 of the suspension with appropriate concentration. The disease development was followed every day and several calculations were made on the intensity of disease attack through estimation of the percentage of infected area, starting few days to 10-15 days after inoculation. For assessment of the effectiveness, the last estimation was taken into consideration. The method of Abbott was used to estimate the effectiveness of the products.

The selection of products was made according to world literature and our personal investigations, including the experiences from the production and protection of tobacco seedlings in Tobacco Institute-Prilep. The list of investigated fungicides is presented in Table 1.

Table 1. Investigated fungicides

Fungicide	Active ingredient	a.i. content	Concentration %
Top - M 70% WP	Thiophanate- methyl	70%	0.1
Pilarić 75% WP	Chlorthalonil	75%	0.2
Bravo 500 SC	Chlorthalonil	515 g/l	0.2
Quadris 25 SC	Azoksistrobin	250 g/l	0.1
Previcur- N	Propamocarb	70 g/dm^3	0.15

RESULTS AND DISCUSSION

Damping off disease emerges on the stem at the ground line, where the tissue necrotizes and dies, making the further development of plant impossible. Infected plants fall down on soil surface as if "cut off". Sometimes, in conditions of higher seedbed moisture, a whitish mold can be observed on the seedlings. The disease also spreads on adjacent plants, resulting in infected infected patches which coalesce and cover large areas of the bed. For this reason, the major part of the seedbed is devastated and the seedlings production is reduced (Fig. 1).

Fig. 1. Symptoms of damping off disease on tobacco seedbeds

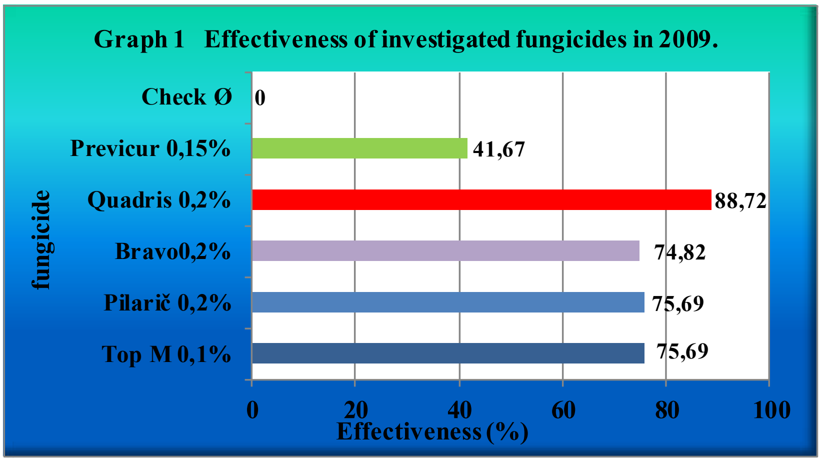
In artificial inoculation with the pathogen in 2009, the percentage of infected area in the check variant ranged from 37.75 to 72.00% (Table 2). The lowest intensity of attack was recorded with application of Quadris 0.2% and Top M 0.1% and the highest intensity with Previcur 0.15%, The latter showed the highest percentage of infected area compared to other products investigated.

With further analysis, the intensity of the disease increases. The highest percentage of infected area in the next two analyses was observed with application of Previcur 0.15% and the lowest with Quadris 0.2%. With Top M 0.1% the percentage of infected area significantly increases, compared to the first analysis. Similar is the case with Pilarić 0.2%, while with Bravo 0.2% it has been held in almost the same level.

According to the intensity of attack, the lowest effectiveness (41.67%) was obtained with Previcur 0.15% and the highest with Quadris 0.2% (88.72%). Other products showed similar effectiveness, ranging about 74 - 75% (Table 2, Graph 1).

Table 2. Effectiveness of fungicides investigated in 2009

Fungicide	infected area % Date of estimation			Effectiveness %
	16.06	20.06	25.06	-
Top M 0.1%	3.29	12.50	17.50	75.69
Pilarič 0.2%	6.32	6.25	17.50	75.69
Bravo 0.2%	16.25	17.50	18.13	74.82
Quadris 0.2%	3.13	5.63	8,13	88.72
Previcur 0.15%	22.50	25.63	42.00	41.67
Контрола Ø	37.75	50.00	72.00	-



In 2010 lower intensity of the disease attack was manifested both in the check (21.88 - 55.63%) and in the variants treated with fungicides (3.13% with Bravo 0.2% and 4.38% with Pilarić 0.2%). In application of Top M 0.1% and Quadris 0.2% no occurrence of the disease was observed. The highest intensity of the attack appears with Previcur 0.15% (Table 3).

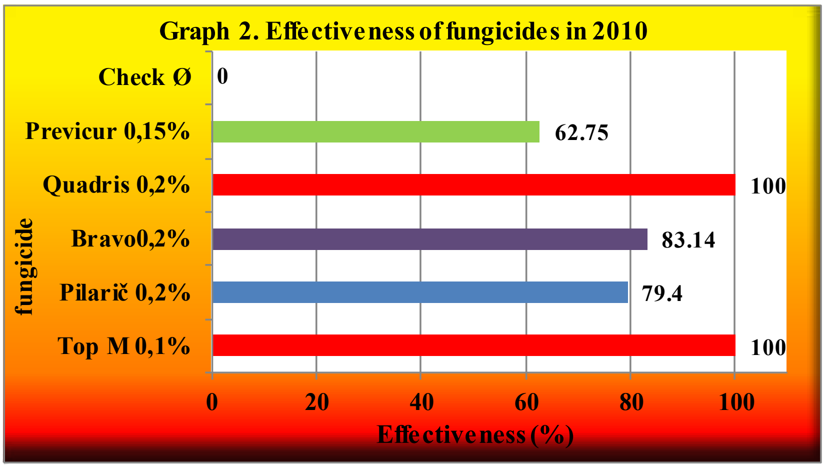
In the following analysis, again, there is no presence of the disease in plants treated with the fungicides M Top 0.1% and Quadris 0.2%. With application of Pilarić 0.2% and Bravo 0.2%, a more significant increase of the infected area was observed in the last analysis. It certainly had an impact on their effectiveness, which ranged 79.40% and 83.14%, respectively.

Top M 0.1% and Quadris 0.2% showed 100% effectiveness in the control of the pathogen *R. solani* (Table 3, Graph 2).

Table3. Effectiveness of fungicides investigated in 2010

Fungicide	infected area % Date of estimation			Effectiveness %
	21.05	25.05	31.05	
Top M 0.1%	-	+-	-	100.00
Pilarič 0.2%	4.38	6.25	11.46	79.40
Bravo 0.2%	3.13	3.22	9.38	83.14
Quadris 0.2%	-	-	-	100.00
Previcur 0.15%	10.63	13.06	20.63	62.75
Контрола Ø	21.88	26.94	55.63	-

- no occurrence of disease
+- insignificant occurrence



The lowest effectiveness, just as in 2009, was determined with Previcur 0.15% (Graph 2). These results are justified because these product is characterized by a specific activity toward several fungi of the class Oomycetes (Extoxnet 1997, Bayer Crop Science, 2009). In practice it is used in control of the pathogen *Pythium debarianum*, which place in the classification of phytopathogenic fungi is completely different than that of *R. solani*. For a long time it has been considered as the only causing agent of the disease, due to the resemblance of symptoms in tobacco seedlings. For this reason it was included in the list of selected products. Results of these investigations imposed the need for correct determination of the causing agent prior to the use of chemicals in disease control (Pataky, 1988).

The problem with the presence of two pathogens (in this case *Pythium ultimum* and *R. solani*) is also emphasized in investigations of other authors. Most of the fungicides give efficient control of one pathogen only and therefore great attention should be paid to the choice of most appropriate product (Mueller, 1996).

Figure 2. The effectiveness of Previcur (0.15%)

Figures 3, 4. Effectiveness of the products with active ingredient chlorothalonil

The products Pilarić and Bravo (a.i. chlorothalonil) showed 74.82% to 83.14% effectiveness in both years (Figures 3 and 4). In 2009 their effectiveness was the same with that of M Top, and in 2010 it was lower. Compared to Quadris, their effectiveness is lower in both years. Also in the investigations of Frisina (1988) on pure cultures of *R. solani* (with mean value of ED50 for the investigated isolates) the degree of inhibition of chlorothalonil was 40-65%. This active ingredient is recommended for control of *R. solani* by spraying the seedlings of horticultural plants prior to or after transplanting (UC Pest Management Guidelines, 2009).

The effectiveness of Top M in 2010 was 100% (Figure 5). In 2009 it was lower, just as that of other products, taking into consideration the higher intensity of attack in the check variety. Compared to Quadris, its effectiveness in the same year was lower, which is in accordance with investigations on sugar beet carried out by William et al. (2004), according to which thiophanate-methyl was less effective than strobilurins.

Nevertheless, the mean value of the effectiveness in both years was 87.85%. Therefore, this product provides good protection from damping off disease and is applied in seedlings production in Tobacco Institute-Prilep. Products with active ingredient thiophanate-methyl are recommended for control of *R. solani* in a number of programs for protection of various crops (Wong 2008, UC Pest Management Guidelines 2009, Schwartz and Gent 2012).

Figure 5. The effectiveness of Top M (0.1%)

In both years, the product Quadris gave excellent results in the control of *R. solani* (Figure 6). According to *in vitro* investigations of Gveroska (2008) on a number of fungicides against the pathogen *R. solani*, Quadris had shown no reducing effect, which is contrary to the results obtained with application of another isolate. Blazier and Conway (2004) reported that isolates of a different, and even of the same anastomosis group are characterized by different susceptibility to azoxystrobin.

In vitro activity of azoxystrobin is not so effective as in field conditions and in biological laboratory. Mycelial growth on PDA can be only 30% at 1000 mg / ml a.i. *R. solani* belongs to the fungi that are using the alternative oxidase enzyme (AOX). It functions well in *in vitro* conditions, but not in the tissues treated with strobilurins. Additionally, the antioxidants of the host-plant as flavones may interfere with the alternative way *in vivo*, which affects the reduction of the intensity of attack. It means that this fungus is more susceptible to strobilurins in natural conditions. In that way, the effectiveness of this active ingredient *in vivo* is higher. Azoxystrobin (Quadris) is a registered product for protection of tobacco from blue mold disease, but it can be also used in treatments of ready-to-transplant tobacco seedlings to protect them from root rot caused by *R. solani* (La Mondia, 2012).

The application of strobilurins during inoculation or half dose in inoculation and the other half two weeks later, provides the most effective protection. In our investigation, too, fungicides were applied the next day after inoculation and the product with a.i. azoxystrobin (Quadris) gave excellent results. Even with a non-optimal period of application, azoxystrobin showed higher effectiveness compared to other strobilurins (William et al., 2004).

Figure 6. The effectiveness of Quadris (0.1%)

CONCLUSIONS

- The lowest effectiveness of fungicides in both years of investigation (2009, 2010) was recorded with Previcur (0.15%).
- Effectiveness of the product Pilarić (0.2%) was 75.69%, i.e. 79.40% and that of Bravo (0.2%) - 74.82% and 83.14% in both years.
- The average value of effectiveness of the products with active ingredient chlorothalonil was 78.27%.
- The standard product Top M (0.1%) in 2009 reached 75.69% effectiveness and in 2010 - 100%. The mean value of effectiveness in both years equaled 87.85%.
- The effectiveness of Quadris (0.2%) was 88.72% in 2009 and 100% in 2010. The average effectiveness was 94.36%.
- The highest effectiveness was obtained with the products Quadris (0.2%) and Top M (0.1%).
- Recommended products the control of damping off disease on tobacco seedlings caused by the pathogen *R. solani* are those which showed highest effectiveness in conditions of artificial inoculation.
- Investigations should be continued in natural conditions of seedlings production.

REFERENCES

1. Bayer Crop Science Crop Compendium, 2009. Propamocarb HCl. The Knowledge for Farmers, Scientists and Agro Consultants, http://compendium.bayercropscience.com
2. Blazier S.R., Conway K.E., 2004. Characterization of *Rhizoctonia solani* Isolates Associated with Patch Diseases on Turfgrass. Proc. Okla. Acad. Sci. 84, pp. 41-51.
3. Ceresini P., 1999. *Rhizoctonia solani*- Pathogen profile. NC State University, http://www.cals.ncsu.edu
4. Csinos A.S., Stephenson M.G., 1998. Evaluation of fungicides and tobacco cultivar resistance against Rhizoctonia solani incited target spot. Crop Protection. Vol. 18, Issue 6, pp. 373-377.
5. Extoxnet, 1997. Propamocarb. Extension Toxicology Network, http://pmep.cce.cornell.edu/profiles/extoxnet/metiram-proxur/pr...
6. Frisina T. A., 1988. Sensitivity of Binucleate Rhizoctonia spp. And R. solani to selected Fungicides In Vitro and on Azalea Under Greenhouse Conditions. Plant Disease, /Vol. 72, No 4, pp. 303-306.
7. Grosh R. 2003. Report: Biological control of Rhizoctonia solani in organic farming with bacterial and fungal antagonists. Geschäftsstelle Bundesprogramm Ökologischer Landbau, Bundenstalt für Landwirtschaft und Ernährung (BLE).
8. Гвероска Б., 2008. Испитување на ефикасноста нна некои фунгициди за уништување на причинителот на сечењето (*Rhizoctonia solani)* кај тутунскиот расад. Заштита на растенија, Год. XIX, Vol. XIX стр. 113-119.
9. Hollaway G., McKay A., Gupta V. 2008. Rhizoctonia fact sheet. Grains Research &Development Corporation.
10. Koenning S.2007. Disease Management in Cotton http://209.85.129.104/search?q=cache:wO1M77KYXcgJ:ipm.ncsu.edu/Production_Guide
11. LaMondia J.A., 2012. Efficacy of Azoxystrobin Fungicide against Sore Shin of Shade Tobacco Caused by Rhizoctonia solani. Tobacco Science, 49:1-3.
12. Mocioni M., Titone P., Garibaldi A., Gullino M.L., 2003. Efficasy of diferent fungicides against Rhizoctonia brown patch and Pythium blight on turfgrass in Italy. Commun Agric. Appl. Biol. Sci. 68 (4 Pt B): 511-7.
13. Mueller J., 1996. Cotton seedling Disease Control.Edisto Research & Education Center. http://72.14.221.104/se.../seedling.pdf+Rhizoctonia+solani%2Bchemical&hl=en&ct=clnk&cd=3&ie=UTF-
14. Nunez J. 2005. Many species have wide host plants range: Pythium, Rhizoctonia usual veggie fungi. Western farm press-timely reliable information for western agricultute, 12.
15. Pataky N.R., Damping-off and root rots of house plants and garden flowers. RPD No 615, Department of Crop Sciences, University of Illinois.
16. Schwartz H. F., Gent D. H., 2012. Potato Black Scurf View Black Scurf. High Plains Integrated Pest management.
17. UC Pest Management Guidelines, 2009. Floriculture and Ornamental Nurseries Chrysanthemum (Chrysanthemum grandiflora). University of California Agriculture & Natural Resources.
18. Uchida J.Y., 2007. Rhizoctonia solani. http://www.extento.hawaii.edu/kbase/crop/type/r_solani.htm
19. William L.S., Franc G.D., Harveson R.M., Wilson R.G., 2004. Strobilurin Fungicide Timing for Rhizoctonia Root and Crown Rot Suppression in Sugarbeet. Journal of Sugar Beet Research, Vol41, No 1&2, pp.17-37.
20. Wong F., 2008. Control of Rhizoctonia Diseases on Turfgrass.Proceeding of the UCR Turfgrass and Landscape Research Field Day, September 2008.

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Original scientific paper

MULTIANNUAL INVESTIGATIONS ON MORPHOLOGY AND BIOLOGY OF *EPITRIX HIRTIPENNIS* MELSH ON TOBACCO

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ABSTRACT

Tobacco flea beetle (*Epitrix hirtipennis* Melsh) is one of the most important pests of the Solanaceae family. It can be found in tobacco throughout the whole period of vegetation, causing severe damages both on seedlings and on transplanted tobacco. In the Republic of Macedonia, investigations on the morphology, biology and eradication of this pest were carried out by Krsteska, Stojanoski (5, 6 and 7).

Presently, *E. hirtipennis* is one of the major pests on tobacco in Macedonia, spreading in almost all tobacco producing regions in the country.

Having in mind the high consuming ability and expressed poliphagia of *E. hirtipennis*, the longevity of adults, the great number of generations and resistance to the atmosphere effects, it is necessary to perform protection against tobacco flea beetle. The struggle should be precise in order to achieve visible results.

Key words: tobacco, *Epitrix hirtipennis*, damage, protection

ПОВЕЌЕГОДИШНИ ИСПИТУВАЊА НА МОРФОЛОГИЈАТА И БИОЛОГИЈАТА НА *EPITRIX HIRTIPENNIS* MELSH КАЈ ТУТУНОТ

Тутунската лисна болва *Epithrix hirtipennis* Melsh е еден од економски најважните штетници на фамилијата Solanaceae. Болвата е присутна на тутунот низ целиот вегетационен период и предизвикува големи штети како на расадот така и на расадениот тутун. Крстеска, Стојаноски вршеле проучувања за морфологијата, биологијата и сузбивањето на овој штетник во Република Македонија (5, 6 и 7).

Денес, *E. hirtipennis* е еден од економски најважните штетници на тутунот во Македонија и е констатиран во скоро сите тутуно-производни реони во земјава.

Сузбивање на тутуновата лисна болва се покажа како неопходност, имајќи го во предвид големата консумативна способност и изразитата полифагност на *E. hirtipennis*, долгиот активен живот на имагата, големиот број на генерации, отпорноста кон атмосверските влијанија. Борбата треба да биде прецизна за да се добијат видливи значајни резултати.

Клучни зборови: тутун, *Epitrix hirtipennis*, оштетувања, заштита

INTRODUCTION

Intensification and concentration of tobacco production in many regions of the Republic of Macedonia, as well as its growing in monoculture, make favorable conditions for frequent mass attacks of harmful insect species on tobacco culture.

E. hirtipennis is one of the most important pests, which has been in large expansion on tobacco producing regions of the south Balkans over the past few years.

Originating from North America, this species has a wide area of distribution and is highly adaptable. Today, tobacco flea beetle is present almost everywhere in the world.

The beetle causes severe damages on cultures of the Solanaceae family, especially on tobacco. In years of massive attack it produces serious economic losses.

According to the *Handbook of quarantine plant diseases* of ex Yugoslavia, it belongs to the group of non-European species of quarantine pests of the genus Epitrix. Beside *E. hirtipennis,* tobacco can be attacked by other species of this genus, like *E. cucumeris* Harris (Eastern parts of North America, mainly a pest on potato) *E. parvula* F. (USA- potato and tomato), *E. fascata* Dur. (Puerto Rico – potato, eggplant and tomato) *E. nicotiana* Bryant, *E. argentinensis* Bryant (South America), etc. (4)

A strong infestation of this pest in Italy was first observed in 1983, in the region of Benvento (Sannino et al., 1984) (11, 12).

A detailed study of the pest was made by Sannino et al, in 1985, and afterwards by Sannino L., Piro F., Balbiani A., Biondi M., Piro F., Milano D., Fiorentino F. A variety of commercial foliar insecticides (alphamethrin, esfenvalerat, lambda-cyhalothrin, endosulfan, fluvalinate, carbaryl, acephat etc), granulated soil insecticides (carbosulfan, phorate, chlorpyriphos, phorate+ terbufos, etc), and microbiological products (bio-products) based on Beauveria bassiana, Bacillus thuringiensis were investigated for control of the pest (1, 10, 13, 14,17).

According to references, investigations on tobacco beetle control in USA are dating from 1970, with organophosphorous and carbamate insecticides (Mistric W.J., Smith F.D.; Semtner P.J., Reed T.D.), to the present days with application of systemic product Admire 2F (a.i. imidacloprid) (9, 15). Its commercial name in our country is Confidor SL 200 \Kohinor R 200.

In Albania, *E. hirtipennis* was first reported by Gixhari in 1986 (2) and today it is present in almost all tobacco producing regions. Investigations with foliar application of seven commercial insecticides (alphamethrin, esfenvalerat, lambda-cyhalothrin, deltamethrin, endosulfan, carbaryl, acephat) were carried out for control of this pest on oriental tobacco.

Likouresis D.P., Mentzos G. (1991) the first occurrence of the pest was reported in west Greece, where it was supposed to be transported from Italy. Today it is widely distributed on tobacco plantings in central Greece, too (8).

In Bulgaria, A. Dimitrov (1997) made investigations for successful control of tobacco beetle with Sumi-alpha (a.i. esfenvalerate) (3).

The first report on *E. hirtipennis* in the Republic of Macedonia was made in 1996 (by researches of Scientific tobacco institute Prilep) in the region of Strumica, the village of Kosturino. During 1997/1998 the pest was reported in the regions of Strumica, Radovis, Veles and Prilep, and presently it is distributed in almost all tobacco producing regions in the country.

MATERIAL AND METHOD OF WORK

The occurrence of *E. hirtipennis* on tobacco was monitored from March to October during 2001-2011. Investigations were made during the whole growing period, from seedlings through field tobacco to suckers in the post-harvest period.

Investigations were made on morphology and biology of the pest and its relationship with soil and climate conditions of the region.

Field trials were carried out for the control of *E. hirtipennis* on oriental tobacco with insecticides. The inspection of tobacco plants before treatments revealed a heavy infestation with the pest.

Chemicals were applied foliary, with knapsack sprayer, and the treatment included 400 tobacco plants in flowering stage.

The first field trial was performed in 2001-2002. Two synthetic pyrethroids were applied foliary on the Experimental field of Scientific Tobacco Institute-Prilep: Karate 2.5 EC (a.i. lambda-cyhalothrin) -0.02%, Sumi-alpha 5 EC (a.i. esfenvalerate)- 0.04%.

Effectiveness of the applied insecticides was estimated 1, 4, 7, 13 and 20 days after.

During 2007-2008, second field trials were carried out on the Experimental field of Scientific Tobacco Institute-Prilep, with three systemic insecticides: Confidor SL 200 (a.i. imidacloprid) -0,03%, Bubastar 20 SP (a.i. acetamiprid) 0,02% and Actara 25 WG (a.i. thiamethoxam)-0,02%.

Effectiveness of the applied insecticides was estimated 1, 4, 9, 17 and 24 days after.

RESULTS AND DISCUSSION

Tobacco beetle *Epitrix hirtipennis* Melsh. belongs to the order Coleoptera, family Chrysomelidae, subfamily Halticinae.

The pest is known under different common names among the authors, mostly as tobacco flea beetle or tomato beetle.

It is oligophagous pest, attacking the plants of the Solanaceae family: tobacco, pepper, potato, tomato, egg-plant, etc., and could be find on the weeds of spontaneous flora: *Datura stramonium, Phytolocca decandra, Vigna sinensis, Artropa* sp., *Hyoscyamus* sp., *Lycium* sp, *Petunia* sp., etc (according literature data).

During our investigations, the beetle was also determined on a number of weed species around tobacco plantings, like: *Amaranthus retroflexus*, *Chenopodium album*, *Sinapis arvensis*, *Datura stramonium,* etc.

The pest is present on tobacco during the whole growing period and attacks seedlings as well as transplanted tobacco.

The imago of *E. hirtipennis* is small coleoptera 1.2 -2 mm long, dark-brown to reddish, ovoid in shape. Antennae consist of 11 segments and the mouthpart is adapted for nibbling. Elytrae are spotted along the entire length and darkening is present in their middle part and toward the ends. Legs are modified for jumping and the femur is darker (Fig. 1).

Fig. 1 Imago of *E. hirtipennis*

According to Sannino L. and Balbiani A., female lays in average 150-200 eggs in the soil, in the vicinity of host-plant, individually or in small groups (3-5) The eggs are oval and elongated, 0.43 x 0.16 mm in size. At the moment of emerging they are white, and with time they turn to strawy-yellow. The surface of the horion is smooth and elastic (11).

Hatching takes place after a week. Newly hatched larvae are small and tiny, in white color. The mature larva has cylindrical form, the body is moderately wrinkled and segmented, reaching a size of 3.5 - 4.5 mm. Its color is dirty white and the head is light brown. It has three pairs of short thoracic legs and a pair of abdominal prolegs on the last segment.

The larva goes through three stages and larval development lasts 4 to 5 weeks.

The mature-larva leaves the root system and makes a soil chamber at a depth of 2-4 cm, where it becomes a pupa (pupa liberta), sized 1.9 x 0.75 mm.

The imago eclodes in 4 to 7 days.

Development of each generation from egg to imago lasts approximately 6 weeks.

The adults have a considerably long active life - about 8 weeks (17).

Due to this, their population rapidly increases and causes losses throughout the whole period of tobacco growing.

It usually has 3-5 generations a year, and the adults of different generations mix together. After harvest, the beetle easily adapts to tobacco suckers and shelters underneath leaf debris or other organic matters near tobacco or other plantings where it hibernates.

E. hirtipennis causes three types of damages:

- Damages caused by larvae to underground part of plants;
- Damages caused by imagos feeding on tobacco plants;
- Indirect damages – the pest appears as a vector of a number of viruses.

Larvae feed on the underground parts of plants, boring choridors through the roots and stalks. They cause more severe damage to younger plants.

The most severe damages are caused by adult insects. These damages are characterized by small round holes that give the leaf a sieve-like appearance (Fig. 2).

When higher population of the beetle is present, the little holes on the seedlings and young transplants coalesce and the leaf can disappear in a short time (Fig. 3).

Fig. 2 Adults of *E. hirtipennis*

Fig. 3 Damage caused by tobacco beetle

Gixhari,1983, reports that tobacco beetle in Albania has destroyed the whole yield of transplanted tobacco in only two days (2).

The beetle can consume food ten times of its own weight a day and is easily adaptable to various environmental conditions (12).

Imagos can cause economically important damages both to young seedlings and to plants transplanted in field.

Fig. 4 Damage caused by tobacco beetle

Fig. 5 Damage caused by tobacco beetle

Holes in the leaf present an entrance for a great number of pathogenic microorganisms. For damaged plants it is difficult to adapt after transplanting and very often they die. The damages are particularly pronounced in dry and windy weather.

Damages on plants transplanted in field depend on the intensity of attack. The increased number of leaf holes reduces the assimilation capacity of leaves, by which they slow down plant growth and reduce the quality of tobacco.

Fig. 6 Damage caused by tobacco beetle

The most severe damages are expected the first three weeks after tobacco transplanting.

Unfortunately, tobacco beetle attacks all leaf insertions, including the suckers and the top leaves – the best for their aroma and quality.

Tobacco beetle can be potential vector of the virus disease TRSV (tobacco ring spot virus).

E. hirtipennis hibernates as adult insect under plant debris or in the stalks remaining in tobacco fields after harvest. In spring, the imagos emerge and feed initially on weed plants and then migrate to tobacco seedbeds. They attack the germinated plants and lay their eggs on soil surface near the roots of the plants.

Tobacco beetle in field is spread through transplantation of seedlings attacked by larvae. In the same manner, the ecloded imagos from tobacco seedbeds and those from adjacent weeds continue to hatch on transplanted tobacco in field. The attack proceeds to the end of harvest.

The control of this pest is very difficult, not only because it is Coleoptera (sheathed wings) but also because it is Halticinae, finding food by hopping from one plant to another in short jumps.

Regarding the climate effects, cold winters result in higher morbidity of the hibernating adults. In the growing period of tobacco, in sunny days, when the temperature is moderate, the adult of *E. hirtipennis* is active and dwells both on front and reverse side of leaf (preferring the latter one). In a period of strong heat (about 35^0C), it hides in shady places or in the soil.

For successful protection of tobacco from this harmful pest and, which is more important, for obtaining higher yields of a good qualty tobacco, it is necessary to have basic knowledge on its morphology, life cycle, dwelling place and environment, damages caused on tobacco, presence or absence of natural enemies and, finally, monitoring. Continuous visual monitoring is necessary during the vegetation period, after tobacco transplanting in field.

Preventive measures are of major importance in the control or reduction of beetle attack. Monoculture production of tobacco should be avoided and crop rotation is recommended instead. Crops in which tobacco beetle can hibernate and survive, should not be used as a precrop. Wheat or sunflower are good precrop for tobacco.

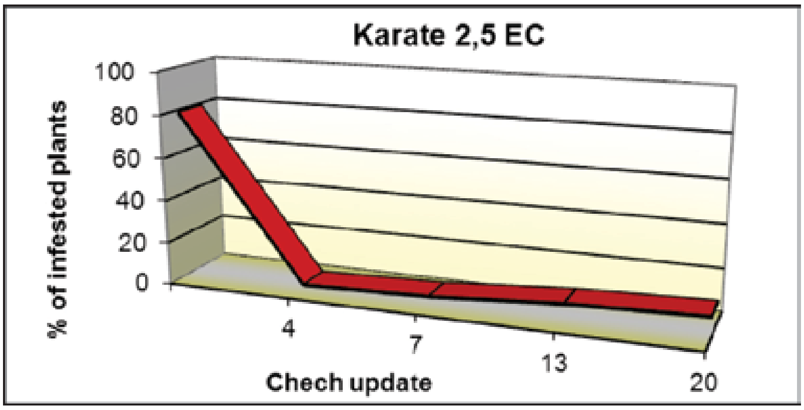
Plants of the Solanaceae family should be avoided to be grown near tobacco seedbeds and plantings.

Chemical control is still indispensable in keeping the population rate in economically acceptable frames.

In the first trial (2001-2002) plot treated with Karate 0.02%, infestation was noticed on 315 plants, i.e. the beetle was identified on 78.75% of the treated stalks.

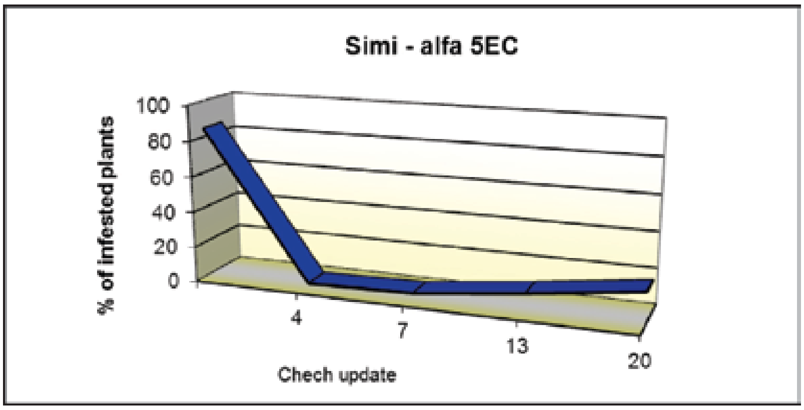
In plot treated with Sumi-alpha 0.04%, the infestation was observed in 339 out of the 400 plants (84.75%).

The percentage of effectiveness was high. Thus, four days after treatment with Karate tobacco beetle was observed in 1.75% of the plants, and 20 days after in 9.75% (Graph 1).



Graf. 1 Control of *E. hirtipennis*- plot treated with Karate 2,5 EC

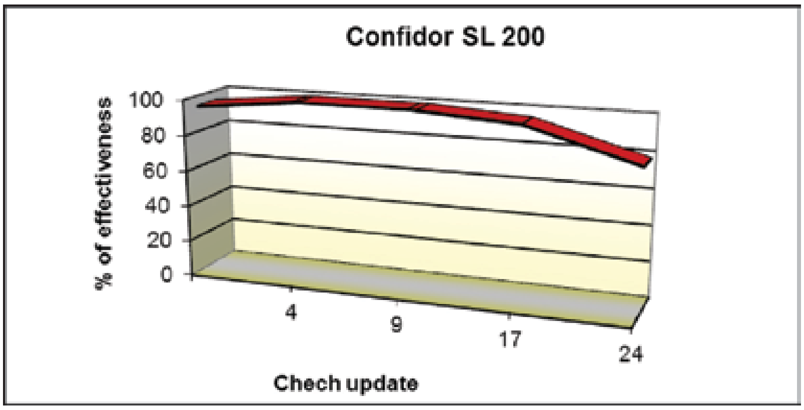
With Sumi-alpha, four days after treatment tobacco beetle survived in only 1% of tobacco stalks, and 20 days after in 17.75% (Graph 2).



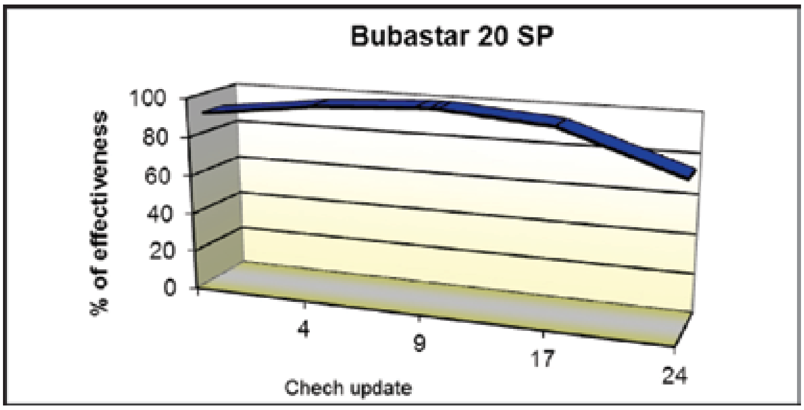
Graf. 2 Control of *E. hirtipennis*- plot treated with Sumi-alpha 5 EC

In about 24 hours, with this active ingredient, the reduction of infestation is 94 - 99%, which is in accordance with data reported by Sannino et al. (13) and Gixhari (2).

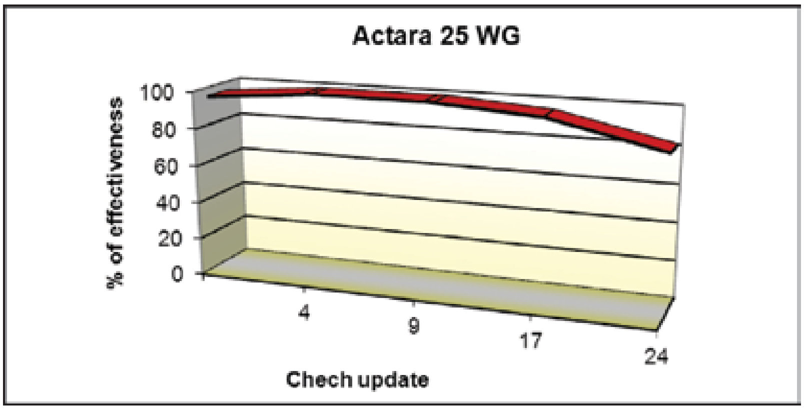
In the second trial (2007-2008), the applied insecticides showed high effectiveness in tobacco beetle control, which was visible from the first check 35 hours after their application up to the 24-st day (Graf 3, 4 and 5).



Graf. 3 Confidor SL 200- effectiveness in tobacco beetle control



Graf. 4 Bubastar 20 SP- effectiveness in tobacco beetle control



Graf. 5 Actara 25 WG- effectiveness in tobacco beetle control

Our investigations on the effectiveness of chemicals in the control of *E. hirtipennis* correspond to those of other authors in the world (from America, Italy, Greece, Bulgaria, Albania).

To avoid the possibility of resistance, a change of chemicals during the same growing period is recommended.

In spring, weeds near tobacco seedbeds and in tobacco farms should be treated with adequate insecticides. Pest treatment with insecticides should be made when first holes appear on leaves in seedbeds or in field and when first imagos are observed.

For treatment of tobacco in seedbeds or during its transplantation in field heavily infested with tobacco beetle in the previous year, it is necessary to apply systemic insecticides (imidacloprid, acetamiprid, or thiamethoxam) in order to provide a long-term protection of young plants in the most critical moment of transplantation.

Similarly, if strong attack of adults is observed in field during the growing period, a treatment with pyrethroid (esfenvalerate, lambda-cyhalothrin), is recommended, which will provide immediate initial control.

In the repeated occurrence of the pest, some of the systemic products (imidacloprid, acetamiprid, or thiamethoxam) can be applied.

CONCLUSION

The high consuming ability and expressed poliphagia of *Epitrix hirtipennis*, the longevity of imagos, the great number of generations and resistance to the atmosphere effects make this insect difficult for control.

Having in mind that imagos are present on tobacco throughout the whole period of growing (April - October), it is necessary to perform a multiannual integral protection of tobacco, making great efforts and applying precise measures to achieve visible results.

REFERENCES

1. Balbiani A., Sannino L., Piro F., 1992. Insetticidi ad applicazione fogliare per il controllo dell' altica del tabacco *(Epitrix hirtipennis Melsh.)*. L'informatore Agrario XLIX, 14, 91–93.
2. Gixhari B., 1995-1996. Hemiska kontrola na tutunskata bolva (*Epitrix hirtipennis - Melsh.*) na orientalski tutun vo pole, 1995-1996 Tobacco Institute of Cerrik – Albanija.
3. Dimitrov A., 1997. Narucnik po rastitelna zasita na tutuna (za diagnosticirane na bolestite i neprijatelite), Institut po tutuna i tutunevite izdelia-Plovdiv, SumiAgro, Sofia.
4. Grupa avtori, 1980, Priracnik o karantinskim biljnim bolestima i stetocinama SFR Jugoslavije, Fakultet poljoprivrednih znanosti sveucilista u Zagrebu, Institut za zastitu bilja.
5. Krsteska V., Stojanoski P., 2002. *Epithrix hirtipennis* Melsh. on tobacco in the Republic of Macedonia. Vtora balkanska naucna konferencia: Kacestvo i efektivnost na proizvodstvoto, obrabotkata i prerabotkata na tutuna, Sbornik na rezometata, p. 85, Plovdiv, septemvri, 2002.
6. Krsteska V., Dimeska V., Stojanoski P., 2008. *Epithrix hirtipennis* Melsh. na tutunot. Izvestaj za proektnite zadaci na naucno-istrazuvackata rabota vo 2008 god., str. 48-51, UKLO Naucen Institut za tutun-Prilep.
7. Krsteska V., Dimeska V., Stojanoski P., 2009, Epitrix hirtipennis Melsh on tobacco, CORESTA -Agronomy meetings and phytopathology study groups, p.65, APPOST 22, Rovinj, 4-8 October, Hrvatska.
8. Lykouressis D. P., Mentzos G., 1992. The abundance of *Epitrix hirtipennis (Melsheimer)* and damage to tobacco in Greece. Abstr Bul.. Spéc. CORESTA , 1992, Congrès Jerez de la Frontera, p. 141, abstr. P1., ISSN.0525-6240 Laboratory of Agricultural Zoology and Entomology, Agricultural University of Athens, 75 Iera Odos, Gr-11855, Athens, Greece.
9. Mistric W. J., Smith F. D., 1970. Organophosphorus and Carbamate Insecticides as Substitutes for DDT in Controlling the Tobacco Flea Beetle on Flue-Cured Tobacco, Department of Entomology, North Carolina State University, Raleigh, N. C., USA; Beitrage zur Tabakforschung, Band 5, Heft 4, August 1970.
10. Piro F., Sannino L., Balbiani A., 1991. Confronto di insetticidi ad applicazione fogliare per il controllo dell' altica del tabacco *(Epitrix hirtipennis Melsh.)*. Ann. Ist. Sper. Tabacco, XV-XVI, 113 – 123.
11. Sannino L., Balbiani A., Espinosa B., 1984. Un nuovo fitofago devasta il tabacco nel brnventano: *Epitrix hirtipennis Melsh.* (Coleoptera: Chrysomelidae) - Nota preliminare - Infor. Agrar., 29 : 55 - 57.
12. Sannino L., Balbiani A., Biondi M., 1985. *Epitrix hirtipennis* (Melsheimer, 1847) (Coleoptera: Chrysomelidae): Considerazioni tassonomiche, ecologiche ed etologiche, Atti XIV Congr. naz. ital. entom., Palermo, Frice, Bagheria, pp. 285 -292.
13. Sannino L., Balbiani A., Piro F., Leone A. P., 1986. Risultati di prove di lotta chimica contro *Epitrix hirtipennis (Melsh.)* (Coleoptera: Chrysomelidae) su tabacco-Atti Giorn. Fitopat., 1: 153 -158.
14. Sannino, L., 1997, Bio-etiology, damages and protection against *Epitrix hirtipennis* (Melsh.) in Italy Annali dell Il Tabacco vol 5, Instituto Sperimentale per il Tobacco, Italy.
15. Semtner P. J., Reed T. D., 1997. Imidacloprid for the management of *Myzus nicotianae* and *Epitrix hirtipennis* on tobacco, abstr CORESTA Meet. Agro-Phyto Groups/ Réunion Montreux, AP21. Virginia Tech, Southern Piedmont Agricultural Experiment Station, Blackstone, VA.
16. Tirelli, M., 1953. Patologia del Tabacco, vol I, Parti Prima, Instituto scientifico sperimentale per i tabacchi – Roma, Collana di pubblicazioni scientifiche e techniche, Augusto Vallerini Editore.
17. Fiorentina F., Milano D., De Pietro S., 1987. Combattere la pulce, Speciale. Tera e Vita No. 37, 54 -56.

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INVESTIGATION OF CHLORIDE CONCENTRATION IN BURLEY TOBACCO VARIETIES

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ABSTRACT

The effects of different rates of chloride application on growth parameters, yield and Cl accumulation in cured leaves of six Burley cultivars have been studied in stationary field trial. Chloride treatments were 0, 65 and 130 kg Cl ha^{-1}. The chloride source was KCl. The accumulation rates of chloride in the leaves of six Burley tobacco varieties (Burley 1317, Burley 1000, Burley 1351 from Bulgaria and TN 86, Banket 102, Kentucky 908 from the United States) were compared.

Statistically significant linear dependence was noted between chloride treatments and Cl concentration in the leaves. The concentrations of chloride in cured leaves were 0.10-0.40%, 0.30-0.99% and 0.37-1.87% respectively for 0, 65 and 130 kg Cl ha^{-1} treatments. In most cases the chloride content increased above 1% at the 130 kg ha^{-1} Cl level. There were significant variety differences in chloride accumulation and significant variety x soil chloride application interactions. Therefore, in order to limit adverse effects of fertilization with high KCl rates the appropriate varieties should be carefully selected.

Key words: Burley tobacco, chloride

ИСТРАЖУВАЊЕ НА КОНЦЕНТРАЦИЈАТА НА ХЛОРИД ВО ТУТУНСКИТЕ СОРТИ ОД ТИПОТ БЕРЛЕЈ

Во стациониран полски опит извршени се проучувања на ефектите од различни применети дози на хлорид врз параметрите на развој – приносот и акумулацијата на Cl во сувите листови од шест берлејски сорти. Третирањата со хлорид беа во дози од 0, 65 и 130 kg Cl ha^{-1}. Изворот на хлорид беше KCl. Компарирани се акумулативните дози на хлорид во тутунските листови од шест берлејски сорти (Берлеј 1317, Берлеј 1000 и Берлеј 1351 берлеј од Бугарија и ТН 86, Банкет 102 и Кентаки 908 од САД). Констатирана е статистички значајна линеарна зависност помеѓу третирањата со хлорид и концентрацијата на Cl во листовите. Концентрациите на хлорид во сувите листови беа 0.10-0.40%, 0.30-0.99% и 0.37-1.87% соодветно за третирањата од 0, 65 и 130 kg Cl ha^{-1} . Во повеќето случаи содржината на хлорид се покачи над 1% при третирање со 130 kg Cl ha^{-1}. Постојат значајни разлики меѓу сортите во однос на акумулацијата на хлорид, како и значајни интеракции помеѓу сортата и почвата во поврзани со примената на хлоридот. Оттаму, за да се намалат непожелните ефекти од ѓубрењето со високи дози на KCl, потребно е внимателно да се одбираат најсоодветните сорти.

Клучни зборови: берлејски тутун, хлорид

INTRODUCTION

Chloride is an essential micronutrient and there is considerable evidence that beneficial effects arise with tobacco from the presence of small amounts of chloride in the fertilizer (McCants & Woltz, 1967). Elliot and Vickery (1961) reported that chlorine from 20 to 50 pounds per acre had no effect on yield or leaf-grade quality of flue-cured tobacco, but increased applications of chlorine increased the chlorine content, moisture uptake, and decreased the burning rate of the cured leaf. Mulchi (1982) found significant reductions in average price, quality index, and leaf burning at chloride application rates above 44 kg/ha or above 0.53% chloride in the cured Maryland tobacco. Various soil and fertilization conditions, as well as various types of tobacco, varieties and methods of harvesting might contribute to the differences in absorption and distribution of chloride with respect to stalk positions and the total leaf chloride content (Tso, 1990). The quality of tobacco is known to be adversely affected by excessive amounts of chloride in the media and fertilization with KCl leads to accumulation of unacceptable levels of chloride, which impairs the quality of the cured leaf. The preferred source of potassium is potassium sulfate but the cost of fertilization tempts tobacco growers to use the cheaper muriate of potash source (Palmer and Pearce, 1999). Gul et al. (2006) illustrated that tobacco cultivars showed different response to the KCl induced chloride toxicity. The same authors reported that chloride, reducing sugars, nitrogen, nicotine and K contents of leaves increased with increasing KCl. Darvishzadeh et al. (2011) observed that Cl uptake by oriental tobacco varied with genotypes and suggested that Cl accumulation is genetically controlled. According to Karaivazoglou et al. (2004) the differentiation among varieties in accumulation rate of chloride with increased chloride level indicate that, when the chloride concentration in the available irrigation water is high, a choice exists to select among Oriental, Virginia and Burley tobacco varieties that exhibit the less adverse effect from chloride.

The objective of the current study was to investigate the influence of Cl addition on growth parameters, yield of cured leaves and Cl accumulation in cured leaves from different primings of six Burley cultivars.

MATERIAL AND METODS

The effect of chloride applied in the soil on the growth, yield and chloride concentration in Burley tobacco genotypes was investigated in 2006. The study was conducted in a stationary field trial. The soil used was sandy loam, alluvial-meadow with low chlorine content - 1,78 mg kg^{-1}; humus – 1,82%; total nitrogen - 0.076%; P_2O_5 - 16 mg kg^{-1}; K_2O - 266,5 mg kg^{-1}; bulk density - 1,3 g/cm^3; $pH_{(H2O)}$ - 8,43.

The stationary plots (1,5 m long and 1,5 meters wide) had a depth of 0,5 m and area of 2,25 m^2. During field preparation, each plot received P broadcast at a rate equal to 100 kg ha^1, using triple superphosphate as fertilizer. Nitrogen was applied at a rate of 140 kg ha^{-1} as ammonium nitrate. Chloride treatments were 0, 65 and 130 kg Cl ha^{-1}. The chloride source was KCl. Before transplanting, the fertilizers (ammonium nitrate and potassium chloride) were uniformly broadcast over the soil surface of each plot and incorporated into the soil.

The accumulation rates of chloride in the leaves of six Burley varieties - Burley 1317, Burley 1000 and Burley 1351 from Bulgaria and TN 86, Banket 102 and Kentucky 908 from the United States were compared.

Cultural practices were applied according to the recommendations for commercial plantations.

The following measurements were taken: plant height and number of leaves per plant (as measured at flowering), yield of cured leaves per plant and chemical analyses of the leaves. Cured leaves at first (lower leaves), second (middle leaves) and third (upper leaves) priming were collected for analyses. All samples were oven-dried at 65^0C and ground. The chlorine content was determined by titration using the silver nitrate method.

The analysis of variance was used for statistical processing of data. Regression analysis was used to establish relationships between chloride treatments rates and changes in chloride contents in cured leaves.

RESULTS AND DISCUSSION

Chloride toxicity in green tobacco has been characterized as thickened leaves which are exceedingly brittle with upward curling of leaf margins (McCants & Woltz, 1967). No symptoms of chloride toxicity appeared under our experimental conditions during the growing season.

Growth parameters including plant height and leaf number per plant were not significantly affected by Cl addition (Table 1). Genotype differences were significant, but varieties x soil chloride application interactions were significant for plant height only. Differences in crop yields were not significant among rates of chloride applications and variety x soil chloride application interaction. However, genotype differences for yield of cured leaves were statistically significant. Gul et al. (2006) in outdoor pot experiment with tobacco cultivars observed that all growth parameters increased with the initial dose of 4 mmol kg^{-1} of KCl, but then decreased with higher levels. Our results indicate that chloride treatments up to 130 kg Cl ha^{-1} did not influence growth processes of Burley tobacco plants and yield of cured leaves.

Table 1. Effect of soil chloride application and variety on growth parameters and yield of cured leaves

Soil chloride application, kg ha^{-1}	Variety	Plant height, cm	Total number of leaves per plant	Yield, g $plant^{-1}$
0	Burley 1317	123.2	23.8	78.0
	Burley 1000	140.2	23.8	80.2
	Burley 1351	141.2	25.2	84.2
	TN 86	138.6	23.6	74.0
	Banket 102	135.0	25.0	85.0
	Kentucky 908	121.6	23.6	74.2
65	Burley 1317	122.5	23.5	80.2
	Burley 1000	137.1	23.6	83.0
	Burley 1351	137.5	25.4	81.4
	TN 86	135.0	23.5	81.0
	Banket 102	134.5	25.3	78.4
	Kentucky 908	120.0	23.8	76.0
130	Burley 1317	125.3	22.9	76.4
	Burley 1000	145.0	24.0	78.8
	Burley 1351	141.5	25.6	87.8
	TN 86	139.7	24.0	80.4
	Banket 102	130.5	24.5	80.2
	Kentucky 908	113.8	24.0	77.4
Significance of F-tests				
Cl Application (Cl A)		NS	NS	NS
Variety (V)		**	**	*
Cl x V		*	NS	NS

*, ** Significant at the 0.05 and 0.01 probability levels, respectively; NS – not significant

Variations of Cl concentration in tobacco leaves due to Cl addition and variety differences are shown in Table 2. The chloride content in tobacco leaves varied from 0.10 to 1.87%. According to Flower (1999), the highest concentration of chloride is found in the lower leaves and it decreases progressively to the top of the plant. Generally, the chlorine concentration in our investigation was the highest in the lower priming and diminished with successive primings. The variation in Cl contents in the second and third priming was not related to leaves' stalk position, which could be explained by the fact that Cl^- is mobile within the plant.

The CL concentration in cured leaves from control treatment is not high - from 0.10 to 0.40%. With the increase of chloride application rates the Cl content in leaves from the first, second and third priming was significantly increased. Chloride concentrations in tobacco leaves grown under the highest KCl rates were 4-5 times higher than those grown on plots without Cl addition. Chloride concentrations in excess of 1% can produce poor quality tobacco (Flower, 1999). In our experiment, the tobacco from 0 and 65 kg Cl ha^{-1} treatments did not reach this detrimental accumulation rate. In most cases the chloride content rose above 1% at the 130 kg ha^{-1} Cl level.

Table 2. Effect of soil chloride application and variety on chloride concentration in cured leaves (% of dry weight)

Soil chloride application, kg ha^{-1}	Variety	Lower leaves	Middle leaves	Upper leaves	Average
0	Burley 1317	0.25	0.12	0.12	0.16
	Burley 1000	0.20	0.10	0.13	0.14
	Burley 1351	0.40	0.23	0.14	0.25
	TN 86	0.22	0.21	0.17	0.20
	Banket 102	0.31	0.17	0.16	0.21
	Kentucky 908	0.21	0.22	0.15	0.20
65	Burley 1317	0.54	0.99	0.42	0.65
	Burley 1000	0.61	0.70	0.58	0.63
	Burley 1351	0.98	0.30	0.53	0.60
	TN 86	0.76	0.43	0.38	0.52
	Banket 102	0.69	0.49	0.50	0.56
	Kentucky 908	0.73	0.38	0.80	0.64
130	Burley 1317	1.51	0.72	0.84	1.02
	Burley 1000	1.29	0.53	0.37	0.73
	Burley 1351	1.04	0.62	1.21	0.96
	TN 86	1.62	1.17	1.24	1.34
	Banket 102	1.75	1.19	1.54	1.49
	Kentucky 908	1.87	1.37	1.85	1.70
Significance of F-tests					
Cl Application (Cl A)		**	**	**	
Variety (V)		*	**	*	
Cl x V		*	**	*	

*, ** Significant at the 0.05 and 0.01 probability levels, respectively

Statistically significant linear dependence was noted between chloride treatments and the concentration of Cl in the leaf tissues. The equations in Table 3 can be used to predict the maximum chloride rate to keep leaf chloride concentrations from exceeding 1%. If y=1%, the corresponding Cl rates are 79, 136, and 113 kg ha^{-1} for lower, middle and upper leaves, respectively. These rates are higher than the chloride rate of 44 kg ha^{-1} obtained by Mulchi (1982) above which he observed significant reductions in some parameters of cured tobacco. Therefore, the calculated rates point to the highest acceptable levels above which the leaf quality is expected to be reduced. Peedin (1999) emphasize that the ***y*** intercept of the regression equation will vary depending on the amount and distribution of rainfall, soil types and some cultivation practices.

Table 3. Relationships between chloride treatment rates and changes in chloride contents in lower, middle and upper leaves of Burley tobacco

Leaf position	Relationship	Correlation coefficient (r)
Lower leaves	Y=0.21+0.010x	0.927**
Middle leaves	Y=0.18+0.006x	0.794**
Upper leaves	Y=0.10+0.008x	0.820**

** Significant at the 0.01 probability level

Evanylo et al. (1988) established nutrient norms for cured Burley tobacco and observed that the mean chloride concentrations for desirable yield and price were 9.6 and 9.9 mg g^{-1}. The same authors concluded that because of a large SD value for mean Cl concentration in the high-price subpopulation high-quality tobacco may be attained under a fairly wide rage of Cl.

Having in mind the emerging consensus views from the cited literature and the data obtained in our experiments, chloride application of 65 kg ha^{-1} should be considered as optimal for producing high cured-leaf quality under South Bulgarian conditions.

The accumulation rate of chloride in leaves at three chloride levels varied between cultivars (Table 2). There were significant variety differences in chloride accumulation and significant variety x soil chloride application interactions. The average Cl concentration of six genotypes ranged from 0.14 to 0.25% in plots not receiving KCl. In control treatments Burley 1351 was the highest Cl accumulator but the same did not apply to the application of 65 and 130 kg Cl ha^{-1}. Averaged for all primings, Burley 1317 had the highest chloride concentration in leaves as compared to other cultivars when Cl rate was 65 kg ha^{-1}. In the highest Cl treatment the average chloride concentration varied from 0.73 to 1.70% and all Bulgarian cultivars accumulated less chloride in the leaves compared to the three introduced varieties. These results indicate that, when properly applied, the cheaper muriate of potash source can be safely used by tobacco growers to increase the economic efficiency of fertilizer application.

CONCLUSIONS

Our results indicate that KCl applications should be planed carefully to prevent its adverse effects on quality of tobacco even when tobacco is grown on soils with low chlorine content.

Under our experimental conditions, no symptoms of chloride toxicity appeared during the growing season. Growth parameters and yield of cured leaves were not significantly affected by Cl addition but statistically significant linear dependence was noted between chloride treatments and the concentration of Cl in the leaves. The concentrations of chloride in cured leaves were 0.10-0.40%, 0.30-0.99% and 0.37-1.87% respectively for 0, 65 and 130 kg Cl ha^{-1} treatments. In most cases the chloride content rose above 1% at the 130 kg ha^{-1} Cl level.

There were significant variety differences in chloride accumulation and significant variety x soil chloride application interactions. In the highest Cl treatment, cultivars Burley 1317, Burley 1000 and Burley 1351 accumulated less chloride in the leaves compared with the three introduced varieties. Therefore, in order to limit adverse effects of fertilization with high KCl rates the appropriate varieties should be carefully chosen.

REFERENCES

1. Darvishzadeh R., S.R. Alavi, A. Sarafi, 2011. Genetic variability for chlorine concentration in oriental tobacco genotypes. Archives of Agronomy and Soil Science 57 (2) 167-177.
2. Elliot J.M., L.S. Vickery, 1961. The effect of chlorine on the hygroscopicity of flue-cured tobacco. Canadian Journal of Plant Science 41(1) 195-198.
3. Evanylo G.K., J.L. Sims, and J.H. Grove, 1988. Nutrient Norms for Cured Burley Tobacco. Agron. J. 80: 610-614.
4. Flower K.C., 1999. Field Practices. In: Davis D., Nielsen M. (Eds.), Tobacco Production, Chemistry and Technology. Blackwell Science, pp. 76-103.
5. Gul H., R.A. Khattak, D. Muhammad, 2006. Yield and Chemical Composition of Tobacco Leaves of Different Cultivars as Affected by Four Levels of Potassium Chloride. Pak. J. Sci. Ind. Res. 49 (2) 125-133.
6. Karaivazoglou N.A., D.K. Papakosta, S. Divanidis, 2004. Effect of chloride in irrigation water on three tobacco types. Coresta Congress, Kyoto, Japan, Coresta CD-ROM.
7. McCants C.B., W.G. Woltz, 1967. Growth and Mineral Nutrition of Tobacco. Adv. in Agron. 19: 211-265.
8. Mulchi C.L., 1982. Chloride effects on agronomic, chemical and physical properties of Maryland tobacco – I. Response to chloride applied to the soil. Tob. Sci. 26, 113-116.
9. Palmer G.K., R.C Pearce, 1999. Light Air-cured Tobacco. In: Davis D., Nielsen M. (Eds.), Tobacco Production, Chemistry and Technology, Blackwell Science, pp. 143-153.
10. Peedin G.F., 1999. Production Practices, Flue-cured Tobacco. In: Davis D., Nielsen M. (Eds.), Tobacco Production, Chemistry and Technology, Blackwell Science, pp. 104-142.
11. Tso T.C., 1990. Production, Physiology, and Biochemistry of Tobacco Plant. Ideas, Inc., Beltsville.

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STUDY OF TECHNOLOGICALLY UNUSABLE TOBACCO WASTE AND PRACTICAL SOLUTIONS FOR ITS RECOVERY

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ABSTRACT

Tests of biological waste from the tobacco industry were carried out. The test results led to the following waste recovery solutions. The above 1.00 mm fractions /10% of the sample/ ***can be utilized for producing briquettes.*** The carbon monoxide yield as compared to the cigarette smoke was below the standard limit of 10.00 mg/cig. The above 0.4 mm fractions /90% of the sample/ ***are suitable for nicotine extraction.*** An aggregate /mixed/ sample should be used for nicotine extraction. The tobacco dust /fraction under 0.4 mm/ with high iron content ***is suitable for being used as fertilizer following composting in alkaline soils. All fractions are suitable for nicotine extraction and subsequent composting –*** the fractions contain an optimal amount of nutrients and are low in heavy metal content. ***The material of all fractions can undergo pyrolysis to generate gas.***

Key words: *tobacco, tobacco waste, tobacco waste utilization*

ПРОУЧУВАЊЕ НА ТЕХНОЛОШКИ НЕУПОТРЕБЛИВИОТ ТУТУНСКИ ОТПАД И ПРАКТИЧНИ РЕШЕНИЈА ЗА НЕГОВОТО ОБНОВУВАЊЕ

Извршени се испитувања на биолошки отпад од тутунската индустрија. Резултатите од опитот доведоа до следниве решенија за обновување на отпадот. Горните фракции од 1,00 mm/10% од примерокот/ можат да се искористат за производство на брикети. Количината на јаглерод моноксид во споредба со чадот од цигарите е под стандардната граница од 10,00 mg/cig. Горните фракции од 0,4 mm / 90% од примерокот/ се погодни за екстракција на никотин. Агрегат / мешани / примерок треба да се користи за никотин екстракција. Тутунската прашина /фракцијата под 0,4 mm/ заради високата содржина на железо е погодна да се користи како ѓубре по компостирањето на алкалните почви. Сите фракции се погодни за екстракција на никотинот и последователно компостирање - фракциите содржат оптимална количина на хранливи материи и се со ниска содржина на тешки метали. Материјалот од сите фракции може да се подложи на пиролиза за добивање на гас.

Клучни зборови: тутун, тутунски отпад, искористување на тутунскиот отпад

INTRODUCTION

Currently there are four cigarette factories operating in Sofia, Plovdiv, Blagoevgrad and Stara Zagora, one tobacco processing plant in the village of Yasen and a considerable number of companies involved in tobacco purchasing and preparation. Impurities in the tobacco products are not allowed. Impurities could be organic, inorganic and miscellaneous materials on tobacco leaves and between them [1]. This is why throughout the technological processing of tobacco, it is cleaned and dedusted numerous times. Pneumatic tubes, separation machines and aspiration systems are applied for the purpose. Particles of tobacco origin are being in this way also captured and respectively discarded. This tobacco waste consists of tobacco dust, cigarette machine crumbs, mid-rib and leaf blade pieces, even leaves. Such waste is referred to as "technologically unusable". It is disposed of in regulated waste depots. Its quantity is significant – more than 100 tons per month. It is not being utilized at this point. Moreover, it creates a burden for the processing plants and cigarette factories. There are companies which offer briquetting of waste to reduce the volume and facilitate storage and transport. Even as a waste tobacco is an organic material in which physical, chemical, biochemical and autolytic processes continue to flow. For this reason it heats up and combusts, i.e. the fire risks with tobacco waste storage are significant. On the other hand Bulgaria is legally obliged to comply with environmental requirements in tobacco growing and tobacco products manufacturing [2]. It had been proven that the so called "technologically unusable waste" has ample potential. Biologically active substances and energy could be derived from it and it can be used for fertilizers [3].

The objective of the study was to explore the opportunities for utilizing the biological waste from the tobacco industry.

MATERIAL AND METHODS

We analyzed samples broken down by type of waste and aggregate /mixed/ samples containing different ingredients /dust, crumbs, briquettes, leaf parts, etc./. The tobacco industry biological waste sample underwent fractionation. The fraction analysis aimed at fractionating the sample based on the size of the constituent particles. The fractional composition was established as per BSS 8026-88 using sieves with mesh size of respectively 3.0 mm, 2.0 mm, 1,0 mm and 0.4 mm. In this way we obtained five fractions with the fifth one being in the below 0.4 size range or the so called tobacco dust. Tobacco dust is particles of tobacco ribs and leaf which were sifted through a 0.4 mm sieve. Usually it also contains sand. Sand is composed of inorganic particles stuck on the tobacco leaf or impurities in the dust fraction derived from the tobacco dedusting process. The subsequent tests were carried out by fractions to determine the basic chemical properties of tobacco. The content of the analyzed harmful substances – nicotine, tar and CO – was established by testing lab-manufactured cigarettes. The nicotine content in the extract was analyzed, as well as the content of micro and macro elements. Standardized methodology was applied for the analysis and data processing.

RESULTS AND DISCUSSION

The results of the fractional composition of the aggregate /mixed/ sample are shown in Table 1.

Table 1 Fractional composition

Fraction	Result, %
I fraction - above 3 mm	0.58
II fraction - above 2 mm	2.04
III fraction - above 1 mm	7.13
IV fraction - above 0.4 mm	27.82
V fraction - below 0.4 mm	62.43

The results show the highest percentage was registered in the dust fraction. The lowest was for the particles above 3.0 mm. The sample consisted mainly of particles below 1.0 mm.

Due to the small amount of the I and II fractions, the samples were collected for future studies. Sand was found only in fraction V – 8.52%.

The results are shown in Figure 1.

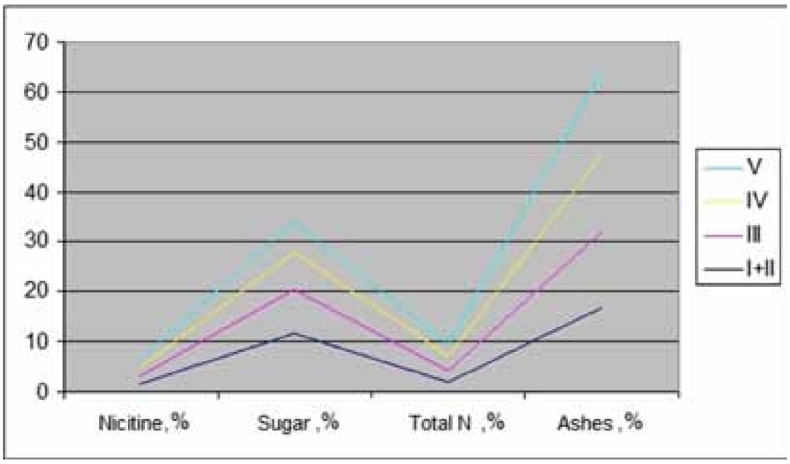

Fig.1 Basic chemical properties of tobacco

It is evident that the first and second fractions contain the highest amounts of carbohydrates. In terms of nicotine and ash contents they are similar to the fifth fraction. The third and fourth fractions exhibit the highest nicotine levels and the lowest ash content. The total nitrogen values are relatively similar.

The test results so far serve for informational purposes and as a basis for outlining further action.

Lab samples of cigarettes containing waste from fractions I, II and III were made for testing the tobacco smoke yields – nicotine, tar and CO, for two reasons:

1. The relatively similar composition of the fractions in terms of the basic properties of tobacco – nicotine, carbohydrates and total nitrogen.

2. The relatively larger size of the particles. It is not possible to make cigarettes using tobacco particles smaller than 1 mm.

Our objective was mainly to establish the carbon monoxide yields.

The tests were carried out on filterless cigarettes with a weight of 0.650 g, paper air permeability – 43.00 CU and cigarette stub length - 23.00 mm.

We assigned arbitrary numbers to the samples, e.g. Sample 1 – fractions I and II together, Sample 2 – fraction III.

The test results are shown in Fig. 2.

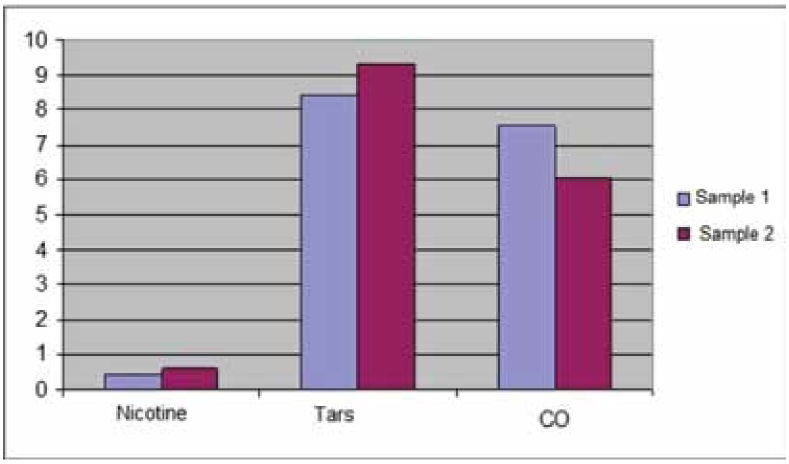

Fig.2 Harmful substance in tobacco smoke, mg/cig

The results are typical and meet the legislative requirements for manufactured cigarettes. Pursuant to TTPA the maximum carbon monoxide yields are 10.00 mg/cig.

Further tests were made to determine the nicotine content in an extract of fraction III /above 1 mm/, fraction IV /above 0,4 mm/ of the aggregate starting sample and separately for briquette, tobacco dust and tobacco leaf and crumb extracts.

The results are shown in Fig. 3.

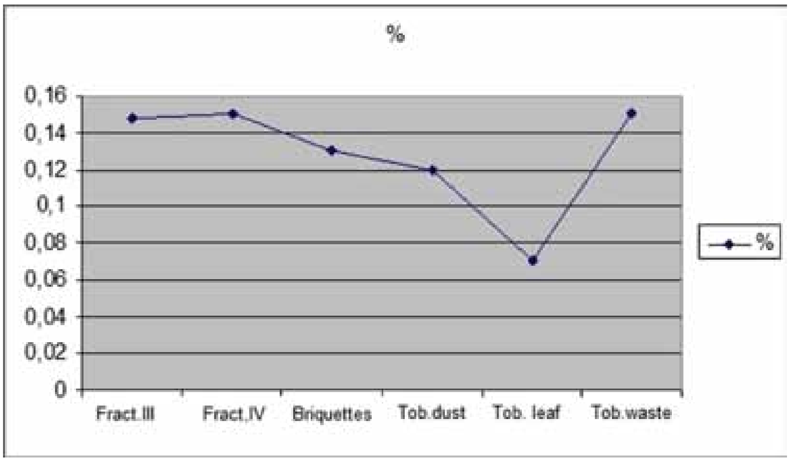

Fig.3 Nicotine content in the extract

The results are indicative of the nicotine variation limits in the extract in the case of separate waste use. The variation is from 0.07 % to 0.15 %. This large value fluctuation does not allow for nicotine extraction from the separate types of waste. In order to retrieve a relatively constant amount of nicotine the biological waste needs to be mixed.

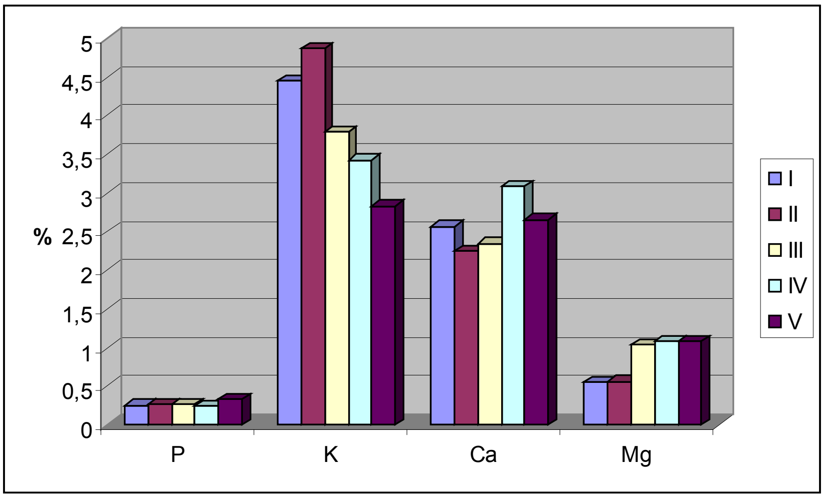

Figure 4. Content of macro-elements in the separate fractions

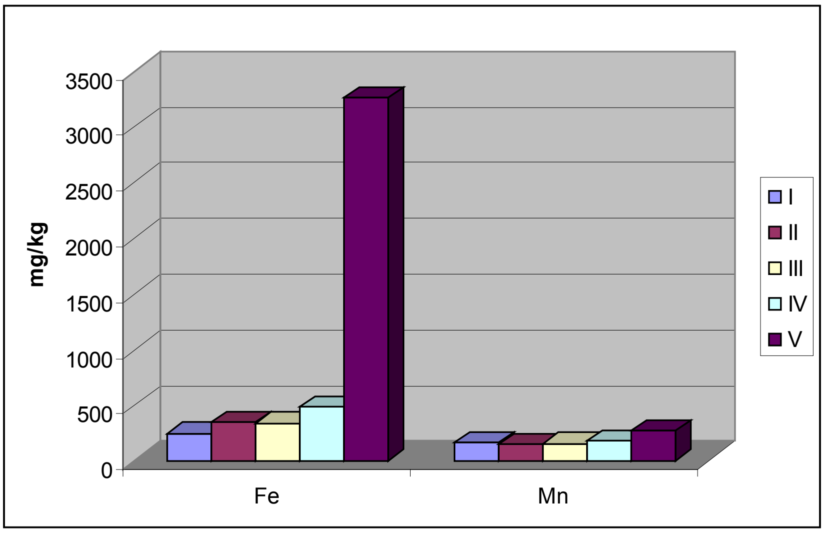

Figure 5. Fe and Mn content in the separate fractions

The nutrients content in the five tested samples was at the normal levels for tobacco and corresponded to the levels cited in the scientific literature (Jones et al., 1991; Drossopoulos, 1992; Campbell, 2000; Zapryanova and Bozhilova, 2009). The iron content registered higher values. According to other publications, the optimal iron content in tobacco leaf is 50 to 300 mg/kg (Campbell, 2000). The iron content found in the four tobacco fractions (from I to IV) is within this range. A concentration of 40-50 mg/kg is deemed to be low (Jones et al., 1991). Data collected by Tso (1990) shows that the iron content in the Virginia tobacco varies from 132 to 595 mg/kg, whereas in the Burley it is 200 to 650 mg/kg. Radojicic et al. (2003) cite levels of 170.72 to 995.87 mg/kg for the Virginia tobacco, Stoilova and Zapryanova (2003) established iron content levels in the Burley tobacco of up to 2257 mg/kg, and Pelivanoska (2007) found iron content levels in the Oriental tobacco from 271 mg/kg to 2532 mg/kg. The iron content in the fifth fraction was 3200 mg/kg.

The heavy metals content in all tested fractions was lower than the critical levels in tobacco (Bozhinova et al., 1995).

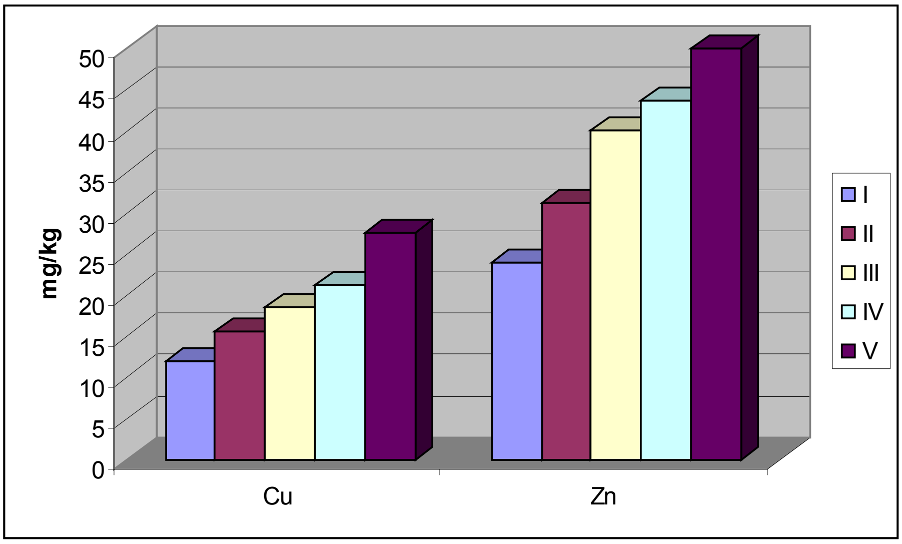

Figure 6. Cu and Zn contents in the separate fractions

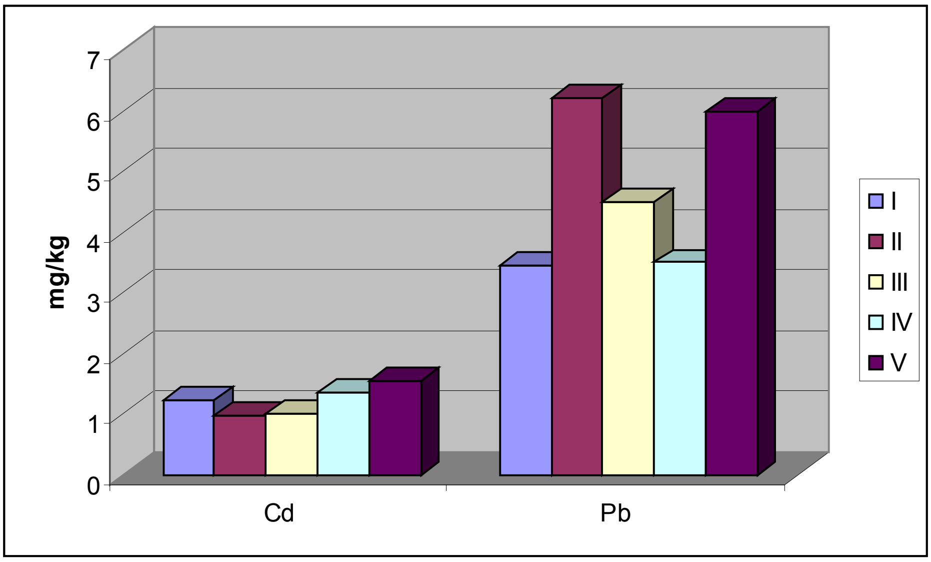

Figure 7. Cd and Pb in the separate fractions

CONCLUSIONS

The test results lead to the following conclusions in terms of utilization of biological waste from tobacco industry.

1. The fractions in the above 1.0 mm range /10 % of the sample/ ***can be used for producing briquettes***. The carbon monoxide emission as compared to the cigarette smoke was below the standard limit of 10.00 mg/cig.

2. The above 0.4 mm fractions /90% of the sample/ ***are suitable for nicotine extraction.*** An aggregate /mixed/ sample should be used for nicotine extraction.

3. The tobacco dust /fraction under 0.4 mm/ with high iron content ***is suitable for being used as a fertilizer following composting in alkaline soils.***

4. ***All fractions are suitable for nicotine extraction and subsequent composting –*** the fractions contain an optimal amount of nutrients and are low in heavy metal content.

5. ***The material of all fractions can undergo pyrolysis to generate gas.***

REFERENCES

1. BSS 8391-85 ”Tobacco and Tobacco Products. Terminology”:
2. Bozhinova P., H. Chuldjian, V Krastevs, B. Georgiev, 1995. Heavy metal pollution in the region of the Lead and Zink Plant – Kardjali and land use opportunities. SUB, Kurdjali. Scientific conference, 3-10.
3. Bozukov, H. 2012. Tobacco as energy generating plant with high environmental potential. Presentation during the International Business Forum organized by ALDE «New Technologies for Environment Protection», Sofia, Sheraton Hotel.
4. Zapryanova P., R. Bozhinova, 2009. Nutrients content and heavy metal content in the Oriental tobaccos and Virginia tobacco, Ecologia i badeshte , 4, 20-29.
5. Pelivanoska V., 2007. Heavy metal content tests in the tobacco of the Prilep region, Tobacco, 33-41.
6. Framework Convention on Tobacco Control of the World Health Organization, Art. 18 Environment Protection.
7. Stoilova A., P. Zapryanova, 2003. Analyzing the mineral content of Bulgarian tobaccos using Atomic Absorption Spectrometry. Food Technology University _ Plovdiv. Scientific publication, Vol. L, 3, 333-339.
8. Campbell C., 2000. Reference Sufficiency Ranges Field Crops, Tobacco, Flue-cured, www.Ncargr.com/agronomi/saaesd/fluecure.htm.
9. Drossopoulos J. B., A. J. Karamanos, G.G. Kouchaji, 1992. A survey of selected nutrient levels at different leaf position of oriental filed-grown tobacco plants, *Tob. Sci.,* 36, 10-15.
10. Jones J., Benjamin Wolf, Harry A. Mills, 1991. Plant Analysis Handbook, Micro-Macro Publishing Inc..
11. Radojicic V. O., Cvetkovic O., Dukic M., 2003. Uticaj agroekoloskih uslova gajenja na sadrzaj mineralnih materija u duvanu lowera Virdzinija, Tutun/Tobacco,53,3-4,96-104.
12. Tso T. C., 1990. Production, physiology and biochemistry of tobacco plant. Inst. of Int. Development & Ed. in Agr. and L. Sciences, B. Maryland, USA.

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A NEW MATHEMATICAL MODEL OF CIGARETTE COMBUSTION

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ABSTRACT

The global cigarette combustion problem, which is one of the hardest scientific problems, until now is not fully solved, because its solution is connected with numerous technical and scientific difficulties. For better understanding of this problem, it should be considered as an open multidisciplinary problem. In the offered research work, this problem is treated only from the mathematical point of view, for whom is given a completely new approach toward its solution. It is considered as a complex mathematical problem, which is composed from the following few subproblems: the problem of balancing chemical reaction of tobacco combustion, field temperature problem in the combustion zone, smoke filtration problem, and the problem of groups' formation of reaction coefficients. For solution of these problems were used theories of ordinary and partial differential equations as well as theory of linear vector spaces and theory of groups. These problems are particularly solved for the certain simulation conditions. For instance, the smoke infiltration problem is founded and solved by virtue of partial differential equation of first order. The field temperature problem in the combustion zone is modeled by the two-dimensional heat transfer equation which is solved by quadratures. The chemical reaction which describes tobacco combustion is a completely new reaction and it includes all important alkaloids and toxins. This reaction belongs to the class of continuum chemical reactions with integer oxidation numbers. In fact, it is a very hard chemical reaction, which cannot be balanced by a computer, because right now in the theory of computer sciences there is not powerful software, which can be used for its balance. The unique way to balance this reaction is by the usage of mathematical method. For that particular case we chose a new algebraic method developed by the author. Since, the reaction of tobacco combustion is very complicated we found only its general solution and one particular solution. This reaction spans real vector spaces. For the reaction coefficients are calculated a symmetric group S_{49}, an alternating group A_{49} and 38 primitive groups.

Keywords: *cigarette, cigarette combustion, smoke filtration, field temperature, groups of reaction coefficients.*

НОВ МАТЕМАТИЧКИ МОДЕЛ ЗА ГОРЕЊЕТО НА ЦИГАРАТА

Глобалниот проблем на горење на цигарата, кој е еден од најтешките научни проблеми, до сега не е решен целосно, бидејќи неговото решение е поврзано со низа технички и научни тешкотии. За подобро разбирање на овој проблем, тој би требало да се разгледува како отворен мулти-дисциплинарен проблем. Во предложенава истражувачка работа, овој проблем е третиран само од математичка гледна точка, за кого е даден комплетно нов пристап во правец на негово решавање. Тој е разгледан како комплексен математички проблем, кој е составен од следниве неколку подпроблеми: проблемот на изедначување на хемиската реакција за горење на тутунот, проблемот за температурното поле во зоната на горење, проблемот за филтрација на чадот и проблемот на формирање на групите од коефициентите на реакцијата. За решавање на овие проблеми беа употребени теориите на обични и парцијални диференцијални равенки, како и теоријата на линеарните векторски простори и теоријата на групи. Овие проблеми се партикуларно решени за извесни симулациони услови. На пример, проблемот за филтрација на чадот е заснован и решен врз основа на парцијални диференцијални равенки од прв ред. Проблемот за температурното поле во зоната на горење е моделиран со две-димензионалната равенка за пренос на топлина која е решена со квадратури. Хемиската равенка што го опишува горењето на тутунот е комплетно нова реакција и таа ги вклучува сите битни алкалоиди и токсини. Оваа реакција спаѓа во класата на континуум хемиски реакции со цели оксидациони боеви. Всушност, таа е многу тешка хемиска реакција, која не може да биде изедначена со компјутер, бидејќи сега во теоријата на информатиката нема моќен програм, што може да биде употребен за нејино изедначување. Единствен начин да се изедначи оваа реакција е употреба на математички метод. За овој партикуларен случај избравме нов алгебарски метод развиен од авторот. Бидејќи реакцијата за горење на тутунот е многу комплицирана најдовме само нејзино општо решение и едно партикуларно решение. Оваа реакција разапнува реален векторски простор. За коефициентите на реакцијата се пресметани симетричната група S_{49}, алтернативната група A_{49} и 38 примитивни групи.

Клучни зборови: *цигара, горење на цигарата, филтрација на чадот, температурно поле, групи од коефициентите на реакцијата.*

1. INTRODUCTION

Cigarette smoking as a bad habit with dangerous consequences to people health is always in the focus of scientific research. This topic in professional literature, for instance in medicine, chemistry, tobacco science and so on, is considered from different points of view, but unfortunately in mathematics this problem was totally neglected. Why? It is hard to reply! Perhaps one of the main causes is that this problem looks for multidisciplinary treatment, because its formulation depends of other factors which are out of the mathematics sphere.

Really for its formulation is necessary a strong knowledge of contemporary chemical engineering, heat transfer theory and chemical thermodynamics, while its solution belongs only in the sphere of mathematics. Just this, was author's main challenge and motive to solve this problem in this article.

With a goal to shed light on this important subject, this work will introduce a new approach toward solution of cigarette smoking problem by virtue of mathematical research. With an intention for better understanding of this approach, emphasis is made throughout the prism of theory of balancing chemical reaction of tobacco combustion, field temperature in the combustion zone, smoke filtration problem, and the problem of groups' formation of reaction coefficients.

Now, this question arises: how stay things about this topic in scientific literature? Most of published papers deal with cigarette properties [1-11]. One group of publications is written by authors associated with the tobacco industry. They contain substantial information for burn rate and temperature of the burning cigarette. Among the variables discussed are tobacco type, cigarette dimensions and packing density, filter parameters and paper porosity, and additives. The major objective these publications seems to be to obtain basic understanding of the burning cigarette, with emphasis on reduction of tar, nicotine, carbon monoxide, and other smoke components. Some of these papers have two major limitations for the present objective:

a) most of the data are obtained during the puff, and

b) the results are obtained with the cigarette held in air.

In the following, some basic cigarette characteristics will be discussed first.

2. CIGARETTE CHARACTERISTICS

As a support in understanding the general trend of this work, some of the important factors which the author thinks are interconnected.

• *Cigarette length*. The tobacco column length involves the time a cigarette burns and thus the probability of it being dropped (since it takes some aware effort to light a cigarette, one may suppose that a short burn time reduces the probability that the smoker becomes inattentive and drops the cigarette stub.).

• *Burn rate*. This factor is presented as a change in length or mass with time. Whether a cigarette is burning in air or is being puffed, the burn rate involves the remaining cigarette length and thus the probability of dropping the cigarette.

• *Packing density*. Lower packing density (achieved primarily by the use of expanded tobacco but also involved by the tobacco blend and cut width, *i. e.*, the width of the tobacco strands) reduces the mass of available fuel.

• *Tobacco type*. The tobaccos used in various cigarette packing may vary in heat yield and burn rate due to variations in tobacco blend constituents and ratios, as well as in types and concentrations of flavorings and humectants.

• *Paper parameters*. Paper parameters cause differences in cigarette heat yield and burn rate. The paper permeability affects the flow of oxygen from the outside air to the combustion zone and the diffusion of pyrolysis gases from this zone to the outside. Chemicals are added to the cigarette paper as smolder accelerants or retardants and to modify the appearance of the ash.

• *Filter characteristics*. The presence and nature of filter tips also affects the flow of air through the cigarette. Additional perforations is often provided in the paper covering the filter, reducing the flow of air through the tobacco column (ventilation; this is used to reduce the exposure of the smoker to smoke components).

3. A New Chemical Formal System

In this section we shall develop a new chemical formal system founded by virtue of principles of the theory of real finite dimensional vector spaces [12, 13] and group theory [14].

Into a mathematical model must be introduced a whole set of auxiliary definitions to make the chemistry work consistently. Just this kind of set will be constructed below.

Only on this way chemistry will be consistent and resistant to paradoxes appearance.

Here, by $\mathbb{R}$ is denoted the set of real numbers and by $\mathbb{R}^n$ is denoted the Euclidian n-dimensional vector space with real entries. Throughout, the set of $m\times n$ matrices over a field will be denoted by $\mathbb{R}^{m\times n}$.

Definition 3. 1. *A vector space over the field $\mathbb{R}$ consists of a nonempty set V of objects called vectors for which hold the axioms for vector addition*

(A_1) *If $\boldsymbol{u}, \boldsymbol{v} \in V$, then $(\boldsymbol{u} + \boldsymbol{v}) \in V$,*

(A_2) $\boldsymbol{u} + \boldsymbol{v} = \boldsymbol{v} + \boldsymbol{u}$, $\forall \boldsymbol{u}, \boldsymbol{v} \in V$,

(A_3) $\boldsymbol{u} + (\boldsymbol{v} + \boldsymbol{w}) = (\boldsymbol{u} + \boldsymbol{v}) + \boldsymbol{w}$, $\forall \boldsymbol{u}, \boldsymbol{v}, \boldsymbol{w} \in V$,

(A_4) $\boldsymbol{u} + \mathbf{0} = \boldsymbol{u} = \mathbf{0} + \boldsymbol{u}$, $\forall \boldsymbol{u} \in V$,

(A_5) $- \boldsymbol{u} + \boldsymbol{u} = \mathbf{0} = \boldsymbol{u} + (- \boldsymbol{u})$, $\forall \boldsymbol{u} \in V$,

and the axioms for scalar multiplication

(S_1) *If $\boldsymbol{u} \in V$, then $a\boldsymbol{u} \in V$, $\forall a \in \mathbb{R}$,*

(S_2) $a(\boldsymbol{u} + \boldsymbol{v}) = a\boldsymbol{u} + a\boldsymbol{v}$, $\forall \boldsymbol{u}, \boldsymbol{v} \in V \wedge \forall a \in \mathbb{R}$,

(S_3) $(a + b)\boldsymbol{u} = a\boldsymbol{u} + b\boldsymbol{u}$, $\forall \boldsymbol{u} \in V \wedge \forall a,b \in \mathbb{R}$,

(S_4) $a(b\boldsymbol{u}) = (ab\boldsymbol{u})$, $\forall \boldsymbol{u} \in V \wedge \forall a,b \in \mathbb{R}$,

(S_5) $1\boldsymbol{u} = \boldsymbol{u}$, $\forall \boldsymbol{u} \in V$.

Remark 3. 2. *The content of axioms* (A_1) *and* (S_1) *is described by saying that V is closed under vector addition and scalar multiplication. The element* **0** *in axiom A_4 is called the zero vector.*

Definition 3. 3. *If V is a vector space over the field $\mathbb{R}$, a subset U of V is called a subspace of V if U is itself a vector space over $\mathbb{R}$, where U uses the vector addition and scalar multiplication of V.*

Definition 3. 4. *Let V be a vector space over the field $\mathbb{R}$, and let $\boldsymbol{v}_i \in V$, $(1 \le i \le n)$. Any vector in V of the form*

$$\boldsymbol{v} = a_1\boldsymbol{v}_1 + a_2\boldsymbol{v}_2 + \cdots + a_n\boldsymbol{v}_n,$$

where $a_i \in \mathbb{R}$, $(1 \le i \le n)$ is called linear combination of $\boldsymbol{v}_i$, $(1 \le i \le n)$.

Definition 3. 5. *The vectors $\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_n$ are said to span or generate V or are said to form a spanning set of V if $V = span\{\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_n\}$. Alternatively, $\boldsymbol{v}_i \in V$, $(1 \le i \le n)$ span V, if for every vector $\boldsymbol{v} \in V$, there exist scalars $a_i \in \mathbb{R}$, $(1 \le i \le n)$ such that*

$$\boldsymbol{v} = a_1\boldsymbol{v}_1 + a_2\boldsymbol{v}_2 + \cdots + a_n\boldsymbol{v}_n,$$

i. e., $\boldsymbol{v}$ is a linear combination of

$$a_1\boldsymbol{v}_1 + a_2\boldsymbol{v}_2 + \cdots + a_n\boldsymbol{v}_n.$$

Remark 3. 6. *If $V = \mathrm{span}\{\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_n\}$, then each vector $\boldsymbol{v} \in V$ can be written as a linear combination of the vectors $\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_n$. Spanning sets have the property that each vector in V has exactly one representation as a linear combinations of these vectors.*

Definition 3. 7. *Let V be a vector space over a field $\mathbb{R}$. The vectors $\boldsymbol{v}_i \in V$, $(1 \le i \le n)$ are said to be linearly independent over $\mathbb{R}$, or simply independent, if it is satisfies the following condition if*

$$s_1\boldsymbol{v}_1 + s_2\boldsymbol{v}_2 + \cdots + s_n\boldsymbol{v}_n = \mathbf{0},$$

then

$$s_1 = s_2 = \cdots = s_n = 0.$$

Otherwise, the vectors that are not linearly independent is said to be linearly dependent, or simply dependent.

Remark 3. 8. *The trivial linear combination of the vectors $\boldsymbol{v}_i$, $(1 \le i \le n)$ is the one with every coefficient zero*

$$0\boldsymbol{v}_1 + 0\boldsymbol{v}_2 + \cdots + 0\boldsymbol{v}_n.$$

Definition 3. 9. *A set of vectors $\{\boldsymbol{e}_1, \boldsymbol{e}_2, \ldots, \boldsymbol{e}_n\}$ is called a basis of V if it satisfies the following two conditions*

1° *$\boldsymbol{e}_1, \boldsymbol{e}_2, \ldots, \boldsymbol{e}_n$ are linearly independent,*

2° *$V = span\{\boldsymbol{e}_1, \boldsymbol{e}_2, \ldots, \boldsymbol{e}_n\}$.*

Definition 3. 10. *A vector space V is said to be of finite dimension n or to be n-dimensional, written $dimV = n$, if V contains a basis with n elements.*

Definition 3. 11. *The vector space* $\{\mathbf{0}\}$ *is defined to have dimension* 0.

Definition 3. 12. *For any matrix $\boldsymbol{A} \in \mathbb{R}^{m\times n}$ we denote*

$$Im\boldsymbol{A} = \{\boldsymbol{y} \in \mathbb{R}^m : \boldsymbol{y} = \boldsymbol{A}\boldsymbol{x} \text{ for some } \boldsymbol{x} \in \mathbb{R}^n\}$$

the image of $\boldsymbol{A}$ *or range of* $\boldsymbol{A}$.

Definition 3. 13. *For any matrix* $\boldsymbol{A} \in \mathbb{R}^{m\times n}$ *we denote*

$$Ker\boldsymbol{A} = \{\boldsymbol{x} \in \mathbb{R}^n\colon \boldsymbol{Ax} = \boldsymbol{0}\}$$

the kernel of $\boldsymbol{A}$ *or null space of* $\boldsymbol{A}$.

Definition 3. 14. *If U and W are subspaces of a vector space V the sum*

$$U + W = \{\boldsymbol{u} + \boldsymbol{w}\colon \boldsymbol{u} \in U, \boldsymbol{w} \in W\}.$$

Definition 3. 15. *The vector space V is said to be direct sum of its subspaces U and W, denoted by*

$$V = U \oplus W,$$

if every vector $\boldsymbol{v} \in V$ *can be written in one and only one way as* $\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{w}$, *where* $\boldsymbol{u} \in U$ *and* $\boldsymbol{w} \in W$.

Definition 3. 16. *Let V and U be vector spaces over the field* $\mathbb{R}$. *A mapping* $F\colon V \to U$ *is called a linear mapping* (*or linear transformation or vector space homomorphism*) *if it satisfies the following two conditions*

1º $\forall \boldsymbol{u}, \boldsymbol{v} \in V, F(\boldsymbol{u} + \boldsymbol{v}) = F(\boldsymbol{u}) + F(\boldsymbol{v})$,

2º $\forall k \in \mathbb{R}, \forall \boldsymbol{u} \in V, F(k\boldsymbol{u}) = kF(\boldsymbol{u})$.

Definition 3. 17. *Let* $F\colon V \to U$ *be a linear mapping. The kernel of F, written KerF, is the set of elements in V which map into*

$$\boldsymbol{0} \in U\colon KerF = \{\boldsymbol{v} \in V\colon F(\boldsymbol{v}) = \boldsymbol{0}\}.$$

Definition 3. 18. *Let* $F\colon V \to U$ *be a linear mapping. The image of F, written ImF, is the set of image points in U*:

$$ImF = \{\boldsymbol{u} \in U\colon \exists \boldsymbol{v} \in V \text{ for which } F(\boldsymbol{v}) = \boldsymbol{u}\}.$$

Definition 3. 19. *The rank of a linear map* $F\colon V \to U$ *is defined to be the dimension of its image, i. e.,*

$$rankF = dim(ImF).$$

Definition 3. 20. *The nullity of a linear map* $F\colon V \to U$ *is defined to be the dimension of its kernel, i. e.,*

$$nullityF = dim(KerF).$$

Definition 3. 21. *A linear mapping* $F\colon V \to U$ *is said to be singular if the image of some nonzero vector under F is* $\boldsymbol{0}$, *i. e., if there exists* $\boldsymbol{v} \in V$ *for which* $\boldsymbol{v} \neq \boldsymbol{0}$ *but* $F(\boldsymbol{v}) = \boldsymbol{0}$. *Thus* $F\colon V \to U$ *is nonsingular if only* $\boldsymbol{0} \in V$ *maps into* $\boldsymbol{0} \in U$ *or equivalently, if its kernel consists only of the zero vector,* $KerF = \{\boldsymbol{0}\}$.

Definition 3. 22. *A mapping* $F\colon V \to U$ *is called an isomorphism if F is linear and if F is bijective, i. e., if F is one-to-one and onto.*

Definition 3. 23. *A vector space V is said to be isomorphic to a vector space U, written* $V \simeq U$, *if there is an isomorphism* $F\colon V \to U$.

Definition 3. 24. *An inner product on a vector space V is a function that assigns a number* $\langle \boldsymbol{u}, \boldsymbol{v}\rangle$ *to every pair* $\boldsymbol{u}, \boldsymbol{v}$ *of vectors in V in such a way that the following axioms are satisfied*

(P_1) $\langle \boldsymbol{u}, \boldsymbol{v}\rangle$ *is a real number,* $\forall \boldsymbol{u}, \boldsymbol{v} \in V$,

(P_2) $\langle \boldsymbol{u}, \boldsymbol{v}\rangle = \langle \boldsymbol{v}, \boldsymbol{u}\rangle, \forall \boldsymbol{u}, \boldsymbol{v} \in V$,

(P_3) $\langle \boldsymbol{u} + \boldsymbol{v}, \boldsymbol{w}\rangle = \langle \boldsymbol{u}, \boldsymbol{w}\rangle + \langle \boldsymbol{v}, \boldsymbol{w}\rangle, \forall \boldsymbol{u}, \boldsymbol{v}, \boldsymbol{w} \in V$,

(P_4) $\langle r\boldsymbol{u}, \boldsymbol{v}\rangle = r\langle \boldsymbol{u}, \boldsymbol{v}\rangle, \forall \boldsymbol{u}, \boldsymbol{v} \in V \wedge \forall r \in \mathbb{R}$,

(P_5) $\langle \boldsymbol{u}, \boldsymbol{u}\rangle > 0, \forall \boldsymbol{u} \neq \boldsymbol{0} \in V$.

A vector space V with an inner product $\langle\ ,\ \rangle$ *will be called an inner product space.*

Remark 3. 25. *A real inner product space* $\mathbb{R}^n$ *with the dot product as inner product* $\langle \boldsymbol{u}, \boldsymbol{v}\rangle = \boldsymbol{u} \cdot \boldsymbol{v}$ *is called a Euclidean space.*

Definition 3. 26. *If* $\langle\ ,\ \rangle$ *is an inner product on a space V, the norm or length* $\|\boldsymbol{v}\|$ *of a vector* $\boldsymbol{v} \in V$, *is defined by* $\|\boldsymbol{v}\| = \langle \boldsymbol{v}, \boldsymbol{v}\rangle^{1/2}$.

Definition 3. 27. *Two vectors* $\boldsymbol{u}$ *and* $\boldsymbol{v}$ *in an inner product space V are said to be orthogonal, written* $\boldsymbol{u} \perp \boldsymbol{v}$, *if* $\langle \boldsymbol{u}, \boldsymbol{v}\rangle = 0$.

Definition 3. 28. *Two vectors* $\boldsymbol{u}$ *and* $\boldsymbol{v}$ *in an inner product space V are said to be orthogonal, written* $\boldsymbol{u} \perp \boldsymbol{v}$, *if* $\langle \boldsymbol{u}, \boldsymbol{v}\rangle = 0$.

Definition 3. 29. *A set* $\{\boldsymbol{e}_1, \boldsymbol{e}_2, \ldots, \boldsymbol{e}_n\}$ *of vectors is called an orthogonal set of vectors if each* $\boldsymbol{e}_i \neq \boldsymbol{0}$, $(1 \leq i \leq n)$ *and* $\langle \boldsymbol{e}_i, \boldsymbol{e}_j\rangle = 0, \forall i \neq j$.

Definition 3. 30. *If, in addition,* $\|\boldsymbol{e}_i\| = 1, \forall i$, *the set* $\{\boldsymbol{e}_1, \boldsymbol{e}_2, \ldots, \boldsymbol{e}_n\}$ *is called an orthonormal set.*

Definition 3. 31. *Let U be a subspace of an inner product space V. The orthogonal complement* $U^{\perp}$ *of U in V is defined by*

$$U^{\perp} = \{\boldsymbol{v}\colon \boldsymbol{v} \in V, \langle \boldsymbol{v}, \boldsymbol{u}\rangle = 0, \forall \boldsymbol{u} \in U\}.$$

Definition 3. 32. *Let* $S = \{\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_n\}$ *be a set of vectors in an inner product space V, then S is said to be orthogonal if each of its vectors are nonzero and if its vectors are mutually orthogonal, i. e., if* $\langle \boldsymbol{v}_i, \boldsymbol{v}_i\rangle \neq 0$ *but* $\langle \boldsymbol{v}_i, \boldsymbol{v}_j\rangle = 0, \forall i \neq j$.

Definition 3. 33. *A permutation* σ, *with a notation* $\sigma = (j_1, j_2, \ldots, j_n)$, *where* $j_i = \sigma(i)$, $(1 \leq i \leq n)$ *of a finite set* $\mathfrak{I}$ *is a one-to-one mapping of* $\mathfrak{I}$ *into itself.*

Definition 3. 34. *In the particular case where* $\mathfrak{I} = \{1, 2, \ldots, n\}$, *we write* $\mathfrak{I} = S_n$, *then* S_n *is called the symmetric group of degree n.*

Definition 3. 35. *The alternating group of degree n, denoted by* A_n, *is a set of even permutation in* S_n.

Definition 3. 36. *By an inversion in* σ *we mean a pair of integers* (i, k) *such that* $i > k$, *but i precedes k in* σ.

Definition 3. 37. *The sign of* σ, *written* $sgn\sigma$, *is defined by* $sgn\sigma = (-1)^k$, *where k is a total number of inversions in* σ.

Definition 3. 38. *A permutation σ is said to be even or odd according as there is an even or odd total number of inversions in σ.*

Definition 3. 39. *A transposition is a permutation τ which interchanges two numbers, i and j > i, and lives the other numbers fixed*:

$$\tau = (1\ 2\ \ldots\ (i - 1)j(i + 1)\ \ldots\ (j - 1)i(j + 1)\ \ldots\ n).$$

Definition 3. 40. *If a_i, $(1 \le i \le m)$ are distinct integers in S_n, $(a_1, a_2, \ldots, a_m)$ stands for the permutation that maps each integer in S_n - $\{a_1, a_2, \ldots, a_m\}$ to itself, and maps $a_1 \to a_2$, $a_2 \to a_3$, ..., $a_{m-1} \to a_m$, $a_m \to a_1$, we call such a permutation an orbit $\varnothing$ of length m.*

Definition 3. 41. *Let n be a positive integer and σ be a permutation, such that $\sigma^n = \iota$, where ι is an identity permutation, then the permutation σ is of order n.*

Let G be a finite group, of order $|G|$.

Definition 3. 42. *The center of G is the set of elements which commute with all elements of G.*

It is a normal subgroup of G. The center of G equals G if and only if G is abelian.

Definition 3. 43. *Two elements g_1 and g_2 of the group G are conjugate, if there is an element $h \in G$ such that*

$$hg_1h^{-1} = g_2.$$

Definition 3. 44. *The conjugacy class of an element $g \in G$ is the set of elements conjugate to g.*

Definition 3. 45. *In the same way, two subgroups H_1 and H_2 of G are conjugate, if there is an element $h \in G$ such that*

$$hH_1h^{-1} = H_2.$$

Definition 3. 46. *A subgroup H is normal if there is no other subgroup conjugate to it.*

Definition 3. 47. *G is a simple group if it contains no normal subgroup other than G and the trivial subgroup.*

Definition 3. 48. *The commutator subgroup or derived subgroup of G,* $[G, G]$, *is the subgroup generated by all the commutators* $g_1g_2g_1^{-1}g_2^{-1}$.

It is a normal subgroup of G, the smallest such that the quotient group is abelian. $[G, G]$ is trivial if and only if G is abelian.

Definition 3. 49. *G is a perfect group if* $[G, G] = G$.

Definition 3. 50. *The derived series of G is the series of subgroups $N_1 \supset N_2 \supset \cdots \supset N_k$, where $N_1 = [G, G]$ (commutator subgroup), and $N_i = [N_{i-1}, N_{i-1}]$ for i > 1.*

And the series stops at N_k such that $N_k = [N_k, N_k]$. All the terms N_i in the derived series are normal subgroups of G.

Definition 3. 51. *G is a solvable group if the derived series stops at the trivial subgroup.*

Definition 3. 52. *The exponent of G is the lcm of orders of all elements of G. It divides $|G|$.*

Definition 3. 53. *The lower central series of G is the series of subgroups $N_1 \supset N_2 \supset \cdots \supset N_k$, where $N_1 = [G, G]$, and $N_i = [G, N_{i-1}]$ for i > 1.*

And the series stops at N_k such that $N_k = [G, N_k]$.

All the terms N_i in the lower central series are normal subgroups of G.

Definition 3. 54. *The normal closure of a subgroup H is the subgroup N of G generated by elements in H and all its conjugates.*

N is a normal subgroup of G.

Definition 3. 55. *The normalizer of a subgroup H is the largest subgroup N of G containing H, such that H is a normal subgroup of N.*

Let p be a prime factor of $|G|$.

Definition 3. 56. *A p-Sylow subgroup of G is a maximal subgroup H whose order is a power of p.*

$|H|$ equals the largest power of p dividing $|G|$.

A p-Sylow subgroup needs not to be normal, but all p-Sylow subgroups are conjugate to each other.

Definition 3. 57. *The upper central series of G is the series of subgroups $N_1 \supset N_2 \supset \cdots \supset N_k$, where N_k is the center of G, and N_i is the center of the quotient group G/N_{i+1} for i < k.*

And either $N_1 = G$, or G/N_1 has trivial center. All the terms N_i in the upper central series are normal subgroups of G.

Definition 3. 58. *G is a nilpotent group if $N_1 = G$.*

An essential role for every chemical equation plays its stability. For that goal here is introduced a new criterion for stability of chemical equations.

Let an arbitrary chemical reaction is given in its algebraically free form

$$\sum_{j=1}^{n} x_jM_j \to 0, \tag{3. 1}$$

where x_j, $(1 \le j \le n)$ are required rational coefficients and M_j, $(1 \le j \le n)$ are molecules, then its stability array can be constructed on this manner

$$\begin{array}{cccccc}
x_n & x_{n-1} & x_{n-2} & \cdots & x_2 & x_1 \\
x_1 & x_2 & x_3 & \cdots & x_{n-1} & x_n \\
a_1 & a_2 & a_3 & \cdots & a_{n-1} & 0 \\
b_1 & b_2 & b_3 & \ldots & 0 & 0 \\
\vdots & & & & & \\
z_1 & z_2 & 0 & \ldots & 0 & 0
\end{array}$$

The elements in third rows are calculated as shown below

$$a_1 = (x_1x_{n-1} - x_2x_n)/x_1,$$
$$a_2 = (x_1x_{n-2} - x_3x_n)/x_1,$$
$$a_3 = (x_1x_{n-3} - x_4x_n)/x_1,$$
$$\vdots$$
$$a_{n-1} = (x_1x_1 - x_nx_n)/x_1,$$

while the elements in fourth row are

$$b_1 = (a_1x_2 - x_1a_2)/a_1,$$
$$b_2 = (a_1x_3 - x_1a_3)/a_1,$$
$$b_3 = (a_1x_4 - x_1a_4)/a_1,$$
$$\vdots$$
$$b_{n-2} = (a_1x_{n-1} - x_1a_{n-1})/a_1,$$
$$\vdots$$

and the elements in the last row are

$$z_1 = (q_1p_2 - p_1q_2)/q_1$$

and

$$z_2 = (q_1p_3 - p_1q_3)/q_1.$$

Definition 3. 59. *For chemical reaction* (3. 1) *to be stable the primary requirement is the elements in first column of the above array to have the same sign.*

Other results for stability criteria are obtained in works [15, 16] for some general classes of complex vector functional equations [17-19].

Let $\mathscr{X}$ be a finite set of molecules.

Definition 3. 60. *A chemical reaction on* $\mathscr{X}$ *is a pair of formal linear combinations of elements of* $\mathscr{X}$, *such that*

$$\rho: \sum_{j=1}^{r} a_{ij}x_j \to \sum_{j=1}^{s} b_{ij}y_j,\ (1 \le i \le m) \quad (3.\ 2)$$

with $a_{ij}, b_{ij} \ge 0$.

Definition 3. 61. *Chemical equation is a numerical quantification of a chemical reaction*.

In [20] is proved the following proposition.

Proposition 3. 62. *Any chemical equation may be presented in this algebraic form*

$$\sum_{j=1}^{s} x_j \prod_{i=1}^{m} \Psi^{i}{}_{a_{ij}} = \sum_{j=s+1}^{n} x_j \prod_{i=1}^{m} \Psi^{i}{}_{b_{ij}}, \quad (3.\ 3)$$

where x_j, $(1 \le j \le n)$ *are unknown rational coefficients,* $\Psi^{i}{}_{a_{ij}}$ and $\Psi^{i}{}_{b_{ij}}$, $(1 \le i \le m)$ *are chemical elements in reactants and products, respectively,* a_{ij} *and* b_{ij}, $(1 \le i \le m;\ 1 \le j \le n;\ m < n)$ *are numbers of atoms of elements* $\Psi^{i}{}_{a_{ij}}$ *and* $\Psi^{i}{}_{b_{ij}}$, *respectively, in j-th molecule.*

Definition 3. 63. *Each chemical reaction* ρ *has a domain*

$$Dom\rho = \{x \in \mathscr{X} \mid a_{ij} > 0\}. \quad (3.\ 4)$$

Definition 3. 64. *Each chemical reaction* ρ *has an image*

$$Im\rho = \{y \in \mathscr{X} \mid b_{ij} > 0\}. \quad (3.\ 5)$$

Definition 3. 65. *Chemical reaction* ρ *is generated for some* $x \in \mathscr{X}$, *if both* $a_{ij} > 0$ *and* $b_{ij} > 0$.

Definition 3. 66. *For the case as the previous definition, we say x is a generator* of ρ.

Definition 3. 67. *The set of generators of* ρ *is thus* $Dom\rho \cap Im\rho$.

Often chemical reactions are modeled like pairs of multisets, corresponding to integer stoichiometric constants.

Definition 3. 68. *A stoichiometrical space is a pair* $(\mathscr{X}, \mathscr{R})$, *where* $\mathscr{X}$ *is a set of chemical reactions on* $\mathscr{X}$. *It may be symbolized by an arc-weighted bipartite directed graph* $\Gamma(\mathscr{X}, \mathscr{R})$ *with vertex set* $\mathscr{X} \cup \mathscr{R}$, *arcs* $x \to \rho$ *with weight* a_{ij} *if* $a_{ij} > 0$, *and arcs* $\rho \to y$ *with weight* b_{ij} *if* $b_{ij} > 0$.

Let us now consider an arbitrary subset $\mathscr{A} \subseteq \mathscr{X}$.

Definition 3. 69. *A chemical reaction* ρ *may take place in a reaction combination composed of the molecules in* A *if and only if* $Dom\rho \subseteq \mathscr{A}$.

Definition 3. 70. *The collection of all possible reactions in the stoichiometrical space* $(\mathscr{X}, \mathscr{R})$, *that can start from* $\mathscr{A}$ *is given by*

$$\mathscr{R}_{\mathscr{A}} = \{\rho \in \mathscr{R} \mid Dom\rho \subseteq \mathscr{A}\}. \quad (3.\ 6)$$

4. REACTION OF TOBACCO COMBUSTION

Begin of this section we shall create with a theoretical approach, *i. e.*, we shall give a completely new method for balancing chemical reactions.

We shall do that, because most of the current chemical ways for balancing chemical reactions are out of order, or they are useless for complex reactions.

Theorem 4. 1. *Any chemical equation may be presented in a free algebraic form on this way*

$$\sum_{j=1}^{n} x_j \prod_{i=1}^{m} \Psi^{i}{}_{a_{ij}} = 0, \quad (4.\ 1)$$

where x_j, $(1 \le j \le n)$ *are unknown rational coefficients,* Ψ^{i}, $(1 \le i \le m)$ *are chemical* *elements and* a_{ij}, $(1 \le i \le m,\ 1 \le j \le n)$ *are atom numbers of i-th element* Ψ^i *in j-th molecule.*

Proof. Let there exists an arbitrary chemical equation from m distinct elements and n molecules

$$\sum_{j=1}^{n} x_j \boldsymbol{v}_j = 0, \qquad (4.\ 2)$$

where

$$\boldsymbol{v}_j = \Psi^1{}_{a_{1j}}\Psi^2{}_{a_{2j}} \cdots \Psi^m{}_{a_{mj}},\ (1 \le j \le n).$$

Then previous expression becomes

$$\sum_{j=1}^{n} \Psi^1{}_{a_{1j}}\Psi^2{}_{a_{2j}} \cdots \Psi^m{}_{a_{mj}} = 0. \qquad (4.\ 3)$$

If we write the above equation in a compact form, then immediately follows (4. 1). □

Theorem 4. 2. *The chemical equation* (4. 1) *reduces to the following matrix equation*

$$\boldsymbol{Ax} = \boldsymbol{0}, \quad (4.\ 4)$$

where $\boldsymbol{A} = [a_{ij}]_{m\times n}$, $(1 \le i \le m,\ 1 \le j \le n)$ *is a reaction matrix,* $\boldsymbol{x}^{\mathrm{T}} = (x_1, x_2, \ldots, x_n)$ *is a column vector of the coefficients* x_j, $(1 \le j \le n)$ *and* $\boldsymbol{0}^{\mathrm{T}} = (0, 0, \ldots, 0)$ *is a null column vector of order m, and* T *denotes transpose*.

Proof. If we develop the molecules of the reaction (4. 1) in an explicit form, then we obtain the reaction matrix $\boldsymbol{A}$ shown below

$$\begin{array}{ccccc}
 & \boldsymbol{v}_1 = \Psi^1{}_{a_{11}}\Psi^2{}_{a_{21}}\cdots\Psi^m{}_{a_{m1}} & \boldsymbol{v}_2 = \Psi^1{}_{a_{12}}\Psi^2{}_{a_{22}}\cdots\Psi^m{}_{a_{m2}} & \ldots & \boldsymbol{v}_n = \Psi^1{}_{a_{1n}}\Psi^2{}_{a_{2n}}\cdots\Psi^m{}_{a_{mn}} \\
\Psi^1 & a_{11} & a_{12} & \ldots & a_{1n} \\
\Psi^2 & a_{21} & a_{22} & \ldots & a_{2n} \\
\vdots & & & & \\
\Psi^m & a_{m1} & a_{m2} & \ldots & a_{mn}
\end{array}$$

From the above development we obtain that

$$\boldsymbol{v}_j = \sum_{i=1}^{m} a_{ij}\Psi^i,\ (1 \le j \le n). \qquad (4.\ 5)$$

If we substitute (4. 5) into (4. 2), follows

$$\sum_{j=1}^{n} x_j \sum_{i=1}^{m} a_{ij}\Psi^i = 0, \qquad (4.\ 6)$$

or

$$\sum_{i=1}^{m} \Psi^i \sum_{j=1}^{n} a_{ij}x_j = 0, \qquad (4.\ 7)$$

i.e.,

$$\sum_{j=1}^{n} a_{ij}x_j = 0,\ (1 \le i \le m). \qquad (4.\ 8)$$

Last equation if we present in a matrix form, actually we obtain (4. 4). □

According to [21], the deterministic approach is important, since it enables us to classify the chemical reaction as:

1º *impossible*, when the system (4. 8) is inconsistent.

2º *unique*, (within relative proportions) when the system (4. 8) has unique solution.

3º *non-unique*, when the system (4. 8) has an infinite number of solutions.

Last kind of the reactions exhibit infinite linearly independent solutions all of which satisfy the chemical balance, and yet they are not all chemically feasible solutions for a given set of experimental conditions. A unique solution is obtained by imposing a chemical constraint, namely, that reactants have to react only in certain proportions.

The coefficients satisfy three basic principles

- the law of conservation of atoms,
- the law of conservation of mass, and
- the time-independence of the reaction.

Theorem 4. 3. *Suppose that chemical equation* (4. 2) *is a vector space V over the field* ℝ *spanned by the vectors of the molecules* $\boldsymbol{v}_i$ $(1 \le i \le n)$. *If any set of m vectors of the molecules in V is linearly independent, then m* $\le$ *n.*

Proof. Let

$$V = \mathrm{span}\{\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_n\}.$$

We must show that every set $\{\boldsymbol{u}_1, \boldsymbol{u}_2, \ldots, \boldsymbol{u}_m\}$ of vectors in V with $m > n$ fails to be linearly independent. This is accomplished by showing that numbers $x_1, x_2, \ldots, x_m$ can be found, not all zero, such that

$$\sum_{j=1}^{m} x_j\boldsymbol{u}_j = x_1\boldsymbol{u}_1 + x_2\boldsymbol{u}_2 + \cdots + x_m\boldsymbol{u}_m = \boldsymbol{0}.$$

Since V is spanned by the vectors $\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_n$, each vector $\boldsymbol{u}_j$ can be expressed as a linear combination of $\boldsymbol{v}_i$

$$\boldsymbol{u}_j = a_{1j}\boldsymbol{v}_1 + a_{2j}\boldsymbol{v}_2 + \cdots + a_{nj}\boldsymbol{v}_n = \sum_{i=1}^{n} a_{ij}\boldsymbol{v}_j.$$

Substituting these expressions into the preceding equation gives

$$\boldsymbol{0} = \sum_{j=1}^{m} x_j \Big(\sum_{i=1}^{n} a_{ij}\boldsymbol{v}_j\Big) = \sum_{i=1}^{n} \Big(\sum_{j=1}^{m} a_{ij}x_j\Big)\boldsymbol{v}_i.$$

This is certainly the case if each coefficient of $\boldsymbol{v}_i$ is zero, *i. e.*, if

$$\sum_{j=1}^{m} a_{ij}x_j = 0,\ (1 \le i \le n).$$

But this is a system of n equations in the m variables $x_1, x_2, \dots, x_m$, so because $m > n$, it has a nontrivial solution. This is what we wanted.□

Now we shall prove the following results.

Theorem 4. 4. *Let U be a subset of a vector space V of the chemical equation* (4. 2) *over the field* $\mathbb{R}$. *Then U is a subspace of V if and only if it satisfies the following conditions*

$$\mathbf{0} \in U,\ \mathbf{0} \text{ is the zero vector of } V, \quad (4.\ 9)$$

$$\text{If } \boldsymbol{u}_1, \boldsymbol{u}_2 \in U, \text{ then } (\boldsymbol{u}_1 + \boldsymbol{u}_2) \in U, \quad (4.\ 10)$$

$$\text{If } \boldsymbol{u} \in U, \text{ then } a\boldsymbol{u} \in U,\ \forall a \in \mathbb{R}. \quad (4.\ 11)$$

Proof. If U is a subspace of V of the chemical equation (4. 1), it is clear by axioms (A_1) and (S_1), that the sum of two vectors in U is again in U and that any scalar multiple of a vector in U is again in U. By other words, U is *closed* under the vector addition and scalar multiplication of V. The nice part is that the converse is also true, *i. e.*, if U is closed under these operations, then all the other axioms are automatically satisfied. For instance, axiom (A_2) asserts that holds $\boldsymbol{u}_1 + \boldsymbol{u}_2 = \boldsymbol{u}_2 + \boldsymbol{u}_1$, $\forall \boldsymbol{u}_1, \boldsymbol{u}_2 \in U$. But, this is clear because the equation is already true in V, and U uses the same addition as V. Similarly, axioms (A_3), (S_2), (S_3), (S_4) and (S_5) hold automatically in U, because they are true in V. All that remains is to verify axioms (A_4) and (A_5).

If (4. 9), (4. 10) and (4. 11) hold, then axiom (A_4) follows from (4. 9) and axiom (A_5) follows from (4. 11), because $-\boldsymbol{u} = (-1)\boldsymbol{u}$ lies in U, $\forall \boldsymbol{u} \in U$. Hence U is a subspace by the above discussion. Conversely, if U is a subspace it is closed under addition and scalar multiplication, and this gives (4. 10) and (4. 11). If $\boldsymbol{z}$ denotes the zero vector of U, then $\boldsymbol{z} = 0\boldsymbol{z}$ in U. But, $0\boldsymbol{z} = \mathbf{0}$ in V, so $\mathbf{0} = \boldsymbol{z}$ lies in U. This proves (4. 9). □

Remark 4. 5. *If U is a subspace of V of the chemical equation* (4. 2) *over the field, then the above proof shows that U and V share the same zero vector. Also, if* $\boldsymbol{u} \in U$, *then* $-\boldsymbol{u} = (-1)\boldsymbol{u} \in U$, *i. e., the negative of a vector in U is the same as its negative in V.*

Proposition 4. 6. *If V is any vector space V of the chemical equation* (4. 2) *over the field* $\mathbb{R}$, *then* $\{\mathbf{0}\}$ *and V are subspaces of V.*

Proof. $U = V$ clearly satisfies the conditions of the Theorem 4. 4. As to $U = \{\mathbf{0}\}$, it satisfies the conditions because

$$\mathbf{0} + \mathbf{0} = \mathbf{0} \text{ and } a\mathbf{0} = \mathbf{0},\ \forall a \in \mathbb{R}. \qquad \square$$

Remark 4. 7. *The vector space* $\{\mathbf{0}\}$ *is called the zero subspace of V of the chemical equation* (4. 2) *over the field* $\mathbb{R}$. *Since all zero subspaces look alike, we speak of the zero vector space and denote it by* 0. *It is the unique vector space containing just one vector.*

Proposition 4. 8. *If* $\boldsymbol{v}$ *is a vector of some molecule in a vector space V of the chemical equation* (4. 2) *over the field* $\mathbb{R}$, *then the set* $\mathbb{R}\boldsymbol{v} = \{a\boldsymbol{v}, \forall a \in \mathbb{R}\}$ *of all scalar multiplies of* $\boldsymbol{v}$ *is a subspace of V.*

Proof. Since $\mathbf{0} = 0\boldsymbol{v}$, it is clear that $\mathbf{0}$ lies in $\mathbb{R}\boldsymbol{v}$. Given two vectors $a\boldsymbol{v}$ and $b\boldsymbol{v}$ in $\mathbb{R}\boldsymbol{v}$, their sum $a\boldsymbol{v} + b\boldsymbol{v} = (a + b)\boldsymbol{v}$ is also a scalar multiple of $\boldsymbol{v}$ and so lies in $\mathbb{R}\boldsymbol{v}$. Therefore $\mathbb{R}\boldsymbol{v}$ is closed under addition. Finally, given $a\boldsymbol{v}$, $r(a\boldsymbol{v}) = (ra)\boldsymbol{v}$ lies in $\mathbb{R}\boldsymbol{v}$, so $\mathbb{R}\boldsymbol{v}$ is closed under scalar multiplication. If we take into account the Theorem 4. 4, immediately follows the statement of the proposition. □

Proposition 4. 9. *Let* $\boldsymbol{A} \in \mathbb{R}^{m\times n}$. *The set* $ImgU = \{\boldsymbol{Ax}: \boldsymbol{x} \in \mathbb{R}^n\}$, *called the range or image of the matrix* $\boldsymbol{A}$ *is a subspace of* $\mathbb{R}^m$.

Proof. Note first that U is in fact a subset of $\mathbb{R}^m$, because $\boldsymbol{A}$ is $m\times n$. Each vector in U is of the form $\boldsymbol{Ax}$ for some vector $\boldsymbol{x} \in \mathbb{R}^n$. To apply the Theorem 4. 4, note that $\mathbf{0} = \boldsymbol{A}\mathbf{0}$ has the required form, so $\mathbf{0}$ lies in U. Similarly, the equation

$$\boldsymbol{Ax} + \boldsymbol{Ay} = \boldsymbol{A}(\boldsymbol{x} + \boldsymbol{y})$$

and

$$r(\boldsymbol{Ax}) = \boldsymbol{A}(r\boldsymbol{x})$$

show that sums and scalar multiplies of vector in U again have the required form. Hence U is a subspace of $\mathbb{R}^m$. □

Proposition 4. 10. *Let* $\boldsymbol{A} \in \mathbb{R}^{m\times n}$. *The set*

$$nullU = KerU = \{\boldsymbol{Ax} = \mathbf{0}: \boldsymbol{x} \in \mathbb{R}^n\},$$

called the null space or kernel of the matrix $\boldsymbol{A}$ *is a subspace of* $\mathbb{R}^n$.

Proof. Here U consists of all columns $\boldsymbol{x}$ in $\mathbb{R}^n$ satisfying the condition that $\boldsymbol{Ax} = \mathbf{0}$. Since $\boldsymbol{A}\mathbf{0} = \mathbf{0}$, it is clear that $\mathbf{0}$ lies in U. If $\boldsymbol{x}$ and $\boldsymbol{y}$ both lie in U, then

$$\boldsymbol{A}(\boldsymbol{x} + \boldsymbol{y}) = \boldsymbol{Ax} + \boldsymbol{Ay} = \mathbf{0} + \mathbf{0} = \mathbf{0}.$$

This shows that $\boldsymbol{x} + \boldsymbol{y}$ qualifies for membership in U, so U is closed under addition. Similarly,

$$\boldsymbol{A}(r\boldsymbol{x}) = r(\boldsymbol{Ax}) = r\mathbf{0} = \mathbf{0},$$

so $r\boldsymbol{x}$ lies in U. Thus U is closed under scalar multiplication and is a subspace of $\mathbb{R}^n$. □

Theorem 4. 11. *Let*

$$U = span\{\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_n\}$$

in a vector space V of the chemical equation (4. 2) *over the field* $\mathbb{R}$. *Then,*

U is a subspace of V containing each of $\boldsymbol{v}_i$, $(1 \le i \le n)$, (4. 12)

U is the smallest subspace in the sense that any subspace of V that contains each of $\boldsymbol{v}_i$, $(1 \le i \le n)$ *must contain U.* (4. 13)

Proof. First we shall proof (4. 12).

Clearly

$$\boldsymbol{0} = 0\boldsymbol{v}_1 + 0\boldsymbol{v}_2 + \cdots + 0\boldsymbol{v}_n$$

belongs to U. If

$$\boldsymbol{v} = a_1\boldsymbol{v}_1 + a_2\boldsymbol{v}_2 + \cdots + a_n\boldsymbol{v}_n$$

and

$$\boldsymbol{w} = b_1\boldsymbol{v}_1 + b_2\boldsymbol{v}_2 + \cdots + b_n\boldsymbol{v}_n$$

are two members of U and $a \in U$, then

$$\boldsymbol{v} + \boldsymbol{w} = (a_1 + b_1)\boldsymbol{v}_1 + (a_2 + b_2)\boldsymbol{v}_2 + \cdots + (a_n + b_n)\boldsymbol{v}_n,$$

$$a\boldsymbol{v} = (aa_1)\boldsymbol{v}_1 + (aa_2)\boldsymbol{v}_2 + \cdots + (aa_n)\boldsymbol{v}_n,$$

so both $\boldsymbol{v} + \boldsymbol{w}$ and $a\boldsymbol{v}$ lie in U. Hence U is a subspace of V. It contains each of $\boldsymbol{v}_i$, $(1 \le i \le n)$. For instance,

$$\boldsymbol{v}_2 = 0\boldsymbol{v}_1 + 1\boldsymbol{v}_2 + 0\boldsymbol{v}_3 + \cdots + 0\boldsymbol{v}_n.$$

This proves (4. 12).

Now, we shall prove (4. 13). Let W be subspace of V that contains each of $\boldsymbol{v}_i$, $(1 \le i \le n)$. Since W is closed under scalar multiplication, each of $a_i\boldsymbol{v}_i$, $(1 \le i \le n)$ lies in W for any choice of a_i, $(1 \le i \le n)$ in $\mathbb{R}$. But, then $a_i\boldsymbol{v}_i$, $(1 \le i \le n)$ lies in W, because W is closed under addition. This means that W contains every member of U, which proves (4. 13). □

Theorem 4. 12. *The intersection of any number of subspaces of a vector space V of the chemical equation* (4. 2) *over the field* $\mathbb{R}$ *is a subspace of V.*

Proof. Let $\{W_i: i \in \boldsymbol{I}\}$ be a collection of subspaces of V and let $W = \cap\ (W_i: i \in \boldsymbol{I})$. Since each W_i is a subspace, then $\boldsymbol{0} \in W_i$, $\forall i \in \boldsymbol{I}$. Thus $\boldsymbol{0} \in W$. Assume $\boldsymbol{u}, \boldsymbol{v} \in W$. Then, $\boldsymbol{u}, \boldsymbol{v} \in W_i$, $\forall i \in \boldsymbol{I}$. Since each W_i is a subspace, then $(a\boldsymbol{u} + b\boldsymbol{v}) \in W_i$, $\forall i \in \boldsymbol{I}$. Therefore $(a\boldsymbol{u} + b\boldsymbol{v}) \in W$. Thus W is a subspace of V of the chemical equation (4. 2). □

Theorem 4. 13. *The union* $W_1 \cup W_2$ *of subspaces of a vector space V of the chemical equation* (4. 2) *over the field* $\mathbb{R}$ *need not be a subspace of V.*

Proof. Let $V = \mathbb{R}^2$ and let $W_1 = \{(a, 0): a \in \mathbb{R}\}$ and $W_2 = \{(0, b): b \in \mathbb{R}\}$. That is, W_1 is the x-axis and W_2 is the y-axis in $\mathbb{R}^2$. Then W_1 and W_2 are subspaces of V of the chemical equation (4. 2). Let $\boldsymbol{u} = (1, 0)$ and $\boldsymbol{v} = (0, 1)$. Then the vectors $\boldsymbol{u}$ and $\boldsymbol{v}$ both belong to the union $W_1 \cup W_2$, but $\boldsymbol{u} + \boldsymbol{v} = (1, 1)$ does not belong to $W_1 \cup W_2$. Thus $W_1 \cup W_2$ is not a subspace of V. □

Theorem 4. 14. *The homogeneous system of linear equations* (4. 8), *obtained from the chemical equation* (4. 1), *in n unknowns* $x_1, x_2, \ldots, x_n$ *over the field* $\mathbb{R}$ *has a solution set W, which is a subspace of the vector space* $\mathbb{R}^n$.

Proof. The system (4. 8) is equivalent to the matrix equation (4. 4). Since $\boldsymbol{A0} = \boldsymbol{0}$, the zero vector $\boldsymbol{0} \in W$. Assume $\boldsymbol{u}$ and $\boldsymbol{v}$ are vectors in W, *i. e.*, $\boldsymbol{u}$ and $\boldsymbol{v}$ are solutions of the matrix equation (4. 4). Then $\boldsymbol{Au} = \boldsymbol{0}$ and $\boldsymbol{Av} = \boldsymbol{0}$.

Therefore, $\forall a, b \in \mathbb{R}$, we have

$$\boldsymbol{A}(a\boldsymbol{u} + b\boldsymbol{v}) = a\boldsymbol{Au} + b\boldsymbol{Av} = a\boldsymbol{0} + b\boldsymbol{0} = \boldsymbol{0} + \boldsymbol{0} = \boldsymbol{0}.$$

Hence $a\boldsymbol{u} + b\boldsymbol{v}$ is a solution of the matrix equation (4. 4), *i. e.*, $a\boldsymbol{u} + b\boldsymbol{v} \in W$. Thus W is a subspace of $\mathbb{R}^n$. □

Theorem 4. 15. *If S is a subset of the vector space V of the chemical equation* (4. 2) *over the field* $\mathbb{R}$, *then*

1° *the set span*$\{S\}$ *is a subspace of V of the chemical equation* (4. 2) *over the field* $\mathbb{R}$ *which contains S.*

2° *span*$\{S\} \subseteq W$, *if W is any subspace of V of the chemical equation* (4. 2) *over the field* $\mathbb{R}$ *containing S.*

Proof. 1°. If $S = \varnothing$, then span$\{S\} = \{\boldsymbol{0}\}$, which is a subspace of V containing the empty set $\varnothing$. Now assume $S \ne \varnothing$. If $\boldsymbol{v} \in S$, then $1\boldsymbol{v} = \boldsymbol{v} \in$ span$\{S\}$, therefore S is a subset of span$\{S\}$. Also, span$\{S\} \ne \varnothing$ because $S \ne \varnothing$. Now assume $\boldsymbol{v}, \boldsymbol{w} \in$ span$\{S\}$; say

$$\boldsymbol{v} = a_1\boldsymbol{v}_1 + \cdots + a_m\boldsymbol{v}_m$$

and

$$\boldsymbol{v} = b_1\boldsymbol{w}_1 + \cdots + b_n\boldsymbol{w}_n$$

where $\boldsymbol{v}_i, \boldsymbol{w}_j \in S$ and a_i, b_j are scalars.

Then

$$\boldsymbol{v} + \boldsymbol{w} = a_1\boldsymbol{v}_1 + \cdots + a_m\boldsymbol{v}_m + b_1\boldsymbol{w}_1 + \cdots + b_n\boldsymbol{w}_n$$

and for any scalar k,

$$k\boldsymbol{v} = k(a_1\boldsymbol{v}_1 + \cdots + a_m\boldsymbol{v}_m) = ka_1\boldsymbol{v}_1 + \cdots + ka_m\boldsymbol{v}_m$$

belong to span$\{S\}$ because each is a linear combination of vectors in S. Thus span$\{S\}$ is a subspace of V of the chemical equation (4. 2) over the field $\mathbb{R}$ which contains S.

2º. If $S = \varnothing$, then any subspace W contains S, and span$\{S\} = \{\mathbf{0}\}$ is contained in W. Now assume $S \neq \varnothing$ and assume $\boldsymbol{v}_i \in S \subset W$, $(1 \leq i \leq m)$. Then all multiples $a_i\boldsymbol{v}_i \in W$, $(1 \leq i \leq m)$ where $a_i \in \mathbb{R}$, and therefore the sum $(a_1\boldsymbol{v}_1 + \cdots + a_m\boldsymbol{v}_m) \in W$. That is, W contains all linear combinations of elements of S. Thus, span$\{S\} \subseteq W$, as claimed. □

Proposition 4. 16. *If W is a subspace of V of the chemical equation* (4. 2) *over the field $\mathbb{R}$, then span$\{W\} = W$.*

Proof. Since W is a subspace of V of the chemical equation (4. 2) over the field $\mathbb{R}$, W is closed under linear combinations. Hence span$\{W\} \subseteq W$. But $W \subseteq$ span$\{W\}$. Both inclusions yield span$\{W\} = W$. □

Proposition 4. 17. *If S is a subspace of V of the chemical equation* (4. 2) *over the field $\mathbb{R}$, then span$\{$span$\{S\}\}$ = span$\{S\}$.*

Proof. Since span$\{S\}$ is a subspace of V, the above Propositions 4. 16 implies that span$\{$span$\{S\}\}$ = span$\{S\}$. □

Proposition 4. 18. *If S and T are subsets of a vector space V of the chemical equation* (4. 2) *over the field $\mathbb{R}$, such that $S \subseteq T$, then span$\{S\} \subseteq$ span$\{T\}$.*

Proof. Assume $\boldsymbol{v} \in$ span$\{S\}$. Then

$$\boldsymbol{v} = a_1\boldsymbol{u}_1 + \cdots + a_r\boldsymbol{u}_r,$$

where $a_i \in \mathbb{R}$, $(1 \leq i \leq r)$ and $\boldsymbol{u}_i \in S$, $(1 \leq i \leq r)$. But $S \subseteq T$, therefore every $\boldsymbol{u}_i \in T$, $(1 \leq i \leq r)$. Thus $\boldsymbol{v} \in$ span$\{T\}$. Accordingly, span$\{S\} \subseteq$ span$\{T\}$. □

Proposition 4. 19. *The span$\{S\}$ is the intersection of all the subspaces of a vector space V of the chemical equation* (4. 2) *over the field $\mathbb{R}$ which contains S.*

Proof. Let $\{W_i\}$ be the collection of all subspaces of a vector space V of the chemical equation (4. 2) containing S, and let $W = \cap W_i$. Since each W_i is a subspace of V, the set W is a subspace of V. Also, since each W_i contains S, the intersection W contains S. Hence span$\{S\} \subseteq W$. On the other hand, span$\{S\}$ is a subspace of V containing S. So span$\{S\} = W_k$ for some k. Then $W \subseteq W_k =$ span$\{S\}$. Both inclusions give span$\{S\} = W$. □

Proposition 4. 20. *If span$\{S\}$ = span$\{S \cup \{\mathbf{0}\}\}$, then one may delete the zero vector from any spanning set.*

Proof. By Proposition 4. 18, span$\{S\} \subseteq$ span$\{S \cup \{\mathbf{0}\}\}$. Assume $\boldsymbol{v} \in$ span$\{S \cup \{\mathbf{0}\}\}$, say

$$\boldsymbol{v} = a_1\boldsymbol{u}_1 + \cdots + a_n\boldsymbol{u}_n + b\cdot\mathbf{0}$$

where $a_i, b \in \mathbb{R}$, $(1 \leq i \leq n)$ and $\boldsymbol{u}_i \in S$, $(1 \leq i \leq n)$. Then

$$\boldsymbol{v} = a_1\boldsymbol{u}_1 + \cdots + a_n\boldsymbol{u}_n,$$

and so $\boldsymbol{v} \in$ span$\{S\}$. Thus span$\{S \cup \{\mathbf{0}\}\} \subseteq$ span$\{S\}$. Both inclusions give span$\{S\}$ = span$\{S \cup \{\mathbf{0}\}\}$. □

Proposition 4. 21. *If the vectors $\boldsymbol{v}_i \in V$, $(1 \leq i \leq n)$ span a vector space V of the chemical equation* (4. 2) *over the field $\mathbb{R}$, then for any vector $\boldsymbol{w} \in V$, the vectors $\boldsymbol{w}$, $\boldsymbol{v}_i$,* $(1 \leq i \leq n)$ *span V.*

Proof. Let $\boldsymbol{v} \in V$. Since the $\boldsymbol{v}_i$, $(1 \leq i \leq n)$ span V, there exist scalars a_i, $(1 \leq i \leq n)$ such that

$$\boldsymbol{v} = a_1\boldsymbol{v}_1 + \cdots + a_n\boldsymbol{v}_n + 0\boldsymbol{w}.$$

Thus $\boldsymbol{w}$, $\boldsymbol{v}_i$, $(1 \leq i \leq n)$ span V. □

Proposition 4. 22. *If $\boldsymbol{v}_i$,* $(1 \leq i \leq n)$ *span a vector space V of the chemical equation* (4. 2) *over the field $\mathbb{R}$, and for $k > 1$, the vector $\boldsymbol{v}_k$ is a linear combination of the preceding vectors $\boldsymbol{v}_i$,* $(1 \leq i \leq k - 1)$ *then $\boldsymbol{v}_i$ without $\boldsymbol{v}_k$ span V, i. e.,*

$$\text{span}\{\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_{k-1}, \boldsymbol{v}_{k+1}, \ldots, \boldsymbol{v}_n\} = V.$$

Proof. Let $\boldsymbol{v} \in V$. Since the $\boldsymbol{v}_i$, $(1 \leq i \leq n)$ span V, there exist scalars a_i, $(1 \leq i \leq n)$ such that

$$\boldsymbol{v} = a_1\boldsymbol{v}_1 + \cdots + a_n\boldsymbol{v}_n.$$

Since $\boldsymbol{v}_k$ is a linear combination of $\boldsymbol{v}_i$, $(1 \leq i \leq k - 1)$ there exist scalars b_i, $(1 \leq i \leq k - 1)$ such that

$$\boldsymbol{v}_k = b_1\boldsymbol{v}_1 + \cdots + a_{k-1}\boldsymbol{v}_{k-1}.$$

Thus

$$\boldsymbol{v} = a_1\boldsymbol{v}_1 + \cdots + a_k\boldsymbol{v}_k + \cdots + a_n\boldsymbol{v}_n$$

$$= a_1\boldsymbol{v}_1 + \cdots + a_k(b_1\boldsymbol{v}_1 + \cdots + b_{k-1}\boldsymbol{v}_{k-1}) + \cdots + a_n\boldsymbol{v}_n$$

$$= (a_1 + a_kb_1)\boldsymbol{v}_1 + \cdots + (a_{k-1} + a_kb_{k-1})\boldsymbol{v}_{k-1}$$

$$+ a_{k+1}\boldsymbol{v}_{k+1} + \cdots + a_n\boldsymbol{v}_n.$$

Therefore,

$$\text{span}\{\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_{k-1}, \boldsymbol{v}_{k+1}, \ldots, \boldsymbol{v}_n\} = V. \quad □$$

Proposition 4. 23. *If W_i,* $(1 \leq i \leq k)$ *are subspaces of a vector space V of the chemical equation* (4. 2) *over the field $\mathbb{R}$, for which $W_1 \subset W_2 \subset \cdots \subset W_k$ and $W = W_1 \cup W_2 \cup \cdots \cup W_k$, then W is a subspace of V.*

Proof. The zero vector $\mathbf{0} \in W_1$, hence $\mathbf{0} \in W$. Assume $\boldsymbol{u}, \boldsymbol{v} \in W$. Then, there exist j_1 and j_2 such that $\boldsymbol{u} \in W_{j_1}$ and $\boldsymbol{v} \in W_{j_2}$. Let $j = \max(j_1, j_2)$. Then $W_{j_1} \subseteq W_j$ and $W_{j_2} \subseteq W$, and so $\boldsymbol{u}, \boldsymbol{v} \in W_j$. But W_j is a subspace. Therefore $(\boldsymbol{u} + \boldsymbol{v}) \in W_j$ and for any scalar s the multiple $s\boldsymbol{u} \in W_j$. Since $W_j \subseteq W$, we have $(\boldsymbol{u} + \boldsymbol{v})$, $s\boldsymbol{u} \in W$.

Thus W is a subspace of V. □

Proposition 4. 24. *If W_i, $(1 \le i \le k)$ are subspaces of a vector space V of the chemical equation* (4. 2) *over the field $\mathbb{R}$ and S_i, $(1 \le i \le k)$ span W_i, $(1 \le i \le k)$ then*

$$S = S_1 \cup S_2 \cup \cdots \cup S_k$$

spans W.

Proof. Let $\boldsymbol{v} \in W$. Then there exists j such that $\boldsymbol{v} \in W_j$. Then $\boldsymbol{v} \in \text{span}\{S_j\} \subseteq \text{span}\{S\}$. Therefore $W \subseteq \text{span}\{S\}$. But $S \subseteq W$ and W is a subspace. Hence $\text{span}\{S\} \subseteq W$. Both inclusions give $\text{span}\{S\} = W$, *i. e.*, S spans W. □

Theorem 4. 25. *Let $\{\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_n\}$ be a linearly independent set of vectors in a vector space V of the chemical equation* (4. 2) *over the field $\mathbb{R}$, then the following conditions*

1º *$\{\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_n\}$ is a linearly independent set,*

2º *$\boldsymbol{v}$ does not lie in $\{\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_n\}$, are equivalent for a vector $\boldsymbol{v}$ in V.*

Proof. Assume 1º is true and assume, if possible, that $\boldsymbol{v}$ lies in $\text{span}\{\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_n\}$, say,

$$\boldsymbol{v} = a_1\boldsymbol{v}_1 + \cdots + a_n\boldsymbol{v}_n.$$

Then

$$\boldsymbol{v} - a_1\boldsymbol{v}_1 + \cdots + a_n\boldsymbol{v}_n = \boldsymbol{0}$$

is a nontrivial linear combination, contrary to 1º. So 1º implies 2º. Conversely, assume that 2º holds and assume that

$$a\boldsymbol{v} + a_1\boldsymbol{v}_1 + \cdots + a_n\boldsymbol{v}_n = \boldsymbol{0}.$$

If $a \ne 0$, then

$$\boldsymbol{v} = (- a_1/a)\boldsymbol{v}_1 + \cdots + (- a_n/a)\boldsymbol{v}_n,$$

contrary to 2º. So $a = 0$ and

$$a_1\boldsymbol{v}_1 + \cdots + a_n\boldsymbol{v}_n = \boldsymbol{0}.$$

This implies that

$$a_1 = \cdots = a_n = 0,$$

because the set $\{\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_n\}$ is linearly independent. This proves that 2º implies 1º. □

Proposition 4. 26. *Let $\{\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_n\}$ be a linearly independent in a vector space V of the chemical equation* (4. 2) *over the field $\mathbb{R}$, then $\{a_1\boldsymbol{v}_1, a_2\boldsymbol{v}_2, \ldots, a_n\boldsymbol{v}_n\}$ is also linearly independent, such that the numbers a_i, $(1 \le i \le n)$ are all nonzero.*

Proof. Suppose a linear combination of the new set vanishes

$$s_1(a_1\boldsymbol{v}_1) + s_2(a_2\boldsymbol{v}_2) + \cdots + s_n(a_n\boldsymbol{v}_n) = \boldsymbol{0},$$

where s_i, $(1 \le i \le n)$ lie in $\mathbb{R}$.

Then

$$s_1a_1 = s_2a_2 = \cdots = s_na_n$$

by the linear independence of $\{\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_n\}$. The fact that each $a_i \ne 0$ $(1 \le i \le n)$ now implies that

$$s_1 = s_2 = \cdots = s_n = 0. \quad □$$

Proposition 4. 27. *No linearly independent set of vectors of molecules can contain the zero vector.*

Proof. The set $\{\boldsymbol{0}, \boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_n\}$ cannot be linearly independent, because

$$1 \cdot \boldsymbol{0} + 0\boldsymbol{v}_1 + \cdots + 0\boldsymbol{v}_n = \boldsymbol{0},$$

is a non-trivial linear combination that vanishes. □

Theorem 4. 28. *A set $\{\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_n\}$ of vectors of molecules in a vector space V of the chemical equation* (4. 2) *over the field $\mathbb{R}$ is linearly dependent if and only if some $\boldsymbol{v}_i$ is a linear combination of the others.*

Proof. Assume that $\{\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_n\}$ is linearly dependent. Then, some nontrivial linear combination vanishes, *i. e.*,

$$a_1\boldsymbol{v}_1 + a_2\boldsymbol{v}_2 + \cdots + a_n\boldsymbol{v}_n = \boldsymbol{0},$$

where some coefficient is not zero.

Suppose $a_1 \ne 0$. Then,

$$\boldsymbol{v}_1 = (- a_2/a_1)\boldsymbol{v}_2 + \cdots + (- a_n/a_1)\boldsymbol{v}_n$$

gives $\boldsymbol{v}_1$ as a linear combination of the others. In general, if $a_i \ne 0$, then a similar argument expresses $\boldsymbol{v}_i$ as linear combination of the others.

Conversely, suppose one of the vectors is a linear combination of the others, *i. e.*,

$$\boldsymbol{v}_1 = a_2\boldsymbol{v}_2 + \cdots + a_n\boldsymbol{v}_n.$$

Then, the nontrivial linear combination $1\boldsymbol{v}_1 - a_2\boldsymbol{v}_2 - \cdots - a_n\boldsymbol{v}_n$ equals zero, so the set $\{\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_n\}$ is not linearly independent, *i. e.*, it is linearly dependent. A similar argument works if *any* $\boldsymbol{v}_i$, $(1 \le i \le n)$ is linear combinations of the others. □

Theorem 4. 29. *Let $V \ne 0$ be a vector space of the chemical equation* (4. 2) *over the field $\mathbb{R}$, then*

1º *each set of linearly independent vectors is part of a basis of V,*

2º *each spanning set V contains a basis of V,*

3º *V has a basis and $\dim V \le n$.*

Proof. 1º Really, if V is a vector space that is spanned by a finite number of vectors, we claim that *any* linearly independent subset $S = \{\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_k\}$ of V is contained in a basis of V. This is certainly true if $V = \text{span}\{S\}$ because then S is *itself* a basis of V. Otherwise, choose $\boldsymbol{v}_{k+1}$ outside $\text{span}\{S\}$. Then

$$S_1 = \{\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_k, \boldsymbol{v}_{k+1}\}$$

is linearly independent by Theorem 4. 25. If $V = \text{span}\{S_1\}$ then S_1 is the desired basis containing S. If not, choose $\boldsymbol{v}_{k+2}$ outside $\text{span}\{\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_k, \boldsymbol{v}_{k+1}\}$ so that

$$S_2 = \{\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_k, \boldsymbol{v}_{k+1}, \boldsymbol{v}_{k+2}\}$$

is linearly independent. Continue this process. Either a basis is reached at some stage or, if not, arbitrary large independent sets are found in V. But this later possibility cannot occur by the Theorem 4. 3 because V is spanned by a finite number of vectors.

2º Let

$$V = \text{span}\{\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_m\},$$

where (as $V \neq 0$) we may assume that each $\boldsymbol{v}_i \neq \mathbf{0}$. If $\{\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_m\}$ is linearly independent, it is itself a basis and we are finished. If not, then according to the Theorem 4. 28, one of these vectors lies in the span of the others. Relabeling if necessary, assume that $\boldsymbol{v}_1$ lies in $\text{span}\{\boldsymbol{v}_2, \ldots, \boldsymbol{v}_m\}$ so that

$$V = \text{span}\{\boldsymbol{v}_2, \ldots, \boldsymbol{v}_m\}.$$

Now repeat argument. If the set $\{\boldsymbol{v}_2, \ldots, \boldsymbol{v}_m\}$ is linearly independent, we are finished. If not, we have (after possible relabeling)

$$V = \text{span}\{\boldsymbol{v}_3, \ldots, \boldsymbol{v}_m\}.$$

Continue this process. If a basis is encountered at some stage, we are finished. If not, we ultimately reach

$$V = \text{span}\{\boldsymbol{v}_m\}.$$

But then $\{\boldsymbol{v}_m\}$ is a basis because $\boldsymbol{v}_m \neq \mathbf{0}$ ($V \neq 0$).

3º V has a spanning set of n vectors, one of which is nonzero because $V \neq 0$. Hence 3º follows from 2º. □

Corollary 4. 30. *A nonzero vector space V of the chemical equation* (4. 2) *over the field* ℝ *is finite dimensional if only if it can be spanned by finitely many vectors.* □

Theorem 4. 31. *Let V be a vector space of the chemical equation* (4. 2) *over the field* ℝ *and* $\dim V = n > 0$, *then*

1º *no set of more than n vectors in V can be linearly independent,*

2º *no set of fewer than n vectors can span V.*

Proof. V can be spanned by n vectors (any basis) so 1º restates the Theorem 4. 3. But the n basis vectors are also linearly independent, so no spanning set can have fewer than n vectors, again by Theorem 4. 3. This gives 2º. □

Theorem 4. 32. *Let V be a vector space of the chemical equation* (4. 2) *over the field* ℝ *and* $\dim V = n > 0$, *then*

1º *any set of n linearly independent vectors in V is a basis* (*that is, it necessarily spans V*),

2º *any spanning set of n nonzero vectors in V is a basis* (*that is, it necessarily linearly independent*).

Proof. 1º If the n independent vectors do not span V, they are part of a basis of more than n vectors by property 1º of the Theorem 4. 29. This contradicts Theorem 4. 31.

2º If the n vectors in a spanning set are not linearly independent, they contain a basis of fewer than n vectors by property 2º of Theorem 4. 29, contradicting Theorem 4. 31. □

Theorem 4. 33. *Let V be a vector space of dimension n of the chemical equation* (4. 2) *over the field* ℝ *and let U and W denote subspaces of V, then*

1º *U is finite dimensional and* $\dim U \leq n$,

2º *any basis of U is part of a basis for V,*

3º *if* $U \subseteq W$ *and* $\dim U = \dim W$, *then* $U = W$.

Proof. 1º If $U = 0$, $\dim U = 0$ by Definition 3. 11. So assume $U \neq 0$ and choose $\boldsymbol{u}_1 \neq \mathbf{0}$ in U. If $U = \text{span}\{\boldsymbol{u}_1\}$, then $\dim U = 1$. If $U \neq \text{span}\{\boldsymbol{u}_1\}$, choose $\boldsymbol{u}_2$ in U outside $\text{span}\{\boldsymbol{u}_1\}$. Then $\{\boldsymbol{u}_1, \boldsymbol{u}_2\}$, is linearly independent by Theorem 4. 25. If $U = \text{span}\{\boldsymbol{u}_1, \boldsymbol{u}_2\}$, then $\dim U = 2$. If not, repeat the process to find $\boldsymbol{u}_3$ in U such that $\{\boldsymbol{u}_1, \boldsymbol{u}_2, \boldsymbol{u}_3\}$ is linearly independent. Continue in this way. The process must terminate because the space V (having dimension n) cannot contain more than n independent vectors. Therefore U has a basis of at most n vectors, proving 1º.

2º This follows from 1º and Theorem 4. 29.

3º Let $\dim U = \dim W = m$. Then any basis $\{\boldsymbol{u}_1, \boldsymbol{u}_2, \ldots, \boldsymbol{u}_m\}$ of U is an independent set of m vectors in W and so is a basis of W by Theorem 4. 32. In particular, $\{\boldsymbol{u}_1, \boldsymbol{u}_2, \ldots, \boldsymbol{u}_m\}$ spans W so, because it also spans U, $W = \text{span}\{\boldsymbol{u}_1, \boldsymbol{u}_2, \ldots, \boldsymbol{u}_m\} = U$. This proves 3º. □

Proposition 4. 34. *If U and W are subspaces of a vector space V of the chemical equation* (4. 2) *over the field* ℝ, *then U + W is a subspace of V.*

Proof. Since U and W are subspaces, $\mathbf{0} \in U$ and $\mathbf{0} \in W$. Hence $\mathbf{0} = \mathbf{0} + \mathbf{0} \in U + W$. Assume $\boldsymbol{v}, \boldsymbol{v}' \in U + W$. Then there exist $\boldsymbol{u}, \boldsymbol{u}' \in U$ and $\boldsymbol{v}, \boldsymbol{v}' \in W$ such that $\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{w}$ and $\boldsymbol{v}' = \boldsymbol{u}' + \boldsymbol{w}'$.

Since U and W are subspaces, $\boldsymbol{u} + \boldsymbol{u}' \in U$ and $\boldsymbol{w} + \boldsymbol{w}' \in W$ and for any scalar k, $k\boldsymbol{u} \in U$ and $k\boldsymbol{w} \in W$.

Accordingly,

$$\boldsymbol{v} + \boldsymbol{v}' = (\boldsymbol{u} + \boldsymbol{w}) + (\boldsymbol{u}' + \boldsymbol{w}')$$
$$= (\boldsymbol{u} + \boldsymbol{u}') + (\boldsymbol{w} + \boldsymbol{w}') \in U + W$$

and for any scalar k,

$$k\boldsymbol{v} = k(\boldsymbol{u} + \boldsymbol{w}) = k\boldsymbol{u} + k\boldsymbol{w} \in U + W.$$

Thus $U + W$ is a subspace of V. □

Proposition 4. 35. *If U and W are subspaces of a vector space V of the chemical equation* (4. 2) *over the field $\mathbb{R}$, then U and W are contained in $U + W$.*

Proof. Let $\boldsymbol{u} \in U$. By hypothesis W is a subspace of V and so $\mathbf{0} \in W$. Hence $\boldsymbol{u} = \boldsymbol{u} + \mathbf{0} \in U + W$. Accordingly, U is contained in $U + W$. Similarly, W is contained in $U + W$. □

Proposition 4. 36. *If U and W are subspaces of a vector space V of the chemical equation* (4. 2) *over the field $\mathbb{R}$, then $U + W$ is the smallest subspace of V containing U and V, i. e., $U + W = span\{U, W\}$.*

Proof. Since $U + W$ is a subspace of V containing both U and W, it must also contain the linear span of U and W, *i. e.*, $\text{span}\{U, W\} \subseteq U + W$.

On the other hand, if $\boldsymbol{v} \in U + W$ then

$$\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{w} = 1\boldsymbol{u} + 1\boldsymbol{w},$$

where $\boldsymbol{u} \in U$ and $\boldsymbol{w} \in W$. Hence, $\boldsymbol{v}$ is a linear combination of elements in $U \cup W$ and so belongs to $\text{span}\{U, W\}$.

Therefore

$$U + W \subseteq \text{span}\{U, W\}.$$

Both inclusions give us the required result.□

Proposition 4. 37. *If W is a subspace of a vector space V of the chemical equation* (4. 2) *over the field $\mathbb{R}$, then $W + W = W$.*

Proof. Since W is a subspace of V, we have that W is closed under vector addition.

Therefore

$$W + W \subseteq W.$$

By Proposition 4. 35,

$$W \subseteq W + W.$$

Thus,

$$W + W = W.$$ □

Proposition 4. 38. *If U and W are subspaces of a vector space V of the chemical equation* (4. 2) *over the field $\mathbb{R}$, such that $U = span\{S\}$ and $W = span\{T\}$, then $U + W = span\{S \cup T\}$.*

Proof. Since

$$S \subseteq U \subseteq U + W$$

and

$$T \subseteq W \subseteq U + W,$$

we have

$$S \cup T \subseteq U + W.$$

Hence

$$\text{span}\{S \cup T\} \subseteq U + W.$$

Now assume $\boldsymbol{v} \in U + W$. Then $\boldsymbol{v} = \boldsymbol{u} + W$, where $\boldsymbol{u} \in U$ and $\boldsymbol{w} \in W$.

Since

$$U = \text{span}\{S\} \text{ and } W = \text{span}\{T\},$$

$$\boldsymbol{u} = a_1\boldsymbol{u}_1 + \cdots + a_r\boldsymbol{u}_r \text{ and } \boldsymbol{w} = b_1\boldsymbol{w}_1 + \cdots + b_s\boldsymbol{w}_s,$$

where $a_i, b_j \in \mathbb{R}$, $\boldsymbol{u}_j \in S$, and $\boldsymbol{w}_i \in T$.

Then

$$\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{w} = a_1\boldsymbol{u}_1 + \cdots + a_r\boldsymbol{u}_r + b_1\boldsymbol{w}_1 + \cdots + b_s\boldsymbol{w}_s.$$

Thus,

$$U + W \subseteq \text{span}\{S \cup T\}.$$

Both inclusions yield

$$U + W = span\{S \cup T\}.$$ □

Proposition 4. 39. *If U and W are subspaces of a vector space V of the chemical equation* (4. 2) *over the field $\mathbb{R}$, then $V = U + W$ if every $\boldsymbol{v} \in V$ can be written in the form $\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{w}$, where $\boldsymbol{u} \in U$ and $\boldsymbol{w} \in W$.*

Proof. Assume, for any $\boldsymbol{v} \in V$, we have $\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{w}$ where $\boldsymbol{u} \in U$ and $\boldsymbol{w} \in W$. Then $\boldsymbol{v} \in U + W$ and so $V \subseteq U + W$. Since U and V are subspaces of V, we have $U + W \subseteq V$. Both inclusions imply $V = U + W$. □

Theorem 4. 40. *The vector space V of the chemical equation* (4. 2) *over the field $\mathbb{R}$, is the direct sum of its subspaces U and W, if only if*

$$V = U + W \text{ and } U \cap W = \{\mathbf{0}\}.$$

Proof. Assume $V = U \oplus W$. Then any $\boldsymbol{v} \in V$ can be uniquely written in the form $\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{w}$ where $\boldsymbol{u} \in U$ and $\boldsymbol{w} \in W$. Thus, in particular, $V = U + W$. Now assume $\boldsymbol{v} \in U \cap W$. Then $\boldsymbol{v} = \boldsymbol{v} + \mathbf{0}$, where $\boldsymbol{v} \in U$, $\mathbf{0} \in W$ and $\boldsymbol{v} = \mathbf{0} + \boldsymbol{v}$, where $\mathbf{0} \in U$, $\boldsymbol{v} \in W$. Since such a sum for $\boldsymbol{v}$ must be unique, $\boldsymbol{v} = \mathbf{0}$. Thus, $U \cap W = \{\mathbf{0}\}$.

On the other hand, assume $V = U + W$ and $U \cap W = \{\mathbf{0}\}$. Let $\boldsymbol{v} \in V$. Since $V = U + W$, there exist $\boldsymbol{u} \in U$ and $\boldsymbol{w} \in W$, such that $\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{w}$. We need to show that such a sum is unique. Assume also that $\boldsymbol{v} = \boldsymbol{u}' + \boldsymbol{w}'$ where $\boldsymbol{u}' \in U$ and $\boldsymbol{w}' \in W$. Then $\boldsymbol{u} + \boldsymbol{w} = \boldsymbol{u}' + \boldsymbol{w}'$ and so $\boldsymbol{u} - \boldsymbol{u}' = \boldsymbol{w}' - \boldsymbol{w}$. But, $\boldsymbol{u} - \boldsymbol{u}' \in U$ and $\boldsymbol{w}' - \boldsymbol{w} \in W$. Hence by $U \cap W = \{\mathbf{0}\}$, $\boldsymbol{u} - \boldsymbol{u}' = \mathbf{0}$, $\boldsymbol{w}' - \boldsymbol{w} = \mathbf{0}$ and so $\boldsymbol{u} = \boldsymbol{u}'$, $\boldsymbol{w}' = \boldsymbol{w}$. Thus such a sum for $\boldsymbol{v} \in V$ is unique and $V = U \oplus W$. □

Proposition 4. 41. *Let $W_1, W_2, \ldots, W_r$ be subspaces of a vector space V of the chemical equation* (4. 2) *over the field $\mathbb{R}$, such that*

$$V = W_1 + W_2 + \cdots + W_r$$

and $\mathbf{0} \in V$ and let be written uniquely as a sum

$$\mathbf{0} = \boldsymbol{w}_1 + \boldsymbol{w}_2 + \cdots + \boldsymbol{w}_r,$$

where $\boldsymbol{w}_i \in W_i$, $(1 \le i \le r)$, then

$$V = W_1 \oplus W_2 \oplus \cdots \oplus W_r,$$

i. e., that the sum is direct.

Proof. Since

$$\mathbf{0} = \mathbf{0}_1 + \mathbf{0}_2 + \cdots + \mathbf{0}_r,$$

where $\mathbf{0}_i \in W_i$, $(1 \le i \le r)$, this is the unique sum for $\mathbf{0} \in V$. Let $\boldsymbol{v} \in V$ and assume

$$\boldsymbol{v} = \boldsymbol{u}_1 + \boldsymbol{u}_2 + \cdots + \boldsymbol{u}_r$$

and

$$\boldsymbol{v} = \boldsymbol{w}_1 + \boldsymbol{w}_2 + \cdots + \boldsymbol{w}_r$$

where $\boldsymbol{u}_i, \boldsymbol{w}_i \in W_i$, $(1 \le i \le r)$. Then

$$\begin{aligned}\mathbf{0} &= \boldsymbol{v} - \boldsymbol{v}\\ &= (\boldsymbol{u}_1 - \boldsymbol{w}_1) + (\boldsymbol{u}_2 - \boldsymbol{w}_2) + \cdots + (\boldsymbol{u}_r - \boldsymbol{w}_r),\end{aligned}$$

where $(\boldsymbol{u}_i - \boldsymbol{w}_i) \in W_i$, $(1 \le i \le r)$. Since such a sum for $\mathbf{0}$ is unique, $\boldsymbol{u}_i - \boldsymbol{v}_i = \mathbf{0}$, $(1 \le i \le r)$ and hence $\boldsymbol{u}_i = \boldsymbol{v}_i$, $(1 \le i \le r)$. Thus, such a sum for $\boldsymbol{v}$ is also unique, and so

$$V = W_1 \oplus W_2 \oplus \cdots \oplus W_r. \qquad \square$$

Theorem 4. 42. *Let Π_i, $(1 \le i \le m)$ be the hyperplanes of the chemical equation* (4. 1), *which is reduced to the linear system* (4. 8)*, in an m-dimensional real space $\mathbb{R}^m$ and let W_i, $(1 \le i \le m)$ be directions of these hyperplanes, then*

$$\begin{aligned}\dim\Big(\sum_{1\le i\le m} \Pi_i\Big) &= \sum_{1\le i\le m} \dim\Pi_i \qquad (4.\ 14)\\ &- \sum_{1\le i<j\le m} \dim(\Pi_i \cap \Pi_j)\\ &+ \sum_{1\le i<j<k\le n} \dim(\Pi_i \cap \Pi_j \cap \Pi_k) - \cdots\\ &+ (-1)^{n-1}\dim\Big(\bigcap_{1\le i\le m} \Pi_i\Big),\ (\Pi_i \cap \Pi_j \neq \varnothing,\ i \neq j)\end{aligned}$$

and

$$\begin{aligned}\dim\Big(\sum_{1\le i\le m} \Pi_i\Big) &= \sum_{1\le i\le m} \dim\Pi_i \qquad (4.\ 15)\\ &- \sum_{1\le i<j\le m} \dim(W_i \cap W_j)\\ &+ \sum_{1\le i<j<k\le m} \dim(W_i \cap W_j \cap W_k) - \cdots\\ &+ (-1)^{n-1}\dim\Big(\bigcap_{1\le i\le m} W_i\Big) + 1,\ (\Pi_i \cap \Pi_j = \varnothing,\ i \neq j).\end{aligned}$$

Proof. First we shall proof the identity (4. 14). We apply induction. For $m = 2$, we obtain the Grassmann's formula

$$\begin{aligned}\dim(\Pi_1 + \Pi_2) &= \dim\Pi_1 + \dim\Pi_2 \qquad (4.16)\\ &- \dim(\Pi_1 \cap \Pi_2),\ (\Pi_1 \cap \Pi_2 \neq \varnothing).\end{aligned}$$

The truthfulness of the formula (4. 16) follows from the formula

$$\begin{aligned}\dim(W_1 + W_2) &= \dim W_1 + \dim W_2 \quad (4.17)\\ &- \dim(W_1 \cap W_2),\ (W_1 \cap W_2 \neq \varnothing),\end{aligned}$$

that holds for vector spaces, because

$$\dim(W_1 \cap W_2) = \dim(\Pi_1 \cap \Pi_2), \quad (4.18)$$

where W_1 and W_2 are subspaces of the vector space V.

Now we shall prove the identity (4. 17). If W_1 and W_2, of dimensions $r \le m$ and $s \le m$, respectively, are subspaces of a vector space V of dimension n and $W_1 \cap W_2$ and $W_1 + W_2$ are of dimensions p and t, respectively, then $t = r + s - p$.

Take $A = \{\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_p\}$ as a basis of $W_1 \cap W_2$ and take $B = A \cup \{\boldsymbol{y}_1, \boldsymbol{y}_2, \ldots, \boldsymbol{y}_{r-p}\}$ as a basis of W_1 and $C = A \cup \{\boldsymbol{z}_1, \boldsymbol{z}_2, \ldots, \boldsymbol{z}_{s-p}\}$ as a basis of W_2. Then, any vector of $W_1 + W_2$ can be expressed as a linear combination of the vectors of

$D = \{\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_p, \boldsymbol{y}_1, \boldsymbol{y}_2, \ldots, \boldsymbol{y}_{r-p}, \boldsymbol{z}_1, \boldsymbol{z}_2, \ldots, \boldsymbol{z}_{s-p}\}$.

To show that D is a linearly independent set and, hence, is a basis of $W_1 + W_2$, consider

$$\begin{aligned}&a_1\boldsymbol{x}_1 + a_2\boldsymbol{x}_2 + \cdots + a_p\boldsymbol{x}_p \qquad (4.19)\\ &+ b_1\boldsymbol{y}_1 + b_2\boldsymbol{y}_2 + \cdots + b_{r-p}\boldsymbol{y}_{r-p}\\ &+ c_1\boldsymbol{z}_1 + c_2\boldsymbol{z}_2 + \cdots + c_{s-p}\boldsymbol{z}_{s-p} = \mathbf{0},\end{aligned}$$

where $a_i, b_j, c_k \in \mathbb{R}$.

Set

$$\boldsymbol{\pi} = c_1\boldsymbol{z}_1 + c_2\boldsymbol{z}_2 + \cdots + c_{s-p}\boldsymbol{z}_{s-p}.$$

Now $\boldsymbol{\pi} \in W_2$ and by (4. 19) $\boldsymbol{\pi} \in W_1$. Thus, $\boldsymbol{\pi} \in W_1 \cap W_2$ and is a linear combination of the vectors of A, say

$$\boldsymbol{\pi} = d_1\boldsymbol{x}_1 + d_2\boldsymbol{x}_2 + \cdots + d_p\boldsymbol{x}_p,\ d_i \in \mathbb{R}.$$

Then

$$c_1\boldsymbol{z}_1 + c_2\boldsymbol{z}_2 + \cdots + c_{s-p}\boldsymbol{z}_{s-p} - d_1\boldsymbol{x}_1 - d_2\boldsymbol{x}_2 - \cdots - d_p\boldsymbol{x}_p = \mathbf{0}$$

and, since C is a basis of W_2, each $c_i = 0$ and each $d_i = 0$. With each $c_i = 0$, (4. 19) becomes

$$\begin{aligned}&a_1\boldsymbol{x}_1 + a_2\boldsymbol{x}_2 + \cdots + a_p\boldsymbol{x}_p \qquad (4.20)\\ &+ b_1\boldsymbol{y}_1 + b_2\boldsymbol{y}_2 + \cdots + b_{r-p}\boldsymbol{y}_{r-p} = \mathbf{0}.\end{aligned}$$

Since B is a basis of W_1, each $a_i = 0$ and each $b_i = 0$ in (4. 20). Then D is a linearly independent set and, hence, is a basis of $W_1 + W_2$ of dimension $t = r + s - p$.

We assume that (4. 14) is true for m and we shall use this assumption to deduce that (4. 14) is true for $m + 1$.

Next, we have

$$\begin{aligned}&\dim(\Pi_1 + \Pi_2 + \cdots + \Pi_m + \Pi_{m+1})\\ &= \dim[(\Pi_1 + \Pi_2 + \cdots + \Pi_m) + \Pi_{m+1}]\\ &= \dim(\Pi_1 + \Pi_2 + \cdots + \Pi_m) + \dim(\Pi_{m+1})\\ &- \dim[(\Pi_1 + \Pi_2 + \cdots + \Pi_m) \cap \Pi_{m+1}]\\ &= \dim\Pi_1 + \dim\Pi_2 + \cdots + \dim\Pi_m\\ &- \dim(\Pi_1 \cap \Pi_2) - \dim(\Pi_1 \cap \Pi_3) - \cdots\\ &- \dim(\Pi_{m-1} \cap \Pi_m) + \cdots\\ &+ (-1)^{m-1}\dim(\Pi_1 \cap \Pi_2 \cap \cdots \cap \Pi_m)\end{aligned}$$

$$+ \dim(\Pi_{m+1}) - \dim(\Pi_1 \cap \Pi_{m+1} + \Pi_2 \cap \Pi_{m+1} + \cdots + \Pi_m \cap \Pi_{m+1})$$
$$= \dim\Pi_1 + \dim\Pi_2 + \cdots + \Pi_{m+1}$$
$$- \dim(\Pi_1 \cap \Pi_2) - \dim(\Pi_1 \cap \Pi_3) - \cdots$$
$$- \dim(\Pi_{m-1} \cap \Pi_m) + \cdots$$
$$+ (-1)^{m-1}\dim(\Pi_1 \cap \Pi_2 \cap \cdots \cap \Pi_m)$$
$$- \{\dim(\Pi_1 \cap \Pi_{m+1}) + \dim(\Pi_2 \cap \Pi_{m+1}) + \cdots + \dim(\Pi_m \cap \Pi_{m+1})$$
$$- \dim[(\Pi_1 \cap \Pi_{m+1}) \cap (\Pi_2 \cap \Pi_{m+1})]$$
$$- \dim[(\Pi_2 \cap \Pi_{m+1}) \cap (\Pi_3 \cap \Pi_{m+1})] - \cdots$$
$$- \dim[(\Pi_{m-1} \cap \Pi_{m+1}) \cap (\Pi_m \cap \Pi_{m+1})]$$
$$+ \cdots + (-1)^{m-1}\dim[(\Pi_1 \cap \Pi_{m+1}) \cap (\Pi_2 \cap \Pi_{m+1}) \cap \cdots \cap (\Pi_m \cap \Pi_{m+1})]\}$$
$$= \dim\Pi_1 + \dim\Pi_2 + \cdots + \dim\Pi_{m+1}$$
$$- \dim(\Pi_1 \cap \Pi_2) - \dim(\Pi_1 \cap \Pi_3) - \cdots$$
$$- \dim(\Pi_{m-1} \cap \Pi_m) + \cdots$$
$$+ (-1)^{m-1}\dim(\Pi_1 \cap \Pi_2 \cap \cdots \cap \Pi_m)$$
$$- \dim(\Pi_1 \cap \Pi_{m+1}) - \dim(\Pi_2 \cap \Pi_{m+1}) - \cdots$$
$$- \dim(\Pi_m \cap \Pi_{m+1}) + \dim(\Pi_1 \cap \Pi_2 \cap \Pi_{m+1})$$
$$+ \dim(\Pi_2 \cap \Pi_3 \cap \Pi_{m+1}) + \cdots$$
$$+ \dim(\Pi_{m-1} \cap \Pi_m \cap \Pi_{m+1}) - \cdots$$
$$+ (-1)^{m}\dim(\Pi_1 \cap \Pi_2 \cap \cdots \cap \Pi_{m+1})$$
$$= \dim\Pi_1 + \dim\Pi_2 + \cdots + \dim\Pi_{m+1}$$
$$- \dim(\Pi_1 \cap \Pi_2) - \dim(\Pi_1 \cap \Pi_3) - \cdots$$
$$- \dim(\Pi_m \cap \Pi_{m+1}) + \dim(\Pi_1 \cap \Pi_2 \cap \Pi_{m+1})$$
$$+ \dim(\Pi_2 \cap \Pi_3 \cap \Pi_{m+1}) + \cdots$$
$$+ \dim(\Pi_{m-1} \cap \Pi_m \cap \Pi_{m+1}) + \cdots$$
$$+ (-1)^{m}\dim(\Pi_1 \cap \Pi_2 \cap \cdots \cap \Pi_{m+1})$$
$$= \sum_{1 \le i \le m+1} \dim\Pi_i - \sum_{1 \le i < j \le m+1} \dim(\Pi_i \cap \Pi_j)$$
$$+ \sum_{1 \le i < j < k \le m+1} \dim(\Pi_i \cap \Pi_j \cap \Pi_k)$$
$$- \cdots + (-1)^{m}\dim(\bigcap_{1 \le i \le m+1} \Pi_i), \ (\Pi_i \cap \Pi_j \ne \varnothing, \ i \ne j).$$

By this the identity (4. 14) is proved.

Now, we shall continue with induction in order to prove the identity (4.15). For $m = 2$, the identity (4.15) reduces to this form

$$\dim(\Pi_1 + \Pi_2) = \dim\Pi_1 + \dim\Pi_2 - \dim(W_1 \cap W_2) + 1, \ (\Pi_1 \cap \Pi_2 = \varnothing). \quad (4.21)$$

Let $\Pi_1 \cap \Pi_2 = \varnothing$. Let $\dim\Pi_1 = r$ and $\dim\Pi_2 = s$. Let the point $X \in \Pi_1$ and the point $Y \in \Pi_2$ and let the basis of the subspace W_2 is the set $\{\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_s\}$. Obviously, the vector $\boldsymbol{YX} \notin W_2$, because conversely $X \in \Pi_1 \cap \Pi_2$ what is a contradiction.

Therefore, the set

$$S = \{\boldsymbol{YX}, \boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_s\}$$

is a basis of the space $W_3 = [\boldsymbol{XY}] + W_2$ and $\dim W_3 = s + 1$. Hyperplane of the point X, which determine the subspace W_3 is given by $X + \Pi_2$, such that $\dim(X + \Pi_2) = s + 1$.

Since,

$$\Pi_1 \cap (X + \Pi_2) \ne \varnothing,$$

then from (4.16) follows

$$\dim(\Pi_1 + \Pi_2) = \dim[\Pi_1 + (X + \Pi_2)] = r + (s + 1) - \dim[\Pi_1 + (X + \Pi_2)]. \quad (4.22)$$

Next, since (4.18) we obtain

$$\dim[\Pi_1 + (X + \Pi_2)] = \dim(W_1 \cap W_3).$$

Let $Y \in \Pi_2$. Prove that $\boldsymbol{XY} \notin W_1 + W_2$. Assume the contrary, *i. e.*, $\boldsymbol{XY} \in W_1 + W_2$, then there exist two vectors $\boldsymbol{x} \in W_1$ and $\boldsymbol{y} \in W_2$ such that $\boldsymbol{XY} = \boldsymbol{x} + \boldsymbol{y}$. Then, there exists a point $Z \in \Pi_1$ such that $\boldsymbol{XZ} = \boldsymbol{x}$ and according to the definition of affine space, one obtain

$$\boldsymbol{XY} = \boldsymbol{XZ} + \boldsymbol{ZY} = \boldsymbol{x} + \boldsymbol{ZY},$$

i. e., $\boldsymbol{ZY} = \boldsymbol{y} \in W_2$. Therefore $Z \in \Pi_2$, respectively $Z \in \Pi_1 + \Pi_2$, that is a contradiction with the supposition $\Pi_1 \cap \Pi_2 = \varnothing$.

By this we proved that $\boldsymbol{XY} \notin W_1 + W_2$. Since $W_3 = [\boldsymbol{XY}, \boldsymbol{v}_1, \ldots, \boldsymbol{v}_s]$, then holds

$$W_1 \cap W_3 = W_1 \cap W_2,$$

and from (4.22) follows

$$\dim(\Pi_1 + \Pi_2) = r + s + 1 - \dim(W_1 \cap W_2),$$

i. e., holds (4. 15).

Since the identity (4.15) holds for $m = 2$, we suppose that it is true for arbitrary m. By virtue of inductive hypothesis we shall prove its truthfulness for $m + 1$, *i. e.*,

$$\dim(\Pi_1 + \Pi_2 + \cdots + \Pi_m + \Pi_{m+1})$$
$$= \dim[(\Pi_1 + \Pi_2 + \cdots + \Pi_m) + \Pi_{m+1}]$$
$$= \dim(\Pi_1 + \Pi_2 + \cdots + \Pi_m) + \dim\Pi_{m+1}$$
$$- \dim[(W_1 + W_2 + \cdots + W_m) \cap W_{m+1}] + 1$$
$$= \dim(\Pi_1 + \Pi_2 + \cdots + \Pi_m) + \dim\Pi_{m+1}$$
$$- [\dim(W_1 \cap W_{m+1} + W_2 \cap W_{m+1} + \cdots + W_m \cap W_{m+1})] + 1$$
$$= \dim\Pi_1 + \dim\Pi_2 + \cdots + \dim\Pi_m + \dim\Pi_{m+1} - \dim(W_1 \cap W_2)$$
$$- \dim(W_1 \cap W_3) - \cdots - \dim(W_{m-1} \cap W_m) + \cdots$$
$$+ (-1)^{m-1}\dim(W_1 \cap W_2 \cap \cdots \cap W_m) + 1$$
$$- \{\dim(W_1 \cap W_{m+1}) + \dim(W_2 \cap W_{m+1}) + \cdots + \dim(W_m \cap W_{m+1})$$
$$- \dim[(W_1 \cap W_{m+1}) \cap (W_2 \cap W_{m+1})]$$
$$- \dim[(W_2 \cap W_{m+1}) \cap (W_3 \cap W_{m+1})] - \cdots$$
$$- \dim[(W_{m-1} \cap W_{m+1}) \cap (W_m \cap W_{m+1})]$$
$$+ \cdots + (-1)^{m-1}\dim[(W_1 \cap W_{m+1}) \cap (W_2 \cap W_{m+1})$$

$$\cap \cdots \cap (W_m \cap W_{m+1})] + 1\} + 1$$
$$= \dim\Pi_1 + \dim\Pi_2 + \cdots + \dim\Pi_{m+1}$$
$$- \dim(W_1 \cap W_2)$$
$$- \dim(W_1 \cap W_3) - \cdots - \dim(W_{m-1} \cap W_m)$$
$$+ \cdots + (-1)^{m-1}\dim(W_1 \cap W_2 \cap \cdots \cap W_m)$$
$$+ 1 - \dim(W_1 \cap W_{m+1}) - \dim(W_2 \cap W_{m+1}) - \cdots$$
$$- \dim(W_m \cap W_{m+1}) + \dim(W_1 \cap W_2 \cap W_{m+1})$$
$$+ \dim(W_2 \cap W_3 \cap W_{m+1}) + \cdots$$
$$+ \dim(W_{m-1} \cap W_m \cap W_{m+1}) + \cdots$$
$$- (-1)^{m-1}\dim(W_1 \cap W_2 \cap \cdots \cap W_m \cap W_{m+1})$$
$$- 1 + 1 = \dim\Pi_1 + \dim\Pi_2 + \cdots + \dim\Pi_{m+1}$$
$$- \dim(W_1 \cap W_2) - \dim(W_1 \cap W_3)$$
$$- \cdots - \dim(W_m \cap W_{m+1}) + \dim(W_1 \cap W_2 \cap W_{m+1})$$
$$+ \dim(W_2 \cap W_3 \cap W_{m+1}) + \cdots$$
$$+ \dim(W_{m-1} \cap W_m \cap W_{m+1}) + \cdots$$
$$+ (-1)^m\dim(\Pi_1 \cap \Pi_2 \cap \cdots \cap \Pi_{m+1}) + 1$$
$$= \sum_{1\le i\le m+1} \dim\Pi_i - \sum_{1\le i<j\le m+1} \dim(W_i \cap W_j)$$
$$+ \sum_{1\le i<j<k\le m+1} \dim(W_i \cap W_j \cap W_k) - \cdots$$
$$+ (-1)^{m-1}\dim(\bigcap_{1\le i\le m+1} W_i) + 1, \ (\Pi_i \cap \Pi_j = \varnothing, i \neq j).$$

The proof of the theorem is completed. □

Remark 4. 43. *This theorem generalizes well-known Grassmann's formulas given in* [22].

As a natural consequence of proved theorem appears the following corollary.

Corollary 4. 44. *Let* $\Pi_i^{k_i}$, $(1 \le i \le m)$ *be* k_i-*dimensional hyperplanes of the chemical equation* (4. 1), *which is reduced to the linear system* (4. 8), *in an m-dimensional real space* $\mathbb{R}^m$. *Then*

$$\dim(\sum_{1\le i\le m} \Pi_i^{k_i}) \le \sum_{1\le i\le m} k_i + 1. \qquad (4.23)$$

The sign equality in (4.23) *holds if only if the hyperplanes* $\Pi_i^{k_i}$ *and* $\Pi_j^{k_j}$, $(i \neq j)$, *are disjoint and the sum of subspaces* $\Pi_i^{k_i} \preccurlyeq V^m$, $(1 \le i \le m)$ *is direct.* □

Proposition 4. 45. *Let*

$$V = U_1 \oplus U_2 \oplus \cdots \oplus U_r$$

is a vector space of the chemical equation (4. 2) *over the field* $\mathbb{R}$ *and* B_i, $(1 \le i \le r)$ *is a basis of* U_i, $(1 \le i \le r)$, *then* $B = \cup B_i$, $(1 \le i \le r)$ *is a basis of V.*

Proof. B is linearly independent since each B_i is linearly independent. Assume $\boldsymbol{v} \in V$. Then

$$\boldsymbol{v} = \boldsymbol{u}_1 + \boldsymbol{u}_2 + \cdots + \boldsymbol{u}_r,$$

where $\boldsymbol{u}_i \in U_i$, $(1 \le i \le r)$. Then $\boldsymbol{u}_i$, $(1 \le i \le r)$ is a linear combination of the vectors in B_i, $(1 \le i \le r)$. Therefore $\boldsymbol{v}$ is a linear combination of the vectors in B.

Thus B spans V. Since B is linearly independent and spans V, B is a basis of V. □

Proposition 4. 46. *Let*

$$V = U_1 \oplus U_2 \oplus \cdots \oplus U_r$$

is a vector space of the chemical equation (4. 2) *over the field* $\mathbb{R}$, *where* $dimU_i = n_i$, *then*

$$dimV = dimU_1 + dimU_2 + \cdots + dimU_r.$$

Proof. Let B_i be a basis of U_i, $(1 \le i \le r)$. Hence B_i, $(1 \le i \le r)$ has n_i, $(1 \le i \le r)$ elements. Thus $B = \cup B_i$, $(1 \le i \le r)$ has $n_1 + n_2 + \cdots + n_r$ elements. By Proposition 4. 45, B is a basis of V.

Hence

$$\dim V = n_1 + n_2 + \cdots + n_r$$
$$= dimU_1 + dimU_2 + \cdots + dimU_r. \quad □$$

Proposition 4. 47. *If* $\{\boldsymbol{u}_1, \boldsymbol{u}_2, \ldots, \boldsymbol{u}_r, \boldsymbol{w}_1, \boldsymbol{w}_2, \ldots, \boldsymbol{w}_s\}$ *is a linearly independent subset of a vector space V of the chemical equation* (4. 2) *over the field* $\mathbb{R}$, *then*

$$span\{\boldsymbol{u}_i\} \cap span\{\boldsymbol{w}_j\} = \{\boldsymbol{0}\}.$$

Proof. Assume $\boldsymbol{v} \in \text{span}\{\boldsymbol{u}_i\} \cap \text{span}\{\boldsymbol{w}_j\}$. Then there exist scalars a_i, $(1 \le i \le r)$ and b_j, $(1 \le j \le s)$ such that

$$\boldsymbol{v} = a_1\boldsymbol{u}_1 + a_2\boldsymbol{u}_2 + \cdots + a_r\boldsymbol{u}_r$$
$$= b_1\boldsymbol{w}_1 + b_2\boldsymbol{w}_2 + \cdots + b_s\boldsymbol{w}_s.$$

Hence

$$a_1\boldsymbol{u}_1 + a_2\boldsymbol{u}_2 + \cdots + a_r\boldsymbol{u}_r$$
$$- b_1\boldsymbol{w}_1 - b_2\boldsymbol{w}_2 - \cdots - b_s\boldsymbol{w}_s = \boldsymbol{0}.$$

But $\{\boldsymbol{u}_i, \boldsymbol{w}_j\}$ is linearly independent. Hence each $a_i = 0$, $(1 \le i \le r)$ and each $b_j = 0$, $(1 \le j \le r)$. Thus $\boldsymbol{v} = \boldsymbol{0}$. □

Proposition 4. 48. *Let U be a subspace of a vector space V of the chemical equation* (4. 2) *over the field* $\mathbb{R}$, *then exists a subspace W of V, such that* $V = U \oplus W$.

Proof. Let $\{\boldsymbol{u}_1, \boldsymbol{u}_2, \ldots, \boldsymbol{u}_r\}$ be a basis of U. Since $\{\boldsymbol{u}_i\}$, $(1 \le i \le r)$ is linearly independent, it can be extended to a basis of V, *i. e.*, $\{\boldsymbol{u}_1, \boldsymbol{u}_2, \ldots, \boldsymbol{u}_r, \boldsymbol{w}_1, \boldsymbol{w}_2, \ldots, \boldsymbol{w}_s\}$. Let W be the space generated by $\{\boldsymbol{w}_1, \boldsymbol{w}_2, \ldots, \boldsymbol{w}_s\}$. Since $\{\boldsymbol{u}_i, \boldsymbol{w}_j\}$, $(1 \le i \le r, 1 \le j \le s)$ spans V, we have $V = U + W$. On the other hand, by Proposition 4. 47, $U \cap W = \{\boldsymbol{0}\}$. Accordingly, $V = U \oplus W$. □

Proposition 4. 49. *Let B is a linearly independent subset of a vector space V of the* *chemical equation* (4. 2) *over the field* $\mathbb{R}$ *and let* $[B_1, B_2, \ldots, B_r]$ *be a partition of B, then*

$$\mathit{span}\{B\} = \mathit{span}\{B_1\} \oplus \mathit{span}\{B_2\} \oplus \cdots \oplus \mathit{span}\{B_r\}.$$

Proof. Since

$$B = B_1 \cup B_2 \cup \cdots \cup B_r$$

and each $B_i \subseteq B$, $(1 \le i \le r)$ we have

$$\text{span}\{B\} = \text{span}\{B_1 \cup B_2 \cup \cdots \cup B_r\} \subseteq \text{span}\{B_1\} \oplus \text{span}\{B_2\} \oplus \cdots \oplus \text{span}\{B_r\} \subseteq \text{span}\{B\}.$$

Therefore

$$\text{span}\{B\} = \text{span}\{B_1\} \oplus \text{span}\{B_2\} \oplus \cdots \oplus \text{span}\{B_r\}.$$

Assume

$$\mathbf{0} = \sum a_{1,j1}\, \boldsymbol{u}_{1,j1} + \sum a_{2,j2}\, \boldsymbol{u}_{2,j2} + \cdots + \sum a_{r,jr}\, \boldsymbol{u}_{r,jr}, \quad (4.\ 24)$$

where $a_{i,ji}$ are scalars and the $\boldsymbol{u}_{i,ji} \in B_i$, $(1 \le i \le r)$. Since B is linearly independent, each $a_{i,ji} = 0$, $(1 \le i \le r)$ in (4. 24). Thus, can only be written uniquely as

$$0 = 0 + 0 + \cdots + 0.$$

Thus,

$$\text{span}\{B\} = \text{span}\{B_1\} \oplus \text{span}\{B_2\} \oplus \cdots \oplus \text{span}\{B_r\}. \quad \square$$

Proposition 4. 50. *Let*

$$V = U_1 + U_2 + \cdots + U_r$$

be a vector space of the chemical equation (4. 2) *over the field* $\mathbb{R}$ *and let*

$$\mathit{dim}V = \mathit{dim}U_1 + \mathit{dim}U_2 + \cdots + \mathit{dim}U_r,$$

then

$$V = U_1 \oplus U_2 \oplus \cdots \oplus B_r.$$

Proof. Assume $\dim V = n$. Let B_i, $(1 \le i \le r)$ be a basis for U_i, $(1 \le i \le r)$. Then

$$B = B_1 \cup B_2 \cup \cdots \cup B_r$$

has n elements and spans V. Thus B is a basis for V. By Proposition 4. 49

$$V = U_1 \oplus U_2 \oplus \cdots \oplus B_r. \quad \square$$

Theorem 4. 51. *Let V and U be vector spaces over a field* $\mathbb{R}$. *Let* $\{\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_n\}$ *be a basis of a vector space V of the chemical equation* (4. 2) *over the field* $\mathbb{R}$ *and let* $\boldsymbol{u}_1, \boldsymbol{u}_2, \ldots, \boldsymbol{u}_n$ *be any arbitrary vectors in U, then there exists a unique linear mapping* $F: V \to U$ *such that*

$$F(\boldsymbol{v}_1) = \boldsymbol{u}_1,\ F(\boldsymbol{v}_2) = \boldsymbol{u}_2,\ \ldots,\ F(\boldsymbol{v}_n) = \boldsymbol{u}_n.$$

Proof. Now, we shall define required map $F: V \to U$ such that $F(\boldsymbol{v}_i) = \boldsymbol{u}_i$, $(1 \le i \le n)$. Let $\boldsymbol{v} \in V$. Since $\{\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_n\}$ is a basis of V, there exist unique scalars $a_i \in \mathbb{R}$, $(1 \le i \le n)$ for which

$$\boldsymbol{v} = a_1\boldsymbol{v}_1 + a_2\boldsymbol{v}_2 + \cdots + a_n\boldsymbol{v}_n.$$

We define $F: V \to U$ by

$$F(\boldsymbol{v}) = a_1\boldsymbol{u}_1 + a_2\boldsymbol{u}_2 + \cdots + a_n\boldsymbol{u}_n.$$

Since the $a_i \in \mathbb{R}$, $(1 \le i \le n)$ are unique, the mapping F is well-defined.

Now,

$$\boldsymbol{v}_i = 0\boldsymbol{v}_1 + \cdots + 1\boldsymbol{v}_i + \cdots + 0\boldsymbol{v}_n,\ (1 \le i \le n).$$

Therefore

$$F(\boldsymbol{v}_i) = 0\boldsymbol{u}_1 + \cdots + 1\boldsymbol{u}_i + \cdots + 0\boldsymbol{u}_n = \boldsymbol{u}_i.$$

Thus the first step of the proof is completed.

Now, we shall show that F is linear.

Assume

$$\boldsymbol{v} = a_1\boldsymbol{v}_1 + a_2\boldsymbol{v}_2 + \cdots + a_n\boldsymbol{v}_n$$

and

$$\boldsymbol{w} = b_1\boldsymbol{v}_1 + b_2\boldsymbol{v}_2 + \cdots + b_n\boldsymbol{v}_n.$$

Then

$$\boldsymbol{v} + \boldsymbol{w} = (a_1 + b_1)\boldsymbol{v}_1 + (a_2 + b_2)\boldsymbol{v}_2 + \cdots + (a_n + b_n)\boldsymbol{v}_n$$

and

$$k\boldsymbol{v} = ka_1\boldsymbol{v}_1 + ka_2\boldsymbol{v}_2 + \cdots + ka_n\boldsymbol{v}_n,\ \forall k \in \mathbb{R}.$$

By definition of the mapping F, we have

$$F(\boldsymbol{v}) = a_1\boldsymbol{u}_1 + a_2\boldsymbol{u}_2 + \cdots + a_n\boldsymbol{u}_n$$

and

$$F(\boldsymbol{w}) = b_1\boldsymbol{u}_1 + b_2\boldsymbol{u}_2 + \cdots + b_n\boldsymbol{u}_n.$$

Therefore

$$F(\boldsymbol{v} + \boldsymbol{w}) = (a_1 + b_1)\boldsymbol{u}_1 + (a_2 + b_2)\boldsymbol{u}_2 + \cdots + (a_n + b_n)\boldsymbol{u}_n = (a_1\boldsymbol{u}_1 + a_2\boldsymbol{u}_2 + \cdots + a_n\boldsymbol{u}_n) + (b_1\boldsymbol{u}_1 + b_2\boldsymbol{u}_2 + \cdots + b_n\boldsymbol{u}_n) = F(\boldsymbol{v}) + F(\boldsymbol{w})$$

and

$$F(k\boldsymbol{v}) = k(a_1\boldsymbol{u}_1 + a_2\boldsymbol{u}_2 + \cdots + a_n\boldsymbol{u}_n) = kF(\boldsymbol{v}).$$

Thus F is linear.

Now we shall show that F is unique.

Assume $G: V \to U$ is linear and

$$G(\boldsymbol{v}_i) = \boldsymbol{u}_i,\ (1 \le i \le n).$$

If

$$\boldsymbol{v} = a_1\boldsymbol{v}_1 + a_2\boldsymbol{v}_2 + \cdots + a_n\boldsymbol{v}_n,$$

then

$$G(\boldsymbol{v}) = G(a_1\boldsymbol{v}_1 + a_2\boldsymbol{v}_2 + \cdots + a_n\boldsymbol{v}_n) = a_1G(\boldsymbol{v}_1) + a_2G(\boldsymbol{v}_2) + \cdots + a_nG(\boldsymbol{v}_n) = a_1\boldsymbol{u}_1 + a_2\boldsymbol{u}_2 + \cdots + a_n\boldsymbol{u}_n = F(\boldsymbol{v}).$$

Since $G(\boldsymbol{v}) = F(\boldsymbol{v})$, $\forall \boldsymbol{v} \in V$, $G = F$.

Thus F is unique and the theorem is proved. □

Proposition 4. 52. *If* $F: V \to U$ *is a linear mapping, then kernel of F is a subspace of a* *vector space V of the chemical equation* (4. 2) *over the field* $\mathbb{R}$.

Proof. Since $F(\mathbf{0}) = \mathbf{0}$, $\mathbf{0} \in \mathrm{Ker}F$.

Now, assume $\boldsymbol{v}, \boldsymbol{w} \in \mathrm{Ker}F$ and $a, b \in \mathbb{R}$. Since $\boldsymbol{v}$ and $\boldsymbol{w}$ belong to the kernel of F, $F(\boldsymbol{v}) = \mathbf{0}$ and $F(\boldsymbol{w}) = \mathbf{0}$.

Therefore

$$F(a\boldsymbol{v} + b\boldsymbol{w}) = aF(\boldsymbol{v}) + bF(\boldsymbol{w}) = a\mathbf{0} + b\mathbf{0} = \mathbf{0}$$

and so $a\boldsymbol{v} + b\boldsymbol{w} \in \mathrm{Ker}F$. Thus the kernel of F is a subspace of V. □

Proposition 4. 53. *If F: $V \to U$ is a linear mapping, then image of F is a subspace of a vector space V of the chemical equation* (4. 2) *over the field* $\mathbb{R}$.

Proof. Since $F(\mathbf{0}) = \mathbf{0}$, $\mathbf{0} \in \mathrm{Im}F$. Now, assume $\boldsymbol{v}, \boldsymbol{w} \in \mathrm{Im}F$ and $a, b \in \mathbb{R}$. Since $\boldsymbol{v}$ and $\boldsymbol{w}$ belong to the image of F, there exist vectors $\boldsymbol{v}', \boldsymbol{w}' \in V$ such that $F(\boldsymbol{v}') = \boldsymbol{v}$ and $F(\boldsymbol{w}') = \boldsymbol{w}$.

Then

$$F(a\boldsymbol{v}' + b\boldsymbol{w}')$$
$$= aF(\boldsymbol{v}') + bF(\boldsymbol{w}') = a\boldsymbol{v} + b\boldsymbol{w} \in \mathrm{Im}V.$$

Thus the image of F is a subspace of V. □

Proposition 4. 54. *If the vectors $\boldsymbol{v}_1, \boldsymbol{v}_2, ..., \boldsymbol{v}_n$ span a vector space V of the chemical equation* (4. 2) *over the field $\mathbb{R}$ and F: $V \to U$ is a linear mapping, then the vectors $F(\boldsymbol{v}_1)$, $F(\boldsymbol{v}_2)$, ..., $F(\boldsymbol{v}_n) \in U$ span ImF.*

Proof. Assume $\boldsymbol{u} \in \mathrm{Im}F$, then $F(\boldsymbol{v}) = \boldsymbol{u}$ for some vector $\boldsymbol{v} \in V$. Since $\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_n$ span V and since $\boldsymbol{v} \in V$, there exist scalars $a_1, a_2, \ldots, a_n$ for which

$$\boldsymbol{v} = a_1\boldsymbol{v}_1 + a_2\boldsymbol{v}_2 + \cdots + a_n\boldsymbol{v}_n.$$

Accordingly,

$$\boldsymbol{u} = F(\boldsymbol{v}) = F(a_1\boldsymbol{v}_1 + a_2\boldsymbol{v}_2 + \cdots + a_n\boldsymbol{v}_n)$$
$$= a_1F(\boldsymbol{v}_1) + a_2F(\boldsymbol{v}_2) + \cdots + a_nF(\boldsymbol{v}_n).$$

Thus the vectors $F(\boldsymbol{v}_1), F(\boldsymbol{v}_2), \ldots, F(\boldsymbol{v}_n)$ span $\mathrm{Im}F$. □

Proposition 4. 55. *Let a vector space V of the chemical equation* (4. 2) *over the field $\mathbb{R}$ has finite dimension and F: $V \to U$ is linear, then ImF has finite dimension, i. e., dim(ImF) ≤ dimV.*

Proof. Assume $\dim V = n$ and $\dim(\mathrm{Im}F) > \dim V$. Then there exist vectors $\boldsymbol{w}_1, \boldsymbol{w}_2, \ldots, \boldsymbol{w}_{n+1} \in \mathrm{Im}F$ which are linearly independent. Let $\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_{n+1}$ be vectors in V such that $F(\boldsymbol{v}_i) = \boldsymbol{w}_i$, $(1 \le i \le n + 1)$.

Assume

$$a_1\boldsymbol{v}_1 + a_2\boldsymbol{v}_2 + \cdots + a_{n+1}\boldsymbol{v}_{n+1} = \mathbf{0}.$$

Then

$$\mathbf{0} = F(\mathbf{0}) = F(a_1\boldsymbol{v}_1 + a_2\boldsymbol{v}_2 + \cdots + a_{n+1}\boldsymbol{v}_{n+1})$$
$$= a_1F(\boldsymbol{v}_1) + a_2F(\boldsymbol{v}_2) + \cdots + a_{n+1}F(\boldsymbol{v}_{n+1})$$
$$= a_1\boldsymbol{w}_1 + a_2\boldsymbol{w}_2 + \cdots + a_{n+1}\boldsymbol{w}_{n+1}.$$

Since $\boldsymbol{w}_i$, $(1 \le i \le n + 1)$ are linearly independent, $a_i = 0$, $(1 \le i \le n + 1)$. Thus $\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_{n+1}$ are linearly independent.

This contradicts the fact that $\dim V = n$. Thus,

$$\dim(\mathrm{Im}F) \le \dim V. \quad □$$

Theorem 4. 56. *Let a vector space V of the chemical equation* (4. 2) *over the field $\mathbb{R}$ has finite dimension and let F: $V \to U$ be a linear mapping, then*

$$\dim V = \dim(\mathrm{Ker}F) + \dim(\mathrm{Im}F).$$

Proof. Assume

$$\dim(\mathrm{Ker}F) = r \text{ and } \{\boldsymbol{w}_1, \boldsymbol{w}_2, \ldots, \boldsymbol{w}_r\}$$

is a basis of $\mathrm{Ker}F$, and assume $\dim(\mathrm{Im}F) = s$ and $\{\boldsymbol{u}_1, \boldsymbol{u}_2, \ldots, \boldsymbol{u}_s\}$ is a basis of $\mathrm{Im}F$. According to the Proposition 4. 55, $\mathrm{Im}F$ has finite dimension. Since $\boldsymbol{u}_j \in \mathrm{Im}F$, there exist vectors $\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_s$ in V such that

$$F(\boldsymbol{v}_1) = \boldsymbol{u}_1, F(\boldsymbol{v}_2) = \boldsymbol{u}_2, \ldots, F(\boldsymbol{v}_s) = \boldsymbol{u}_s.$$

We claim that the set

$$B = \{\boldsymbol{w}_1, \boldsymbol{w}_2, \ldots, \boldsymbol{w}_r, \boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_s\}$$

is a basis of V, *i. e.*, 1º B spans V and 2º B is linearly independent. Once we prove 1º and 2º, then

$$\dim V = r + s = \dim(\mathrm{Ker}F) + \dim(\mathrm{Im}F).$$

1º B spans V. Let $\boldsymbol{v} \in V$. Then $F(\boldsymbol{v}) \in \mathrm{Im}F$. Since the $\boldsymbol{u}_j$ span $\mathrm{Im}F$, there exist scalars a_i, $(1 \le i \le s)$, such that

$$F(\boldsymbol{v}) = a_1\boldsymbol{u}_1 + a_2\boldsymbol{u}_2 + \cdots + a_s\boldsymbol{u}_s.$$

Set

$$\boldsymbol{v}^* = a_1\boldsymbol{v}_1 + a_2\boldsymbol{v}_2 + \cdots + a_s\boldsymbol{v}_s - \boldsymbol{v}.$$

Then

$$F(\boldsymbol{v}^*) = F(a_1\boldsymbol{v}_1 + a_2\boldsymbol{v}_2 + \cdots + a_s\boldsymbol{v}_s - \boldsymbol{v})$$
$$= a_1F(\boldsymbol{v}_1) + a_2F(\boldsymbol{v}_2) + \cdots + a_sF(\boldsymbol{v}_s) - F(\boldsymbol{v})$$
$$= a_1\boldsymbol{u}_1 + a_2\boldsymbol{u}_2 + \cdots + a_s\boldsymbol{u}_s - F(\boldsymbol{v}) = \mathbf{0}.$$

Thus $\boldsymbol{v} \in \mathrm{Ker}F$. Since the $\boldsymbol{w}_i$ span $\mathrm{Ker}F$, there exist scalars b_i, $(1 \le i \le r)$, such that

$$\boldsymbol{v}^* = b_1\boldsymbol{w}_1 + b_2\boldsymbol{w}_2 + \cdots + b_r\boldsymbol{w}_r$$
$$= a_1\boldsymbol{v}_1 + a_2\boldsymbol{v}_2 + \cdots + a_s\boldsymbol{v}_s - \boldsymbol{v}.$$

Accordingly,

$$\boldsymbol{v} = a_1\boldsymbol{v}_1 + a_2\boldsymbol{v}_2 + \cdots + a_r\boldsymbol{v}_r$$
$$- b_1\boldsymbol{w}_1 - b_2\boldsymbol{w}_2 - \cdots - b_r\boldsymbol{w}_r.$$

Thus B spans V.

2º B is linearly independent. Assume

$$x_1\boldsymbol{w}_1 + x_2\boldsymbol{w}_2 + \cdots + x_r\boldsymbol{w}_r \qquad (4.\ 25)$$
$$+ y_1\boldsymbol{v}_1 + y_2\boldsymbol{v}_2 + \cdots + y_s\boldsymbol{v}_s = \mathbf{0},$$

where $x_i, y_j \in \mathbb{R}$, $(1 \le i \le r, 1 \le j \le s)$.

Then

$$\mathbf{0} = F(\mathbf{0}) = \mathrm{F}(x_1\boldsymbol{w}_1 + x_2\boldsymbol{w}_2 + \cdots + x_r\boldsymbol{w}_r + y_1\boldsymbol{v}_1 + y_2\boldsymbol{v}_2 + \cdots + y_s\boldsymbol{v}_s) \tag{4.26}$$
$$= x_1F(\boldsymbol{w}_1) + x_2F(\boldsymbol{w}_2) + \cdots + x_rF(\boldsymbol{w}_r) + y_1F(\boldsymbol{v}_1) + y_2F(\boldsymbol{v}_2) + \cdots + y_sF(\boldsymbol{v}_s).$$

But $F(\boldsymbol{w}_i) = \mathbf{0}$, $(1 \le i \le r)$ since $\boldsymbol{w}_i \in \mathrm{Ker}F$, $(1 \le i \le r)$ and $F(\boldsymbol{v}_j) = \boldsymbol{u}_j$, $(1 \le j \le s)$. Substitution in (4. 26) gives $y_1\boldsymbol{u}_1 + y_2\boldsymbol{u}_2 + \cdots + y_s\boldsymbol{u}_s = \mathbf{0}$. Since the $\boldsymbol{u}_j$, $(1 \le j \le s)$ are linearly independent, each $y_j = 0$, $(1 \le j \le s)$. Substitution in (4. 25) gives

$$x_1\boldsymbol{w}_1 + x_2\boldsymbol{w}_2 + \cdots + x_r\boldsymbol{w}_r = \mathbf{0}.$$

Since the $\boldsymbol{w}_i$, $(1 \le i \le r)$ are linearly independent, each $x_i = 0$, $(1 \le i \le r)$. Thus B is linearly independent. By this the proof is completed. □

Proposition 4. 57. *Let a vector space V of the chemical equation* (4. 2) *over the field* $\mathbb{R}$ *has finite dimension and let* F: $V \to U$ *be a linear mapping, then V and the image of F have the same dimension if only if F is nonsingular.*

Proof. By Theorem 4. 56, $\dim V = \dim(\mathrm{Im}F) + \dim(\mathrm{Ker}F)$. Therefore V and $\mathrm{Im}F$ have the same dimension if and only if $\dim(\mathrm{Ker}F) = 0$ or $\mathrm{Ker}F = \{\mathbf{0}\}$, *i. e.*, if and only if F is nonsingular. □

Proposition 4. 58. *Let a vector space V of the chemical equation* (4. 2) *over the field* $\mathbb{R}$ *has finite dimension and let* F: $V \to U$ *be a linear mapping, such that maps independent sets into independent sets, then F is nonsingular.*

Proof. Assume $\boldsymbol{v} \in V$ is nonzero vector, then $\{\boldsymbol{v}\}$ is independent. Then $\{F(\boldsymbol{v})\}$ is independent and so $F(\boldsymbol{v}) \ne \mathbf{0}$.

Therefore, F is nonsingular. □

Theorem 4. 59. *Let a vector space V of the chemical equation* (4. 2) *over the field* $\mathbb{R}$ *has finite dimension and let* F: $V \to U$ *be a linear mapping, then the image of any linearly independent set is linearly independent.*

Proof. Assume $\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_n$ are linearly independent vectors in V. We claimed that the vectors $F(\boldsymbol{v}_1), F(\boldsymbol{v}_2), \ldots, F(\boldsymbol{v}_n)$ are also linearly independent.

Assume

$$a_1F(\boldsymbol{v}_1) + a_2F(\boldsymbol{v}_2) + \cdots + a_nF(\boldsymbol{v}_n) = \mathbf{0},$$

where $a_i \in \mathbb{R}$, $(1 \le i \le n)$.

Since F is linear,

$$F(a_1\boldsymbol{v}_1 + a_2\boldsymbol{v}_2 + \cdots + a_n\boldsymbol{v}_n) = \mathbf{0},$$

therefore $a_1\boldsymbol{v}_1 + a_2\boldsymbol{v}_2 + \cdots + a_n\boldsymbol{v}_n$ belongs to $\mathrm{Ker}F$. But F is nonsingular, *i. e.*, $\mathrm{Ker}F = \{\mathbf{0}\}$, therefore

$$a_1\boldsymbol{v}_1 + a_2\boldsymbol{v}_2 + \cdots + a_n\boldsymbol{v}_n = \mathbf{0}.$$

Since the $\boldsymbol{v}_i$ $(1 \le i \le n)$ are linearly independent, all the a_i are 0. Accordingly, the $F(\boldsymbol{v}_i)$, $(1 \le i \le n)$ are linearly independent. Accordingly the theorem is proved. □

Proposition 4. 60. *Let a vector space V of the chemical equation* (4. 2) *over the field* $\mathbb{R}$ *has finite dimension and dimV = dimU, then a linear mapping* F: $V \to U$ *is nonsingular if and only if F is surjective, i. e., maps V onto U.*

Proof. By Theorem 4. 56,

$\dim V = \dim(\mathrm{Ker}F) + \dim(\mathrm{Im}F)$.

Thus F is surjective if and only if

$$\mathrm{Im}F = U$$

if and only if

$$\dim(\mathrm{Im}F) = \dim U = \dim V$$

if and only if

$$\dim(\mathrm{Ker}F) = 0$$

if and only if F is nonsingular. □

Proposition 4. 61. *Let a vector space V of the chemical equation* (4. 2) *over the field* $\mathbb{R}$ *has* $\dim V = n$, *then* $V \simeq \mathbb{R}^n$.

Proof. Let $\{\boldsymbol{w}_1, \boldsymbol{w}_2, \ldots, \boldsymbol{w}_n\}$ be a basis of V. Let $[\boldsymbol{v}]$ denote the coordinates of $\boldsymbol{v} \in V$ relative to the given basis. Then the map F: $V \to \mathbb{R}^n$ defined by $F(\boldsymbol{v}) = [\boldsymbol{v}]$ is an isomorphism. Thus $V \simeq \mathbb{R}^n$. □

Theorem 4. 62. *Let a vector space V of the chemical equation* (4. 2) *over the field* $\mathbb{R}$ *has finite dimension, such that dimV = dimU and let* F: $V \to U$ *be a linear mapping, then F is an isomorphism if and only if F is nonsingular.*

Proof. If F is an isomorphism then only 0 maps onto 0 so F is nonsingular.

Assume F is nonsingular. Then $\dim(\mathrm{Ker}F) = 0$.

By Theorem 4. 56,

$$\dim V = \dim(\mathrm{Ker}F) + \dim(\mathrm{Im}F).$$

Therefore,

$$\dim U = \dim V = \dim(\mathrm{Im}F).$$

Since U has finite dimension, $\mathrm{Im}F = U$ and so F is surjective. Thus F is both one-to-one and onto, *i. e.*, F is an isomorphism. □

Proposition 4. 63. *The orthogonal complement* $W^{\perp}$ *is a subspace of a vector space V of the chemical equation* (4. 2) *over the field* $\mathbb{R}$.

Prof. Obviously, $0 \in W^{\perp}$. Now, we assume $\boldsymbol{u}, \boldsymbol{v} \in W^{\perp}$. Then $\forall a, b \in \mathbb{R} \wedge \forall \boldsymbol{w} \in W$,

$$\langle a\boldsymbol{u} + b\boldsymbol{v}, \boldsymbol{w}\rangle = a\langle \boldsymbol{u}, \boldsymbol{w}\rangle + b\langle \boldsymbol{v}, \boldsymbol{w}\rangle$$
$$= a\cdot 0 + b\cdot 0 = 0.$$

Therefore, $a\boldsymbol{u} + b\boldsymbol{v} \in W^{\perp}$ and thus W is a subspace of V. □

Proposition 4. 64. *If the set of vectors* $\{\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_n\}$ *of the chemical equations* (4. 2) *is orthogonal, then the vectors* v_i, $(1 \le i \le n)$ *are linearly independent.*

Proof. Assume $S = \{\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_n\}$ and assume

$$a_1\boldsymbol{v}_1 + a_2\boldsymbol{v}_2 + \cdots + a_n\boldsymbol{v}_n = \boldsymbol{0}. \quad (4.\ 27)$$

Taking the inner product of (4. 27) with $\boldsymbol{v}_1$ we get

$$0 = \langle \boldsymbol{0}, \boldsymbol{v}_1\rangle = \langle a_1\boldsymbol{v}_1 + a_2\boldsymbol{v}_2 + \cdots + a_n\boldsymbol{v}_n, \boldsymbol{v}_1\rangle$$
$$= a_1\langle \boldsymbol{v}_1, \boldsymbol{v}_1\rangle + a_2\langle \boldsymbol{v}_2, \boldsymbol{v}_1\rangle + \cdots + a_n\langle \boldsymbol{v}_n, \boldsymbol{v}_1\rangle$$
$$= a_1\langle \boldsymbol{v}_1, \boldsymbol{v}_1\rangle + a_2\cdot 0 + \cdots + a_n\cdot 0 = a_1\langle \boldsymbol{v}_1, \boldsymbol{v}_1\rangle.$$

Since S is orthogonal, $\langle \boldsymbol{v}_1, \boldsymbol{v}_1\rangle \ne 0$, therefore $a_1 = 0$. Similarly, for $2 \le i \le n$ taking the inner product of (4. 27) with $\boldsymbol{v}_i$,

$$0 = \langle \boldsymbol{0}, \boldsymbol{v}_i\rangle = \langle a_1\boldsymbol{v}_1 + a_2\boldsymbol{v}_2 + \cdots + a_n\boldsymbol{v}_n, \boldsymbol{v}_i\rangle$$
$$= a_1\langle \boldsymbol{v}_1, \boldsymbol{v}_i\rangle + \cdots + a_i\langle \boldsymbol{v}_i, \boldsymbol{v}_i\rangle + \cdots + a_n\langle \boldsymbol{v}_n, \boldsymbol{v}_i\rangle$$
$$= a_i\langle \boldsymbol{v}_i, \boldsymbol{v}_i\rangle.$$

But, $\langle \boldsymbol{v}_i, \boldsymbol{v}_i\rangle \ne 0$ and therefore $a_i = 0$, $(1 \le i \le n)$.

Thus S is linearly independent. □

Proposition 4. 65. *If the set of vectors* $\{\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_n\}$ *of the chemical equations* (4. 2) *is orthogonal, then* $\{a_1\boldsymbol{v}_1, a_2\boldsymbol{v}_2, \ldots, a_n\boldsymbol{v}_n\}$ *is orthogonal* $\forall a_i \in \mathbb{R}$, $a_i \ne 0$, $(1 \le i \le n)$.

Proof. Since $\boldsymbol{v}_i \ne \boldsymbol{0}$, $(1 \le i \le n)$ and $a_i \ne 0$, $(1 \le i \le n)$, we obtain $a_i\boldsymbol{v}_i \ne 0$, $(1 \le i \le n)$. Also, for $i \ne j$, $\langle \boldsymbol{v}_i, \boldsymbol{v}_j\rangle = 0$ and therefore

$$\langle a_i\boldsymbol{v}_i, a_j\boldsymbol{v}_j\rangle = a_ia_j\cdot\langle \boldsymbol{v}_i, \boldsymbol{v}_j\rangle = a_ia_j\cdot 0 = 0.$$

Thus $\{a_1\boldsymbol{v}_1, a_2\boldsymbol{v}_2, \ldots, a_n\boldsymbol{v}_n\}$ is orthogonal. □

Next, we shall continue this section, with a practical approach regarding the tobacco combustion process. The text which follows, it provides a comprehensive description about that process.

Tobacco is a plant substance which contains approximately 3800 components, ranging from small organic and inorganic molecules to biopolymers [23].

The small molecules belong to various classes of compounds such as acids, alcohols, aldehydes, alkaloids, amino acids, Amadori compounds, carbohydrates, esters, isoprenoids, ketones, nitriles, phenols, quinones, sterols, sulphur compounds, terpenes etc.

The biopolymers contain cellulose, hemicellulose, lignin, nucleic acids, pectin, proteins and peptides, starch, etc. During cigarette smoking, these are all subjected to temperatures up to 950ºC in the presence of varying concentrations of oxygen in the burning zone or other tobacco product [24]. According to the research [25] about 4800 substances in tobacco smoke are identified. Many of the smoke components occur from different phases including distillation from tobacco, combustion, pyrolysis and pyrosynthetic reactions. In tobacco smoke are detected 2800 components, indicating the importance of pyrolysis, pyrosynthetic and combustion formation mechanisms.

Rough structural relations of the main combustion processes involved in smoke can be interpreted by the following block diagram shown by Fig. 1.

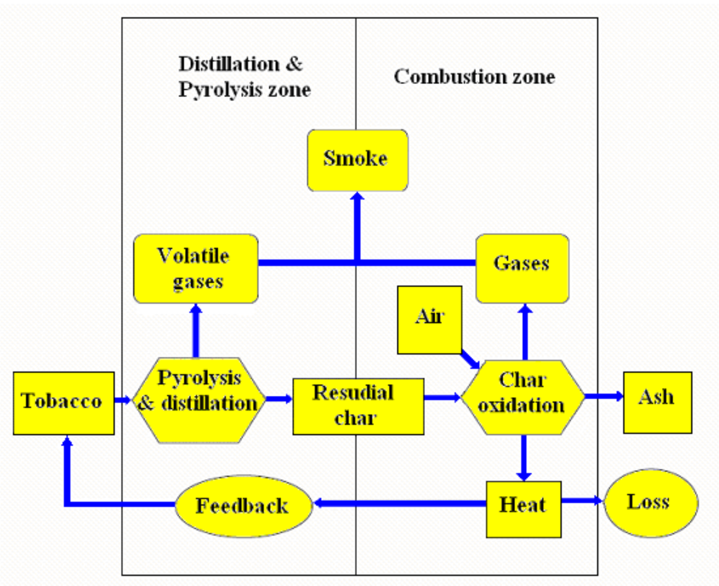

Fig. 1. *A block diagram of cigarette combustion*

In [26] is considered the combustion process and approximately are given the conditions inside the burning zone of the cigarette that influence product formation mechanisms. These include determination of the temperature and gas formation contour distributions at various times in the smoking cycle of a burning cigarette.

A graphic illustration of the major smoke formation mechanisms occurring inside the cigarette is shown by Fig. 2.

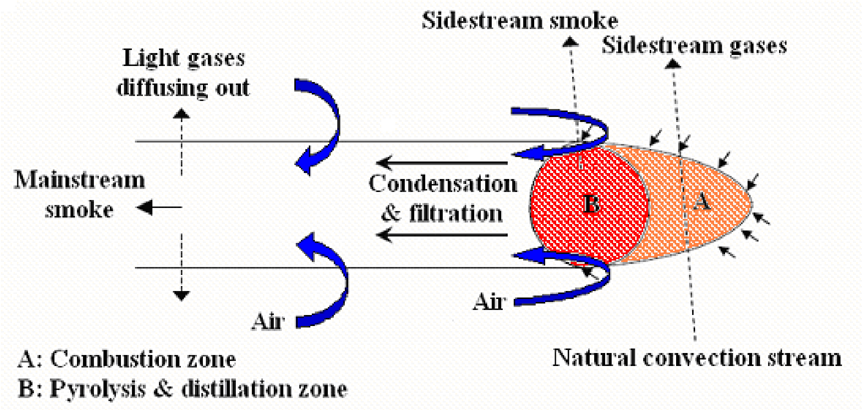

Fig. 2. *The burning cigarette during a puff*

The internal part of the burning region, which has deficit of oxygen and surplus of hydrogen, can be effectively divided into two zones:

A: an exothermic combustion zone, and
B: an endothermic pyrolysis/distillation zone.

Like air is drawn into the cigarette during the puff, oxygen is consumed by combustion with carbonized tobacco and the simple combustion products carbon monoxide, carbon dioxide and water are produced, together with the release of heat which sustains the whole burning process.

Temperature in this region is generated between 700 and 950ºC, and heating rate as high as 500ºC/s is achieved. Instantly downstream of the combustion region is the pyrolysis/distillation zone, where the temperature is between 200 and 600ºC and which is still low in oxygen. Most of smoke products are generated in this region by a variety of mechanisms which are essentially endothermic. A highly concentrated, perhaps supersaturated, vapor is generated and, during a puff, is drawn down the tobacco rod to constitute the mainstream smoke. Its dwelling time in the formation region is very short, only a few milliseconds.

Like the generated vapor is drawn out of the pyrolysis/distillation region during the puff, it cools very rapidly in the presence of diluting air entering at the paper burn line. This brings the vapors of the less volatile compounds rapidly to their infiltration point and condensation takes place like the vapor cools below 350ºC. A dense aerosol consisting of growing droplet particles is produced. The mainstream smoke emerging from the mouth end of the cigarette during a puff is a highly concentrated, dynamic aerosol system. The smoke particles are liquid, with water making up to 20% of the droplet volume and, consequently, they have spherical forms.

Also, the size of smoke particles will increase in humid environments due to absorption of water vapor. This is particularly important in the respiratory tract, where the relative humidity is 99.5%. The relative humidity of mainstream smoke is 60-70% and little particle growth due to water absorption occurs below 90% relative humidity. Particle growth increases sharply with humidity above 90% due to the absorption of water and smoke particles double in size at 99.5% relative humidity. Smoke particle growth by absorption is a complex process dependent on the chemical composition of both the particles and surrounding gas phase. It occurs in milliseconds.

CO and CO_2 are the main products in cigarette smoke and are produced by both thermal decomposition and combustion of many of the components of tobacco: amino acids, carboxylic acids, esters, celluloses, sugars, starch etc. Studies in which the cigarette was smoked or pyrolysed in isotopically labeled oxygen and carbon dioxide [27] have shown that approximately 30% of the carbon monoxide is formed by thermal decomposition of tobacco constituents, about 36% by combustion of tobacco and at least 23% by the endothermic carbonaceous reduction of carbon dioxide: $C + CO_2 \rightarrow 2CO$.

Cigarette smoke is a complex mixture of 4800 identified substances, and 45 are believed by Regulatory Authorities in Canada and the USA to be relevant to smoking-related diseases [28].

If we take into account these substances, together with their approximate amounts in cigarette smoke and their phase in smoke, and phytochemicals that cultivated tobacco or *Nicotiana tabacum* contains, then the following reaction of tobacco combustion will take a place

$$\begin{aligned} & x_1\, C_5H_{14}NO + x_2\, C_8H_{10}N_2O \qquad (4.\ 28) \\ & + x_3\, C_{23}H_{32}N_2O_3 + x_4\, C_4H_9N + x_5\, C_9H_{10}N_2 \\ & + x_6\, C_9H_{12}N_2 + x_7\, C_{10}H_{12}N_2 + x_8\, C_{10}H_{14}N_2 \\ & + x_9\, C_{15}H_{13}N_3 + x_{10}\, C_3H_4O + x_{11}\, C_3H_6O_2 \\ & + x_{12}\, C_7H_6O_2 + x_{13}\, C_{10}H_{12}O + x_{14}\, C_{13}H_{12}O_2 \\ & + x_{15}\, C_{14}H_{10}O_3 + x_{16}\, C_{16}H_{18}O_9 + x_{17}\, C_{21}H_{20}O_{12} \\ & + x_{18}\, C_{25}H_{30}O_{15} + x_{19}\, C_{27}H_{30}O_{16} + x_{20}\, O_2 \\ & \rightarrow x_{21}\, C_5H_5N + x_{22}\, C_9H_7N + x_{23}\, C_{10}H_7N \\ & + x_{24}\, C_{10}H_9N + x_{25}\, C_{15}H_{17}N_3 + x_{26}\, C_6H_5NO_2 \\ & + x_{27}\, C_{10}H_{14}N_2O + x_{28}\, CH_2O + x_{29}\, C_2H_4O \\ & + x_{30}\, C_3H_4O + x_{31}\, C_3H_6O + x_{32}\, C_4H_6O \\ & + x_{33}\, C_4H_8O + x_{34}\, C_6H_6O_2 + x_{35}\, C_{28}H_{48}O \\ & + x_{36}\, C_{29}H_{48}O + x_{37}\, C_{29}H_{50}O + x_{38}\, HCN \\ & + x_{39}\, C_4H_6 + x_{40}\, C_5H_8 + x_{41}\, C_8H_8 + x_{42}\, C_{16}H_{10} \\ & + x_{43}\, C_{20}H_{32} + x_{44}\, CO + x_{45}\, CO_2 + x_{46}\, NO \\ & + x_{47}\, NO_2 + x_{48}\, NH_3 + x_{49}\, H_2O. \end{aligned}$$

Here, the author would like to emphasize very clearly that the organic reaction (4. 28), which describes the tobacco combustion, is a completely new reaction and for the first time it appears in scientific literature. Until now, the tobacco combustion was described by some elementary chemical reactions, which did not satisfy accurately real conditions, and they provided only a rough picture about combustion zone.

Since we have all necessary preconditions for balancing a chemical reaction, now we shall balance the main reaction of tobacco combustion.

This reaction includes all important alkaloids and toxins which tobacco contains.

The reaction (4. 28) belongs to the class of continuum reactions and its balancing is a tough question, which enters deeply in the theory of linear programming, because the necessary and sufficient conditions generate very hard solvable systems of linear inequalities in several variables.

Since, the reaction (4.28) has a stoichiometric matrix

$$\boldsymbol{A} = \left[\begin{array}{cccccccccc} 5 & 8 & 23 & 4 & 9 & 9 & 10 & 10 & 15 & 3 \\ 14 & 10 & 32 & 9 & 10 & 12 & 12 & 14 & 13 & 4 \\ 1 & 2 & 2 & 1 & 2 & 2 & 2 & 2 & 3 & 0 \\ 1 & 1 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right.$$

$$\begin{array}{cccccccccc} 3 & 7 & 10 & 13 & 14 & 16 & 21 & 25 & 27 & 0 \\ 6 & 6 & 12 & 12 & 10 & 18 & 20 & 30 & 30 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 2 & 1 & 2 & 3 & 9 & 12 & 15 & 16 & 2 \end{array}$$

$$\begin{array}{ccccccccc} -5 & -9 & -10 & -10 & -15 & -6 & -10 & -1 & -2 \\ -5 & -7 & -7 & -9 & -17 & -5 & -14 & -2 & -4 \\ -1 & -1 & -1 & -1 & -3 & -1 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -2 & -1 & -1 & -1 \end{array}$$

$$\begin{array}{ccccccccc} -3 & -3 & -4 & -4 & -6 & -28 & -29 & -29 & -1 \\ -4 & -6 & -6 & -8 & -6 & -48 & -48 & -50 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ -1 & -1 & -1 & -1 & -2 & -1 & -1 & -1 & 0 \end{array}$$

$$\left.\begin{array}{ccccccccccc} -4 & -5 & -8 & -16 & -20 & -1 & -1 & 0 & 0 & 0 & 0 \\ -6 & -8 & -8 & -10 & -32 & 0 & 0 & 0 & 0 & -3 & -2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & -2 & -1 & -2 & 0 & -1 \end{array}\right],$$

with a rank$\boldsymbol{A}$ = 4 and 49 molecules, it can be balanced on $49!/[4!(49 - 4)!] = 211876$ different ways. In other words, there are 211876 general solutions, different each other, such that every one generates an infinite number of particular solutions. Only on this way can be fully balanced reaction (4. 28). Really, it is an extremely hard problem. In this work we shall find only one general solution and one particular solution.

The above chemical reaction (4. 28) reduces to the following system of linear equations

$$\begin{aligned} & 5x_1 + 8x_2 + 23x_3 + 4x_4 + 9x_5 + 9x_6 + 10x_7 + 10x_8 \\ & + 15x_9 + 3x_{10} + 3x_{11} + 7x_{12} + 10x_{13} + 13x_{14} \\ & + 14x_{15} + 16x_{16} + 21x_{17} + 25x_{18} + 27x_{19} = 5x_{21} \\ & + 9x_{22} + 10x_{23} + 10x_{24} + 15x_{25} + 6x_{26} + 10x_{27} \\ & + x_{28} + 2x_{29} + 3x_{30} + 3x_{31} + 4x_{32} + 4x_{33} + 6x_{34} \\ & + 28x_{35} + 29x_{36} + 29x_{37} + x_{38} + 4x_{39} \\ & + 5x_{40} + 8x_{41} + 16x_{42} + 20x_{43} + x_{44} + x_{45}, \end{aligned}$$

$$14x_1 + 10x_2 + 32x_3 + 9x_4 + 10x_5 + 12x_6 + 12x_7 + 14x_8 + 13x_9 + 4x_{10} + 6x_{11} + 6x_{12} + 12x_{13} + 12x_{14} + 10x_{15} + 18x_{16} + 20x_{17} + 30x_{18} + 30x_{19} = 5x_{21} + 7x_{22} + 7x_{23} + 9x_{24} + 17x_{25} + 5x_{26} + 14x_{27} + 2x_{28} + 4x_{29} + 4x_{30} + 6x_{31} + 6x_{32} + 8x_{33} + 6x_{34} + 48x_{35} + 48x_{36} + 50x_{37} + x_{38} + 6x_{39} + 8x_{40} + 8x_{41} + 10x_{42} + 32x_{43} + 3x_{48} + 2x_{49},$$

$$x_1 + 2x_2 + 2x_3 + x_4 + 2x_5 + 2x_6 + 2x_7 + 2x_8 + 3x_9 = x_{21} + x_{22} + x_{23} + x_{24} + 3x_{25} + x_{26} + 2x_{27} + x_{38} + x_{46} + x_{47} + x_{48}, \quad (4.\ 29)$$

$$x_1 + x_2 + 3x_3 + x_{10} + 2x_{11} + 2x_{12} + x_{13} + 2x_{14} + 3x_{15} + 9x_{16} + 12x_{17} + 15x_{18} + 16x_{19} + 2x_{20} = 2x_{26} + x_{27} + x_{28} + x_{29} + x_{30} + x_{31} + x_{32} + x_{33} + 2x_{34} + x_{35} + x_{36} + x_{37} + x_{44} + 2x_{45} + x_{46} + 2x_{47} + x_{49}.$$

The general solution of the system (4.29) is given by the following expressions

$$x_1 = (158x_5 + 128x_6 + 166x_7 + 136x_8 + 324x_9 - 66x_{10} - 216x_{11} - 64x_{12} + 80x_{13} + 74x_{14} + 22x_{15} - 742x_{16} - 942x_{17} - 1300x_{18} - 1344x_{19} - 240x_{20} - 98x_{21} - 220x_{22} - 258x_{23} - 228x_{24} - 264x_{25} + 104x_{26} - 16x_{27} + 112x_{28} + 104x_{29} + 66x_{30} + 96x_{31} + 58x_{32} + 88x_{33} + 102x_{34} - 224x_{35} - 262x_{36} - 232x_{37} - 6x_{38} - 62x_{39} - 70x_{40} - 184x_{41} - 458x_{42} - 280x_{43} + 82x_{44} + 202x_{45} + 137x_{46} + 257x_{47} + 62x_{48} + 150x_{49})/157,$$

$$x_2 = (- 95x_5 - 71x_6 - 70x_7 - 46x_8 - 165x_9 - 10x_{10} - 47x_{11} - 43x_{12} + 93x_{13} + 35x_{14} - 49x_{15} - 317x_{16} - 471x_{17} - 530x_{18} - 589x_{19} - 122x_{20} + 47x_{21} + 19x_{22} + 18x_{23} - 6x_{24} + 117x_{25} + 168x_{26} + 107x_{27} + 36x_{28} + 11x_{29} + 10x_{30} - 14x_{31} - 15x_{32} - 39x_{33} + 44x_{34} - 543x_{35} - 544x_{36} - 568x_{37} + 99x_{38} - 76x_{39} - 101x_{40} - 104x_{41} - 136x_{42} - 404x_{43} + 60x_{44} + 121x_{45} + 173x_{46} + 234x_{47} + 76x_{48} + 37x_{49})/157, \quad (4.\ 30)$$

$$x_3 = (21x_5 - 19x_6 - 32x_7 - 30x_8 - 53x_9 - 27x_{10} - 17x_{11} - 69x_{12} - 110x_{13} - 141x_{14} - 148x_{15} - 118x_{16} - 157x_{17} - 175x_{18} - 193x_{19} + 16x_{20} + 17x_{21} + 67x_{22} + 80x_{23} + 78x_{24} + 49x_{25} + 14x_{26} + 22x_{27} + 3x_{28} + 14x_{29} + 27x_{30} + 25x_{31} + 38x_{32} + 36x_{33} + 56x_{34} + 308x_{35} + 321x_{36} + 319x_{37} - 31x_{38} + 46x_{39} + 57x_{40} + 96x_{41} + 198x_{42} + 228x_{43} + 5x_{44} - 3x_{45} - 51x_{46} - 59x_{47} - 46x_{48} - 10x_{49})/157,$$

$$x_4 = (- 240x_5 - 262x_6 - 276x_7 - 298x_8 - 359x_9 + 140x_{10} + 344x_{11} + 288x_{12} - 46x_{13} + 138x_{14} + 372x_{15} + 1612x_{16} + 14x_{17} + 2710x_{18} + 2908x_{19} + 452x_{20} + 127x_{21} + 205x_{22} + 219x_{23} + 241x_{24} + 403x_{25} - 311x_{26} + 72x_{27} - 190x_{28} - 154x_{29} - 140x_{30} - 118x_{31} - 104x_{32} - 82x_{33} - 302x_{34} + 694x_{35} + 708x_{36} + 730x_{37} + 27x_{38} + 122x_{39} + 158x_{40} + 200x_{41} + 334x_{42} + 632x_{43} - 212x_{44} - 438x_{45} - 224x_{46} - 450x_{47} + 35x_{48} - 204x_{49})/157,$$

where x_i, $(5 \le i \le 49)$ are arbitrary real numbers.

Balanced reaction (4. 28) has four generators

$$x_i > 0, \ (1 \le i \le 4), \quad (4.\ 31)$$

which produce an infinite number of particular solutions.

From (4. 31) we obtain the following system of linear inequalities

$$158x_5 + 128x_6 + 166x_7 + 136x_8 + 324x_9 - 66x_{10} - 216x_{11} - 64x_{12} + 80x_{13} + 74x_{14} + 22x_{15} - 742x_{16} - 942x_{17} - 1300x_{18} - 1344x_{19} - 240x_{20} - 98x_{21} - 220x_{22} - 258x_{23} - 228x_{24} - 264x_{25} + 104x_{26} - 16x_{27} + 112x_{28} + 104x_{29} + 66x_{30} + 96x_{31} + 58x_{32} + 88x_{33} + 102x_{34} - 224x_{35} - 262x_{36} - 232x_{37} - 6x_{38} - 62x_{39} - 70x_{40} - 184x_{41} - 458x_{42} - 280x_{43} + 82x_{44} + 202x_{45} + 137x_{46} + 257x_{47} + 62x_{48} + 150x_{49} > 0,$$

$$- 95x_5 - 71x_6 - 70x_7 - 46x_8 - 165x_9 - 10x_{10} - 47x_{11} - 43x_{12} + 93x_{13} + 35x_{14} - 49x_{15} - 317x_{16} - 471x_{17} - 530x_{18} - 589x_{19} - 122x_{20} + 47x_{21} + 19x_{22} + 18x_{23} - 6x_{24} + 117x_{25} + 168x_{26} + 107x_{27} + 36x_{28} + 11x_{29} + 10x_{30} - 14x_{31} - 15x_{32} - 39x_{33} + 44x_{34} - 543x_{35} - 544x_{36} - 568x_{37} + 99x_{38} - 76x_{39} - 101x_{40} - 104x_{41} - 136x_{42} - 404x_{43} + 60x_{44} + 121x_{45} + 173x_{46} + 234x_{47} + 76x_{48} + 37x_{49} > 0, \quad (4.\ 32)$$

$$21x_5 - 19x_6 - 32x_7 - 30x_8 - 53x_9 - 27x_{10} - 17x_{11} - 69x_{12} - 110x_{13} - 141x_{14} - 148x_{15} - 118x_{16} - 157x_{17} - 175x_{18} - 193x_{19} + 16x_{20} + 17x_{21} + 67x_{22} + 80x_{23} + 78x_{24} + 49x_{25} + 14x_{26} + 22x_{27} + 3x_{28} + 14x_{29} + 27x_{30} + 25x_{31} + 38x_{32} + 36x_{33} + 56x_{34} + 308x_{35} + 321x_{36} + 319x_{37} - 31x_{38} + 46x_{39} + 57x_{40} + 96x_{41} + 198x_{42} + 228x_{43} + 5x_{44} - 3x_{45} - 51x_{46} - 59x_{47} - 46x_{48} - 10x_{49} > 0,$$

$$- 240x_5 - 262x_6 - 276x_7 - 298x_8 - 359x_9 + 140x_{10} + 344x_{11} + 288x_{12} - 46x_{13} + 138x_{14} + 372x_{15} + 1612x_{16} + 14x_{17} + 2710x_{18} + 2908x_{19} + 452x_{20} + 127x_{21} + 205x_{22} + 219x_{23} + 241x_{24} + 403x_{25} - 311x_{26} + 72x_{27} - 190x_{28} - 154x_{29} - 140x_{30} - 118x_{31} - 104x_{32} - 82x_{33} - 302x_{34} + 694x_{35} + 708x_{36} + 730x_{37} + 27x_{38} + 122x_{39} + 158x_{40} + 200x_{41} + 334x_{42} + 632x_{43} - 212x_{44} - 438x_{45} - 224x_{46} - 450x_{47} + 35x_{48} - 204x_{49} > 0$$

The inequalities (4. 32) are necessary and suficient conditions to hold (4. 28). In other words (4. 20) holds if and only if are satisfied (4. 32)

If we take into account (4. 30) immediately follows balanced reaction

$$20985498810722\ C_5H_{14}NO \quad (4.33)$$
$$+ 211988664173123\ C_8H_{10}N_2O$$
$$+ 71404734962373\ C_{23}H_{32}N_2O_3$$
$$+ 74612231873620\ C_4H_9N$$
$$+ 64075324647690\ C_9H_{10}N_2$$
$$+ 66410485874690\ C_9H_{12}N_2$$
$$+ 67029087720750\ C_{10}H_{12}N_2$$
$$+ 69364248947750\ C_{10}H_{14}N_2$$

$+ 56492180494820\ C_{15}H_{13}N_3$
$+ 74139764219020\ C_3H_4O$
$+ 67661465635250\ C_3H_6O_2$
$+ 70135873019490\ C_7H_6O_2$
$+ 87997222049440\ C_{10}H_{12}O$
$+ 80852967776850\ C_{13}H_{12}O_2$
$+ 65431891883276\ C_{14}H_{10}O_3$
$+ 28020038320640\ C_{16}H_{18}O_9$
$+ 7007829345630\ C_{21}H_{20}O_{12}$
$+ 5282336567440\ C_{25}H_{30}O_{15}$
$+ 6429296343045\ C_{27}H_{30}O_{16}$
$+ 41657297999398\ O_2$
$\rightarrow 81971892630682\ C_5H_5N$
$+ 76226627132560\ C_9H_7N$
$+ 73010292315826\ C_{10}H_7N$
$+ 74740831521832\ C_{10}H_9N$
$+ 93344031289188\ C_{15}H_{17}N_3$
$+ 206607536035418\ C_6H_5NO_2$
$+ 94371998670282\ C_{10}H_{14}N_2O$
$+ 80908579479454\ CH_2O$
$+ 79543512098852\ C_2H_4O$
$+ 78225105368756\ C_3H_4O$
$+ 77436648403356\ C_3H_6O$
$+ 74952028932304\ C_4H_6O$
$+ 74125308440176\ C_4H_8O$
$+ 80627882148850\ C_6H_6O_2$
$+ 16837231445704\ C_{28}H_{48}O$
$+ 11981945330022\ C_{29}H_{48}O$
$+ 9621876521676\ C_{29}H_{50}O$
$+ 87674677063278\ HCN$
$+ 66578882188196\ C_4H_6$
$+ 64203924295902\ C_5H_8$
$+ 60669668898798\ C_8H_8$
$+ 54853660365650\ C_{16}H_{10}$
$+ 26692479484410\ C_{20}H_{32}$
$+ 84621954002320\ CO$
$+ 93435413813090\ CO_2$
$+ 100038052918110\ NO$
$+ 108851512728880\ NO_2$
$+ 87721851266840\ NH_3$
$+ 82905394621380\ H_2O.$

The system (4. 29) has four (nonzero) linear equations in fourty-nine unknowns; and hence it has 49 - 4 = 45 free variables $x_i > 0$, $(5 \le i \le 49)$. Thus, the dimension of the solution space W of the system (4. 29) is dim $W = 45$. To obtain a basis for W, we set

$$\begin{gathered} x_5 = 1,\ x_6 = \cdots = x_{49} = 0, \\ x_5 = 0,\ x_6 = 1,\ x_7 = \cdots = x_{49} = 0, \\ x_5 = x_6 = 0,\ x_7 = 1,\ x_8 = \cdots = x_{49} = 0, \quad (4.\ 34) \\ \vdots \\ x_5 = \cdots = x_{47} = 0,\ x_{48} = 1,\ x_{49} = 0, \\ x_5 = \cdots = x_{48} = 0,\ x_9 = 1, \end{gathered}$$

in the expressions (4. 30) to obtain the solutions

$\boldsymbol{a}_1$ = (158/157, -95/157, 21/157, -240/157, 1, 0, 0, …, 0, 0),
$\boldsymbol{a}_2$ = (128/157, -71/157, -19/157, -262/157, 0, 1, 0, …, 0, 0),
$\boldsymbol{a}_3$ = (166/157, -70/157, -32/157, -276/157, 0, 0, 1, 0, …, 0, 0),
$\boldsymbol{a}_4$ = (136/157, -46/157, -30/157, -298/157, 0, 0, 0, 1, 0, …, 0, 0),
$\boldsymbol{a}_5$ = (324/157, -165/157, -53/157, -359/157, 0, 0, 0, 0, 1, 0, …, 0, 0),
$\boldsymbol{a}_6$ = (-66/157, -10/157, -27/157, 140/157, 0, 0, 0, 0, 0, 1, 0, …, 0, 0),
$\boldsymbol{a}_7$ = (-216/157, -47/157, -17/157, 344/157, 0, 0, 0, 0, 0, 0, 1, 0, …, 0, 0),
$\boldsymbol{a}_8$ = (-64/157, -43/157, -69/157, 288/157, 0, 0, 0, 0, 0, 0, 0, 1, 0, …, 0, 0),
$\boldsymbol{a}_9$ = (80/157, 93/157, -110/157, -46/157, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, …, 0, 0),
$\boldsymbol{a}_{10}$ = (74/157, 35/157, -141/157, 138/157, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, …, 0, 0),
$\boldsymbol{a}_{11}$ = (22/157, -49/157, -148/157, 372/157, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, …, 0, 0),
$\boldsymbol{a}_{12}$ = (-742/157, -317/157, -118/157, 1612/157, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, …, 0, 0),
$\boldsymbol{a}_{13}$ = (-942/157, -471/157, -157/157, 14/157, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, …, 0, 0),
$\boldsymbol{a}_{14}$ = (-1300/157, -530/157, -175/157, 2710/157, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, …, 0, 0),
$\boldsymbol{a}_{15}$ = (-1344/157, -589/157, -193/157, 2908/157, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, …, 0, 0),
$\boldsymbol{a}_{16}$ = (-240/157, -122/157, 16/157, 452/157, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, …, 0, 0),
$\boldsymbol{a}_{17}$ = (-98/157, 47/157, 17/157, 127/157, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, …, 0, 0),
$\boldsymbol{a}_{18}$ = (-220/157, 19/157, 67/157, 205/157, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, …, 0, 0),
$\boldsymbol{a}_{19}$ = (-258/157, 18/157, 80/157, 219/157, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, …, 0, 0),
$\boldsymbol{a}_{20}$ = (-228/157, -6/157, 78/157, 241/157, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, …, 0, 0),
$\boldsymbol{a}_{21}$ = (-264/157, 117/157, 49/157, 403/157, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, …, 0, 0),
$\boldsymbol{a}_{22}$ = (104/157, 168/157, 14/157, -311/157, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, …, 0, 0),

$\boldsymbol{a}_{23}$ = (-16/157, 107/157, 22/157, 72/157, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, …, 0, 0),
$\boldsymbol{a}_{24}$ = (112/157, 36/157, 3/157, -190/157, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, …, 0, 0),
$\boldsymbol{a}_{25}$ = (104/157, 11/157, 14/157, -154/157, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, …, 0, 0),
$\boldsymbol{a}_{26}$ = (66/157, 10/157, 27/157, -140/157, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, …, 0, 0),
$\boldsymbol{a}_{27}$ = (96/157, -14/157, 25/157, -118/157, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, …, 0, 0),
$\boldsymbol{a}_{28}$ = (58/157, -15/157, 38/157, -104/157, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, …, 0, 0),
$\boldsymbol{a}_{29}$ = (88/157, -39/157, 36/157, -82/157, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, …, 0, 0),
$\boldsymbol{a}_{30}$ = (102/157, 44/157, 56/157, -302/157, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, …, 0, 0),
$\boldsymbol{a}_{31}$ = (-224/157, -543/157, 308/157, 694/157, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, …, 0, 0),
$\boldsymbol{a}_{32}$ = (-262/157, -544/157, 321/157, 708/157, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, …, 0, 0),
$\boldsymbol{a}_{33}$ = (-232/157, -568/157, 319/157, 730/157, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, …, 0, 0),
$\boldsymbol{a}_{34}$ = (-6/157, 99/157, -31/157, 27/157, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, …, 0, 0),
$\boldsymbol{a}_{35}$ = (-62/157, -76/157, 46/157, 122/157, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, …, 0, 0),
$\boldsymbol{a}_{36}$ = (-70/157, -101/157, 57/157, 158/157, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, …, 0, 0),
$\boldsymbol{a}_{37}$ = (-184/157, -104/157, 96/157, 200/157, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, …, 0, 0),
$\boldsymbol{a}_{38}$ = (-458/157, -136/157, 198/157, 334/157, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, …, 0, 0),
$\boldsymbol{a}_{39}$ = (-280/157, -404/157, 228/157, 632/157, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, …, 0, 0),
$\boldsymbol{a}_{40}$ = (82/157, 60/157, 5/157, 212/157, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, …, 0, 0),
$\boldsymbol{a}_{41}$ = (202/157, 121/157, -3/157, -438/157, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, …, 0, 0),
$\boldsymbol{a}_{42}$ = (137/157, 173/157, -51/157, -224/157, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0),
$\boldsymbol{a}_{43}$ = (257/157, 234/157, -59/157, -450/157, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0),
$\boldsymbol{a}_{44}$ = (62/157, 76/157, -46/157, 35/157, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1,0),
$\boldsymbol{a}_{45}$ = (150/157, 37/157, -10/157, -204/157, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1),

The set $\{\boldsymbol{a}_1, \boldsymbol{a}_2, \ldots, \boldsymbol{a}_{45}\}$ is a basis of the solution space W.

The reaction (4. 33) is just one particular reaction obtained from (4. 30). It plays a very important role in tobacco science as well in organic chemistry, because it provides accurate relations among molecules.

By the above organic reaction (4. 33) showed in a molecular form, for the first time in literature the quantitative relations among tobacco alkaloids and toxins are given.

Organic chemistry does not study the quantitative relations among components, and it is one of its biggest shortcomings. By the reactions (4. 33) this gap is filled out.

Now, we shall consider obtained coefficients x_i, $(1 \leq i \leq 49)$ of the reaction (4.33) as elements of a permutation. Let they represent this permutation $(x_1, x_2, \ldots, x_{49})$, which in fact is obtained from the initial permutation $(x_{18}, x_{19}, x_{17}, x_{37}, x_{36}, x_{35}, x_1, x_{43}, x_{16}, x_{20}, x_{42}, x_9, x_{41}, x_5, x_{40}, x_{15}, x_6, x_{39}, x_7, x_{11}, x_8, x_{12}, x_3, x_{23}, x_{33}, x_{10}, x_4, x_{24}, x_{32}, x_{22}, x_{31}, x_{30}, x_{29}, x_{34}, x_{14}, x_{28}, x_{21}, x_{49}, x_{44}, x_{38}, x_{48}, x_{13}, x_{25}, x_{45}, x_{27}, x_{46}, x_{47}, x_{26}, x_2)$ or arranged permutation of the reaction (4.33) is

$\sigma = (x_7, x_{49}, x_{23}, x_{27}, x_{14}, x_{17}, x_{19}, x_{21}, x_{12}, x_{26}, x_{20}, x_{22}, x_{42}, x_{35}, x_{16}, x_9, x_3, x_1, x_2, x_{10}, x_{37}, x_{30}, x_{24}, x_{28}, x_{43}, x_{48}, x_{45}, x_{36}, x_{33}, x_{32}, x_{31}, x_{29}, x_{25}, x_{34}, x_6, x_5, x_4, x_{40}, x_{18}, x_{15}, x_{13}, x_{11}, x_8, x_{39}, x_{44}, x_{46}, x_{47}, x_{41}, x_{38})$.

The parity of the above permutation we shall determine on this way.

Since x_7 is before: x_3, x_1, x_2, x_6, x_5, x_4, then we have 6 inversions;

Since x_{49} is before: x_{23}, x_{27}, x_{14}, x_{17}, x_{19}, x_{21}, x_{12}, x_{26}, x_{20}, x_{22}, x_{42}, x_{35}, x_{16}, x_9, x_3, x_1, x_2, x_{10}, x_{37}, x_{30}, x_{24}, x_{28}, x_{43}, x_{48}, x_{45}, x_{36}, x_{33}, x_{32}, x_{31}, x_{29}, x_{25}, x_{34}, x_6, x_5, x_4, x_{40}, x_{18}, x_{15}, x_{13}, x_{11}, x_8, x_{39}, x_{44}, x_{46}, x_{47}, x_{41}, x_{38}, then we have 47 inversions;

Since x_{23} is before: x_{14}, x_{17}, x_{19}, x_{21}, x_{12}, x_{20}, x_{22}, x_{16}, x_9, x_3, x_1, x_2, x_{10}, x_6, x_5, x_4, x_{18}, x_{15}, x_{13}, x_{11}, x_8, then we have 21 inversions;

Since x_{27} is before: x_{14}, x_{17}, x_{19}, x_{21}, x_{12}, x_{26}, x_{20}, x_{22}, x_{16}, x_9, x_3, x_1, x_2, x_{10}, x_{24}, x_{25}, x_6, x_5, x_4, x_{18}, x_{15}, x_{13}, x_{11}, x_8, then we have 24 inversions;

Since x_{14} is before: x_{12}, x_9, x_3, x_1, x_2, x_{10}, x_6, x_5, x_4, x_{13}, x_{11}, x_8, then we have 12 inversions;

Since x_{17} is before: x_{12}, x_{16}, x_9, x_3, x_1, x_2, x_{10}, x_6, x_5, x_4, x_{15}, x_{13}, x_{11}, x_8, then we have 14 inversions;

Since x_{19} is before: x_{12}, x_{16}, x_9, x_3, x_1, x_2, x_{10}, x_6, x_5, x_4, x_{18}, x_{15}, x_{13}, x_{11}, x_8, then we have 15 inversions;

Since x_{21} is before: x_{12}, x_{20}, x_{16}, x_9, x_3, x_1, x_2, x_{10}, x_6, x_5, x_4, x_{18}, x_{15}, x_{13}, x_{11}, x_8, then we have 16 inversions;

Since x_{12} is before: x_9, x_3, x_1, x_2, x_{10}, x_6, x_5, x_4, x_{11}, x_8, then we have 10 inversions;

Since x_{26} is before: x_{20}, x_{22}, x_{16}, x_9, x_3, x_1, x_2, x_{10}, x_{24}, x_{25}, x_6, x_5, x_4, x_{18}, x_{15}, x_{13}, x_{11}, x_8, then we have 18 inversions;

Since x_{20} is before: x_{16}, x_9, x_3, x_1, x_2, x_{10}, x_6, x_5, x_4, x_{18}, x_{15}, x_{13}, x_{11}, x_8, then we have 14 inversions;

Since x_{22} is before: x_{16}, x_9, x_3, x_1, x_2, x_{10}, x_6, x_5, x_4, x_{18}, x_{15}, x_{13}, x_{11}, x_8, then we have 14 inversions;

Since x_{42} is before: x_{35}, x_{16}, x_9, x_3, x_1, x_2, x_{10}, x_{37}, x_{30}, x_{24}, x_{28}, x_{36}, x_{33}, x_{32}, x_{31}, x_{29}, x_{25}, x_{34}, x_6, x_5, x_4, x_{40}, x_{18}, x_{15}, x_{13}, x_{11}, x_8, x_{39}, x_{41}, x_{38}, then we have 30 inversions;

Since x_{35} is before: x_{16}, x_9, x_3, x_1, x_2, x_{10}, x_{30}, x_{24}, x_{28}, x_{33}, x_{32}, x_{31}, x_{29}, x_{25}, x_{34}, x_6, x_5, x_4, x_{18}, x_{15}, x_{13}, x_{11}, x_8, then we have 23 inversions;

Since x_{16} is before: x_9, x_3, x_1, x_2, x_{10}, x_6, x_5, x_4, x_{15}, x_{13}, x_{11}, x_8, then we have 12 inversions;

Since x_9 is before: x_3, x_1, x_2, x_6, x_5, x_4, x_8, then we have 7 inversions;

Since x_3 is before: x_1, x_2, then we have 2 inversions;

Since x_{10} is before: x_6, x_5, x_4, x_8, then we have 4 inversions;

Since x_{37} is before: x_{30}, x_{24}, x_{28}, x_{36}, x_{33}, x_{32}, x_{31}, x_{29}, x_{25}, x_{34}, x_6, x_5, x_4, x_{18}, x_{15}, x_{13}, x_{11}, x_8, then we have 18 inversions;

Since x_{30} is before: x_{24}, x_{28}, x_{29}, x_{25}, x_6, x_5, x_4, x_{18}, x_{15}, x_{13}, x_{11}, x_8, then we have 12 inversions;

Since x_{24} is before: x_6, x_5, x_4, x_{18}, x_{15}, x_{13}, x_{11}, x_8, then we have 8 inversions;

Since x_{28} is before: x_{25}, x_6, x_5, x_4, x_{18}, x_{15}, x_{13}, x_{11}, x_8, then we have 9 inversions;

Since x_{43} is before: x_{36}, x_{33}, x_{32}, x_{31}, x_{29}, x_{25}, x_{34}, x_6, x_5, x_4, x_{18}, x_{15}, x_{13}, x_{11}, x_8, x_{39}, x_{41}, x_{38}, then we have 18 inversions;

Since x_{48} is before: x_{45}, x_{36}, x_{33}, x_{32}, x_{31}, x_{29}, x_{25}, x_{34}, x_6, x_5, x_4, x_{40}, x_{18}, x_{15}, x_{13}, x_{11}, x_8, x_{39}, x_{44}, x_{46}, x_{47}, x_{41}, x_{38}, then we have 23 inversions;

Since x_{45} is before: x_{36}, x_{33}, x_{32}, x_{31}, x_{29}, x_{25}, x_{34}, x_6, x_5, x_4, x_{40}, x_{18}, x_{15}, x_{13}, x_{11}, x_8, x_{39}, x_{44}, x_{41}, x_{38}, then we have 20 inversions;

Since x_{36} is before: x_{33}, x_{32}, x_{31}, x_{29}, x_{25}, x_{34}, x_6, x_5, x_4, x_{18}, x_{15}, x_{13}, x_{11}, x_8, then we have 14 inversions;

Since x_{33} is before: x_{32}, x_{31}, x_{29}, x_{25}, x_6, x_5, x_4, x_{18}, x_{15}, x_{13}, x_{11}, x_8, then we have 12 inversions;

Since x_{32} is before: x_{31}, x_{29}, x_{25}, x_6, x_5, x_4, x_{40}, x_{18}, x_{15}, x_{13}, x_{11}, x_8, then we have 12 inversions;

Since x_{31} is before: x_{29}, x_{25}, x_6, x_5, x_4, x_{18}, x_{15}, x_{13}, x_{11}, x_8, then we have 10 inversions;

Since x_{29} is before: x_{25}, x_6, x_5, x_4, x_{18}, x_{15}, x_{13}, x_{11}, x_8, then we have 9 inversions;

Since x_{25} is before: x_6, x_5, x_4, x_{18}, x_{15}, x_{13}, x_{11}, x_8, then we have 8 inversions;

Since x_{34} is before: x_6, x_5, x_4, x_{18}, x_{15}, x_{13}, x_{11}, x_8, then we have 8 inversions;

Since x_6 is before: x_5, x_4, then we have 2 inversions;

Since x_5 is before: x_4, then we have 1 inversion;

Since x_{40} is before: x_{18}, x_{15}, x_{13}, x_{11}, x_8, x_{39}, x_{38}, then we have 7 inversions;

Since x_{18} is before: x_{15}, x_{13}, x_{11}, x_8, then we have 4 inversions;

Since x_{15} is before: x_{13}, x_{11}, x_8, then we have 3 inversions;

Since x_{13} is before: x_{11}, x_8, then we have 2 inversions;

Since x_{11} is before: x_8, then we have 1 inversion;

Since x_{39} is before: x_{38}, then we have 1 inversion;

Since x_{44} is before: x_{41}, x_{38}, then we have 2 inversions;

Since x_{46} is before: x_{41}, x_{38}, then we have 2 inversions;

Since x_{47} is before: x_{41}, x_{38}, then we have 2 inversions;

Since x_{41} is before: x_{38}, then we have 1 inversion;

The total number of the inversion is k = 498. According to the Definition 3. 37, the sign of the permutation σ will be sign$\sigma = (-1)^k = (-1)^{498} = 1$, and according to the Definition 3. 38, the permutation σ is even.

Elements of the permutation σ of the chemical equation (4. 33), lie in seven orbits.

The orbits of the permutation σ are:

$\oslash_1 = (x_1, x_2, x_4, x_7, x_8, x_9, x_{12}, x_{15}, x_{16}, x_{18}, x_{19}, x_{21}, x_{22}, x_{25}, x_{27}, x_{29}, x_{30}, x_{32}, x_{33}, x_{37}, x_{38}, x_{39}, x_{40}, x_{43}, x_{44}, x_{45}, x_{49})$ of length $m = 27$,

$\oslash_2 = (x_3, x_5, x_6, x_{14}, x_{17}, x_{23}, x_{24}, x_{28}, x_{35}, x_{36})$ of length $m = 10$,

$\oslash_3 = (x_{10}, x_{11}, x_{13}, x_{20}, x_{26}, x_{41}, x_{42}, x_{48})$ of length $m = 8$,

$\oslash_4 = (x_{31})$ of length $m = 1$,

$\oslash_5 = (x_{34})$ of length $m = 1$,

$\oslash_6 = (x_{46})$ of length $m = 1$

and

$\oslash_7 = (x_{47})$ of length $m = 1$.

Therefore, the permutation can be presented in this form

$$\sigma = \oslash_1 \cup \oslash_2 \cup \oslash_3 \cup \oslash_4 \cup \oslash_5 \cup \oslash_6 \cup \oslash_7.$$

One of the most important criteria for balancing chemical equations is their stability. Stability made analysis shown that this reaction (4. 33) is not stable.

In the next section we shall consider the field temperature problem in the combustion zone by using of two-dimensional heat equation.

5. Field Temperature Problem

Consider the two-dimensional heat equation on the rectangular region $0 < x < \ell$, $0 < y < h$,

$$\partial T/\partial t = k\,(\partial^2 T/\partial^2 x + (\partial^2 T/\partial^2 y), \quad (5.\ 1)$$

with the initial condition

$$T(x, y, 0) = f(x, y), \quad (5.\ 2)$$

and these boundary conditions

$$T(0, y, t) = 0,\ \partial T(\ell, y, t)/\partial x = 0, \quad (5.\ 3)$$

$$\partial T(x, 0, t)/\partial y = 0,\ \partial T(x, h, t)/\partial y = 0,\ t \to \infty.$$

Since the equation and the boundary conditions are linear and homogeneous we can use separation of variables. Assuming a solution of the form $T(x, y, t) = X(x)\cdot Y(y)\cdot T(t)$ and substituting this into the partial differential equation (5. 1) we have

$$T'(t)X(x)Y(y)$$
$$= k\,[X''(x)Y(y)T(t) + Y''(y)X(x)T(t)],$$

so that

$$T'(t)/kT(t) = X''(x)/X(x) + Y''(y)/Y(y) = -\lambda,$$

where λ is the separation constant. This gives

$$X''(x)/X(x) = -\lambda - Y''(y)/Y(y) = -\mu,$$

where μ is another separation constant.

We can satisfy the boundary conditions by requiring that

$$X(0) = X'(\ell) = 0 \text{ and } Y'(0) = Y'(h) = 0,$$

and therefore X and Y satisfy these boundary value problems

$$X''(x) + \mu X(x) = 0,\ 0 < x < \ell, \quad (5.\ 4)$$
$$X(0) = 0 \text{ and } X'(\ell) = 0,$$

and

$$Y''(y) + \alpha Y(y) = 0,\ 0 < y < h, \quad (5.\ 5)$$
$$Y'(0) = 0 \text{ and } Y'(h) = 0,$$

where $\alpha = \lambda - \mu$, while T satisfies the differential equation

$$T' + \lambda k T = 0,\ t > 0. \quad (5.\ 6)$$

The eigenvalues and corresponding eigenfunctions for the problem (5. 4) are

$$\mu_n = [(2n - 1)\pi/2\ell]^2$$

and

$$X_n(x) = \sin[(2n - 1)\pi x/2\ell],\ n = 1, 2, 3, \dots$$

while the eigenvalues and corresponding eigenfunctions for the problem (5. 5) are

$$\alpha_m = (m\pi/h)^2$$

and

$$Y_m(y) = \cos(m\pi y/h),\ m = 0, 1, 2, \dots$$

The corresponding solutions to the time equation (5. 6) are

$$T_{nm} = \exp(-\lambda_{nm} k t),$$

where

$$\lambda_{nm} = \mu_n + \alpha_m = [(2n - 1)\pi/2\ell]^2 + (m\pi/h)^2,$$

we need to use all possible combinations of indices $n = 1, 2, 3, \dots$ and $m = 0, 1, 2, \dots$

The products

$$T_{nm}(x, y, t) = X_n(x)\cdot Y_m(y)\cdot T_{nm}(t)$$
$$= \sin[(2n - 1)\pi x/2\ell]\cdot\cos(m\pi y/h)\cdot\exp(-\lambda_{nm} k t),$$

satisfy the partial differential equation and the boundary conditions, and by the superposition principle, the function

$$T(x, y, t) = \sum_{n=1}^{\infty}\sum_{m=0}^{\infty} C_{nm} \sin[(2n - 1)\pi x/2\ell]$$
$$\times\cos(m\pi y/h)\exp(-\lambda_{nm} k t), \quad (5.\ 7)$$

also satisfies the partial differential equation and all the boundary conditions. In order to satisfy the initial condition, we could use the fact that the eigenfunctions

$$\{\sin[(2n - 1)\pi x/2\ell]\cdot\cos(m\pi y/h)\}_{n\geq 1,\, m\geq 0}$$

form an orthogonal set on the rectangle $[0, \ell]\times[0,h]$ in $\mathbb{R}^2$. However, we use another method which is similar to the methods used for one-dimensional Fourier series expansions. Setting

$t = 0$ in the expression above for $T(x, y, t)$, we want

$$18 = \sum_{n=1}^{\infty} \sum_{m=0}^{\infty} C_{nm} \sin[(2n - 1)\pi x/2\ell]\cdot\cos(m\pi y/h),$$

for $0 \leq x \leq \ell$ and $0 \leq y \leq h$, and writing this as

$$18 = \sum_{n=1}^{\infty} B_n(y)\sin[(2n - 1)\pi x/2\ell].$$

This is a Fourier sine series expansion of 18 on the interval $[0,\ell]$ holding y fixed, and therefore

$B_n(y) = (36/\ell)\ {}_0\!\int^{\ell} \sin[(2n - 1)\pi x/2\ell]dx$, $(n \geq 1)$.
For $n \geq 1$,

$$B_n(y) = \sum_{m=0}^{\infty} C_{nm} \cos(m\pi y/h),$$

is the expansion of $B_n(y)$ on the interval $[0,h]$, so that

$$C_{n0} = (1/h)\ {}_0\!\int^{h} B_n(y)dy$$
$$= (36/\ell h)\ {}_0\!\int^{h}\ {}_0\!\int^{\ell} \sin[(2n - 1)\pi x/2\ell]dxdy,\ (n \geq 1).$$
$$C_{nm} = (2/h)\ {}_0\!\int^{h} B_n(y)\cos(m\pi y/h)dy$$
$$= (72/\ell h)\ {}_0\!\int^{h}\ {}_0\!\int^{\ell} \sin[(2n - 1)\pi x/2\ell] \times\cos(m\pi y/h)dxdy,\ (\forall\ n, m \geq 1).$$

We can note that that in the solution (5. 7) all terms in the sum for which either $n \geq 1$ or $m \geq 1$ contain a factor of $\exp(-\lambda_{nm}kt)$, where $\lambda_{nm} > 0$ and as $t \to \infty$, all these terms vanish, and therefore $\lim_{t \to \infty} T(x, y, t) = 0$.

According to the above obtained results, on the Fig. 3 are presented the particular $\mathbb{R}^2$ isotherms for $T(x, y, 0) = 18$ºC, $0 < x < 8$ mm, $0 < y < 10$ mm, and $k = 1\cdot10^{-7}$ m^2/s in the combustion zone for the free burn profile.

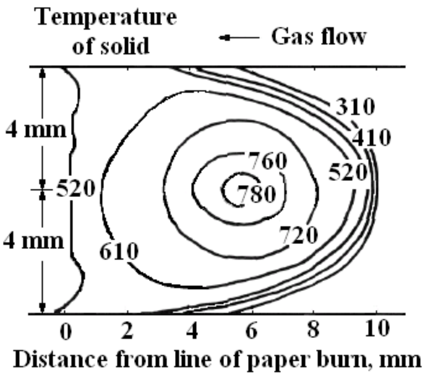

Fig. 3. *Temperature distribution in* $\mathbb{R}^2$, ºC

6. Smoke Filtration Problem

Let us consider a regular cigarette being smoked under the following conditions:

1° In the cigarette smoking process under continuous inhalation (*i. e.*, with a constant speed of the air and burning gases passing through the cigarette and a constant combustion factor) there enters into the cigarette a constant fraction α of any components X_i, $(1 \leq i \leq n)$ of tobacco (volatiles oils, carbohydrates, proteins, nicotine, organic acids, and so on.), while the remaining fraction $1 - \alpha$ escapes into atmosphere.

2° In filtration of any components X_i, $(1 \leq i \leq n)$ of tobacco through the cigarette, the absorption factor of the tobacco β is constant.

3° We ignore the change in length of the cigarette during the time of passage of smoke through it.

Remark. *The coefficients α and β can be naturally distinguished for different components X_i, $(1 \leq i \leq n)$ of tobacco.*

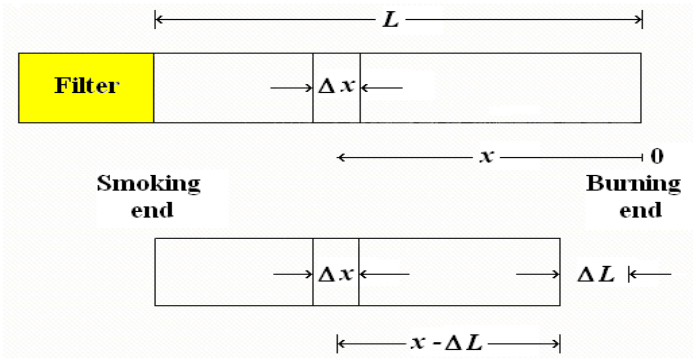

Fig. 4. *A schematic view of a cigarette*

Let $C_i(x, L)$ denote the concentration of components X_i, $(1 \leq i \leq n)$ for a cigarette of length L at a distance x from the burning end as it is showed in Fig. 4.

The amount of the components X_i, $(1 \leq i \leq n)$ transmitted down the cigarette after burning ΔL of cigarette is, to first order terms, $\Delta S =$

$\alpha C_i(0, L)\Delta L$. The fraction of ΔS filtered out by element Δx is

$$\Delta(\Delta S) = \Delta C_i \cdot \Delta x = \alpha\beta e^{-\beta x} C_i(0, L)\Delta L \cdot \Delta x. \quad (6.\ 1)$$

This follows since the filtration law is

$$dS/dx = -\beta S \text{ or } S = S_0 e^{-\beta x},$$

and thus, to first order terms, $\Delta S = \beta S \Delta x$.

Equation (6. 1) can be rewritten as

$$[C_i(x - \Delta L, L - \Delta L) - C_i(x, L)]/\Delta L = \alpha\beta C_i(0, L)e^{-\beta x}.$$

On letting $\Delta L \to 0$, one obtains the mixed partial differential equation of first order

$$\partial C_i(x, L)/\partial x + \partial C_i(x, L)/\partial L = -\alpha\beta C_i(0, L)e^{-\beta x}, \quad (6.\ 2)$$

subject to the initial condition $C_i(0, L_0) = C_0$ (constant), where L_0 is the initial length of the cigarette.

To solve the above partial equation (6. 2) we suppose a solution of the form

$$C_i(x, L) = C_0 + \varphi(L)e^{-\beta x},$$

where $\varphi(L_0) = 0$. This transforms partial equation (6. 2) into following ordinary linear differential equation

$$\varphi'(L) - \beta(1 - \alpha)\varphi(L) = -\alpha\beta C_0.$$

Therefore

$$C_i(x, L) = C_0 + C_0\alpha e^{-\beta x}[1 - e^{-\beta(1 - \alpha)(L_0 - L)}]/(1 - \alpha).$$

The amount of components X_i, $(1 \le i \le n)$ transmitted to the smoker after burning ΔL of the cigarette is, to first order terms,

$$\Delta S_t = \alpha C_i(0, L)e^{-\beta L}\Delta L.$$

Therefore, the total amount transmitted when the cigarette has been burned down to a length L_f is

$$S_t = \int_{L_f}^{L_0} \alpha C_i(0, L)e^{-\beta L} dL \quad (6.\ 3)$$
$$= \alpha C_0 e^{-\beta L_f}[1 - e^{-\beta(1 - \alpha)(L_0 - L_f)}]/(1 - \alpha)\beta$$
$$= [C_i(L_f, L_f) - C_0]/\beta.$$

The total amount of X_i, $(1 \le i \le n)$ destroyed by burning is

$$S_b = \int_{L_f}^{L_0} (1 - \alpha)C_i(0, L)Dl = C_0(L_0 - L_f) - \alpha C_0[1 - e^{-\beta(1 - \alpha)(L_0 - L_f)}]/(1 - \alpha)\beta.$$

The amount of X_i, $(1 \le i \le n)$ left in the cigarette is

$$S_c = \int_0^{L_f} C_i(x, L_f)dx = C_0 L_f + \alpha C_0(1 - e^{-\beta L_f})[1 - e^{-\beta(1 - \alpha)(L_0 - L_f)}]/(1 - \alpha)\beta.$$

It follows immediately that

$$S_t + S_b + S_c = C_0 L_0. \quad (6.\ 4)$$

Equation (4) expresses the material balance on components X_i, $(1 \le i \le n)$.

We can now determine the effect of the initial length L_0 on the amount S_t transmitted to the smoker. Let as consider two cigarettes of lengths L_0 and L_0', and identical otherwise. The amounts S_t and S_t' may be compared on one of the following two principles:

(P_1) both cigarettes burn down the same amount,

(P_2) both cigarettes burn down to the same final length.

In the latter case, we would consider the amount S_t per unite length of cigarette smoked. On either principle, the longer cigarette is more effective in reducing the amount S_t transmitted to the smoker.

We shall now consider the effects of filtration alone on the amount S_t. We let $\alpha \to 1$ (zero burning fraction). Then (6. 3) reduces to

$$S_t = C_0 e^{-\beta L_f}(L_0 - L_f).$$

Using principle (P_1) for a comparison we get

$$S_t/S_t' = e^{-\beta L_f}/e^{-\beta L_f'}. \quad (6.\ 5)$$

Therefore, the initially longer cigarette is more effective in reducing the amount of S_t transmitted. Though, if we now use principle (P_2) we obtain

$$S_t/(L_0 - L_f) = S_t'/(L_0' - L_f). \quad (6.\ 6)$$

In this case, the filtering capacity is independent of the initial length. Since many smokers burn their cigarettes (be they regular or king size) to the same approximate final length before discarding them, principle (P_2) would be indicated.

To approximate more closely to actual smoking conditions, we shall consider a cigarette being smoked in the following way. The cigarette is first smoked a length ΔL_s under steady inhalation and then is allowed to burn a length ΔL_b without inhalation. This process is repeated until the cigarette is discarded.

We now have to solve the partial differential equation (6. 2) subject to the nonconstant initial condition $C_i(x, L_0) = C(x)$, where $C(x)$ is a special function. To solve (6. 2) under this condition, we let

$$C_i(x, L) = e^{-\beta x}[U(x, L) + V(L)].$$

Then (6. 2) becomes

$$\partial U(x, L)/\partial x + \partial U(x, L)/\partial L + V'(L) = \beta[U(x, L) + V(L)] - \alpha\beta[U(0, L) + V(L)].$$

If we let

$$\partial U(x, L)/\partial x + \partial U(x, L)/\partial L = \beta U(x, L), \quad (6.\ 7)$$

subject to the condition

$$U(x, L_0) = e^{\beta x}C(x), \quad (6.\ 8)$$

then $V(L)$ must satisfy

$$dV(L)/dL - \beta(1 - \alpha)V(L) = -\alpha\beta U(0, L), \quad (6.\ 9)$$

subject to the initial condition $V(L_0) = 0$.

The subsidiary equations for solving partial differential equation (7) are

$$dx = dL = dU(x, L)/\beta.$$

Consequently,

$$U(x, L) = e^{\beta x}\zeta(L - x),$$

where $\zeta(x)$ is an arbitrary function of x. To determine $\zeta(x)$, we use (6. 8). Thus we find that

$$U(x, L) = e^{\beta x}C(L_0 - L + x).$$

We can now solve (6. 9), and the solution is

$$V(L) = \alpha\beta e^{\beta(1 - \alpha)L} {}_{L}\int^{L_0} e^{-\beta(1 - \alpha)L}C(L_0 - L)dL,$$

whence

$$C(x, L) = C(L_0 - L + x)$$
$$+ \alpha\beta e^{\beta(1 - \alpha)L - \beta x} {}_{L}\int^{L_0} e^{-\beta(1 - \alpha)L}C(L_0 - L)dL.$$

If we now smoke a length ΔL_s under steady inhalation, the concentration function will be given by

$$C_1(x, L) = C_0$$
$$+ \alpha\beta e^{-\beta x + \beta(1 - \alpha)L_1} {}_{L_1}\int^{L_0} e^{-\beta(1 - \alpha)L}C_0 dL,$$

where $L_1 = L_0 - \Delta L_s$. After burning a length ΔL_b, concentration function will then be

$$C_1^*(x, L_2) = C_1(x + \Delta L_b, L_1).$$

Here we have

$$L_2 = L_1 - \Delta L_b = L_0 - \Delta L_s - \Delta L_b.$$

On repeating another cycle we obtain

$$C_2(x, L_3) = C_1^*(L_2 - L_3 + x, L_2)$$
$$+ \alpha\beta e^{-\beta x + \beta(1 - \alpha)L_3}$$
$$\times {}_{L_3}\int^{L_2} e^{-\beta(1 - \alpha)L}C_1^*(L_2 - L, L_2)dL,$$
$$C_2^*(x, L_4) = C_2(x + \Delta L_b, L_3).$$

By induction, we find that after n cycles,

$$C_n(x, L_{2n-1}) = C_{n-1}^*(L_{2n-2} - L_{2n-1} + x, L_{2n-2})$$
$$+ \alpha\beta e^{-\beta x + \beta(1 - \alpha)L_{2n-1}}$$
$$\times {}_{L_{2n-1}}\int^{L_{2n-2}} e^{-\beta(1 - \alpha)L}C_{n-1}^*(L_{2n-2} - L, L_{2n-2})dL,$$
$$C_n^*(x, L_{2n}) = C_n(x + \Delta L_b, L_{2n-1}),$$

where

$$L_{2n-1} = L_0 - n\Delta L_s - (n - 1)\Delta L_b,$$
$$L_{2n} = L_0 - n\Delta L_s - n\Delta L_b.$$

The amount S_{tn} transmitted to the smoker during the n-th cycle is given by

$$S_{tn} = {}_{L_{2n-1}}\int^{L_{2n-2}} \alpha C_{n-1}(0, L)e^{-\beta L}dL.$$

Thus, the total amount S_T absorbed by the smoker after n cycles is

$$S_T = \sum_{n=1}^{N} S_{tn}$$

$$= \sum_{n=1}^{N} {}_{L_{2n-1}}\int^{L_{2n-2}} \alpha C_{n-1}(0, L)e^{-\beta L}dL.$$

Again, in order to determine the effect of filtration alone, we set $\alpha = 1$. It then can be established by induction that

$$C_N(x, L)/C_0 = 1$$
$$+ \beta\Delta L_s e^{-\beta x}[(e^{-(N - 1)\beta\Delta L_b} - 1)/(1 - e^{\beta\Delta L_b})$$
$$+ (L_0 - L)/\Delta L_s - (N - 1)(1 + \Delta L_b/\Delta L_s)],$$

and that also

$$S_T = C_0\Delta L_s e^{\beta(N\Delta L_s - L_0)}\times[(1 - e^{N\beta\Delta L_b})/(1 - e^{\beta\Delta L_b})].$$

For a comparison of the filtration efficiency of two cigarettes (under discontinuous inhalation), identical except for length, it is reasonable to keep ΔL_s and ΔL_b fixed. Then, using principle (P_1) we get

$$S_T / S_T' = e^{-\beta L_0}/e^{-\beta L_0'},$$

which corresponds to (6. 5), and to which it reduces if $\Delta L_b = 0$. By using principle (P_2) we obtain

$$(S_T/N\Delta L_s)/(S_T'/N'\Delta L_s)$$
$$= [(1 - e^{-N\beta\Delta L_b})/N]/[(1 - e^{-N'\beta\Delta L_b})/N'].$$

Since $(1 - e^{-x})/x$ is a monotonic decreasing function, it follows that under either principles, (P_1) and (P_2), the longer cigarette is a better filter under discontinuous inhalation which more nearly approximates actual smoking conditions than continuous inhalation.

7. Groups' Formation Problem

I. *A symmetric group* S_{49}

Let G be the symmetric group S_{49}. We have $|G|$ = 608281864034267560872252163321295376887552831379210240000000000 $= 2^{46}\times3^{22}\times5^{10}\times7^8\times11^4\times13^3\times17^2\times19^2\times23^2\times29\times31\times37\times41\times43\times47$.

Generators of G:	$a_1 = (1,2,\ldots,49)$ (order 49)
	$a_2 = (1,2)$ (order 2)

The center of G is trivial. The derived subgroup $D = [G, G]$ is a simple group of order 304140932017133780436126081660647688443776415689605120000000000, generated by {(1,8,42,39,49,46,9,24,12,40,23,21,16,44,37,6,7,11,22,4,34,19)(2,36,33,18,47,20,28,10,26,3,25,35,17,32,27)(5,15,31,43,48,14,30,41,13,45)(1,20,18,43,17,25,4,40,45,42,24,36,46,3,22,27,41,13,14,31,44,32,7,33,47)(2,12,35,49,28,21,9,30,16,39,6,34,38,23,8,10,37,19,29,5,26,11,48)} and $G/D \cong C_2$.

Let G be a group generated by 2 permutations. We have

$|G|$ = 304140932017133780436126081660647688443776415689605120000000000 $= 2^{45}\times3^{22}\times5^{10}\times7^8\times11^4\times13^3\times17^2\times19^2\times23^2\times29\times31\times37\times41\times43\times47$.

G is the alternating group A_{49} on the set {1, 2, …, 49}. It's a simple group.

II. *An alternating group* A_{49}

Let G be the alternating group A_{49}. We have

$|G|$ = 304140932017133780436126081660647688443776415689605120000000000

$= 2^{45}\times3^{22}\times5^{10}\times7^{8}\times11^{4}\times13^{3}\times17^{2}\times19^{2}\times23^{2}\times29\times31\times37\times41\times43\times47.$

Generators of G:	a_1 = (1,2,… ,49) (order 49)
	a_2 = (47,48,49) (order 3)

III. *A primitive group of degree* 49

There are in total 40 primitive groups (up to conjugation) on the set {1, 2, …, 49}. Only 38 of them are listed below, the rest being too big (alternating or symmetric).

Number	Order	Nature
1	196	solvable
2	294	solvable
3	392	solvable
4	392	solvable
5	392	solvable
6	588	solvable
7	588	solvable
8	588	solvable
9	784	solvable
10	784	solvable
11	784	solvable
12	882	solvable
13	1176	solvable
14	1176	solvable
15	1176	solvable
16	1176	solvable
17	1176	solvable
18	1176	solvable
19	1568	solvable
20	1764	solvable
21	1764	solvable
22	2352	solvable
23	2352	solvable
24	2352	solvable
25	2352	solvable
26	3528	solvable
27	3528	solvable
28	4704	solvable
29	7056	solvable
30	49	cyclic
31	147	solvable
32	294	solvable
33	98	solvable
34	56448	---
35	12700800	---
36	25401600	---
37	25401600	---
38	50803200	---

1. Let G be a primitive group of degree 49 with 2 generators. We have $|G| = 196 = 2^2\times7^2$.

Generators of G:	a_1 = (2,38,5,20)(3,46,6,28)(4,12,7,30)(8,37,29,19)(9,13,33,31)(10,24,34,48)(11,43,35,22)(14,18,32,42)(15,45,36,27)(16,26,40,44)(17,21,41,39)(23,49,47,25) (order 4)
	a_2 = (1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35) (order 7)

G is a solvable group. G acts transitively on the set of 49 elements {1, 2, …, 49}. The center of G is trivial.

The derived subgroup $D = [G, G]$ is an abelian group of order 49, generated by {(1,16,32,24,48,14,40)(2,18,31,27,49,12,36)(3,20,35,26,43,9,39)(4,17,34,28,47,8,37)(5,15,30,25,45,13,42)(6,21,33,22,44,11,38)(7,19,29,23,46,10,41)(1,26,42,34,10,18,44)(2,22,40,35,13,17,46)(3,25,37,29,12,21,48)(4,23,36,33,14,20,45)(5,28,41,31,11,16,43)(6,24,39,30,8,19,49)(7,27,38,32,9,15,47)} and $G/D \cong C_4$.

Lower central series				
Level	Order	Nature	Quotient	Successive quotient
1	49	abelian	cyclic	C_4

Derived series				
Level	Order	Nature	Quotient	Successive quotient
1	49	abelian	cyclic	C_4
2	1	trivial	solvable	C_7^2

Conjugacy classes of subgroups					
Serial	Order	Nature	Conjugacy classes	Maximal subgroup classes	Generators
1	1	trivial	1	--	---
2	2	cyclic	49	1	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12, 30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42) (19,37)(20,38)(21,39)(22,43)(23,47)(24,48) (25,49)(26,44)(27,45)(28,46)
3	4	cyclic	49	2	(2,20,5,38)(3,28,6,46)(4,30,7,12)(8,19,29,37) (9,31,33,13)(10,48,34,24)(11,22,35,43)(14,42, 32,18)(15,27,36,45)(16,44,40,26)(17,39,41,21) (23,25,47,49)
4	7	cyclic	2	1	(1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10, 24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26, 19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21, 42,49,35)
5	7	cyclic	2	1	(1,9,25,17,41,49,33)(2,11,24,20,42,47,29)(3, 13,28,19,36,44,32)(4,10,27,21,40,43,30)(5,8, 23,18,38,48,35)(6,14,26,15,37,46,31)(7,12, 22,16,39,45,34)
6	7	cyclic	2	1	(1,10,26,18,42,44,34)(2,13,22,17,40,46,35) (3,12,25,21,37,48,29)(4,14,23,20,36,45,33) (5,11,28,16,41,43,31)(6,8,24,19,39,49,30) (7,9,27,15,38,47,32)
7	7	cyclic	2	1	(1,2,4,3,6,7,5)(8,9,11,10,13,14,12)(15,16,18, 17,20,21,19)(22,23,25,24,27,28,26)(29,30,32, 31,34,35,33)(36,37,39,38,41,42,40)(43,44,46, 45,48,49,47)
8	14	dihedral	14	2,7	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12, 30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42) (19,37)(20,38)(21,39)(22,43)(23,47)(24,48) (25,49)(26,44)(27,45)(28,46)(1,2,4,3,6,7,5)(8, 9,11,10,13,14,12)(15,16,18,17,20,21,19)(22, 23,25,24,27,28,26)(29,30,32,31,34,35,33)(36, 37,39,38,41,42,40)(43,44,46,45,48,49,47)
9	14	dihedral	14	2,4	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12, 30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42) (19,37)(20,38)(21,39)(22,43)(23,47)(24,48) (25,49)(26,44)(27,45)(28,46)(1,8,22,15,36,43, 29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31) (4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6, 13,27,20,41,48,34)(7,14,28,21,42,49,35)
10	14	dihedral	14	2,5	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12, 30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42) (19,37)(20,38)(21,39)(22,43)(23,47)(24,48) (25,49)(26,44)(27,45)(28,46)(1,9,25,17,41,49, 33)(2,11,24,20,42,47,29)(3,13,28,19,36,44,32) (4,10,27,21,40,43,30)(5,8,23,18,38,48,35)(6,

					14,26,15,37,46,31)(7,12,22,16,39,45,34)
11	14	dihedral	14	2,6	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46),(1,10,26,18,42,44,34)(2,13,22,17,40,46,35)(3,12,25,21,37,48,29)(4,14,23,20,36,45,33)(5,11,28,16,41,43,31)(6,8,24,19,39,49,30)(7,9,27,15,38,47,32)
12	49	abelian	1	4,5,6,7	(1,2,4,3,6,7,5)(8,9,11,10,13,14,12)(15,16,18,17,20,21,19)(22,23,25,24,27,28,26)(29,30,32,31,34,35,33)(36,37,39,38,41,42,40)(43,44,46,45,48,49,47)(1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35)
13	98	solvable	1	8,9,10,11,12	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35)(1,2,4,3,6,7,5)(8,9,11,10,13,14,12)(15,16,18,17,20,21,19)(22,23,25,24,27,28,26)(29,30,32,31,34,35,33)(36,37,39,38,41,42,40)(43,44,46,45,48,49,47)
14	196	*G*	1	3,13	(2,38,5,20)(3,46,6,28)(4,12,7,30)(8,37,29,19)(9,13,33,31)(10,24,34,48)(11,43,35,22)(14,18,32,42)(15,45,36,27)(16,26,40,44)(17,21,41,39)(23,49,47,25),(1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35)

Proper normal subgroups					
Serial	Order	Nature	Quotient	Subgroups	Generators
1	49	abelian	cyclic	--	(1,11,27,19,37,45,35)(2,10,28,15,39,48,33)(3,14,22,18,41,47,30)(4,13,26,16,38,49,29)(5,9,24,21,36,46,34)(6,12,23,17,42,43,32)(7,8,25,20,40,44,31), (1,43,15,8,29,36,22)(2,44,16,9,30,37,23)(3,45,17,10,31,38,24)(4,46,18,11,32,39,25)(5,47,19,12,33,40,26)(6,48,20,13,34,41,27)(7,49,21,14,35,42,28)
2	98	solvable	cyclic	1	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(1,19,35,27,45,11,37)(2,15,33,28,48,10,39)(3,18,30,22,47,14,41)(4,16,29,26,49,13,38)(5,21,34,24,46,9,36)(6,17,32,23,43,12,42)(7,20,31,25,44,8,40),(1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35)

Sylow subgroups						
p	Order	Conjug. classes	Nature	Normalizer	Normal closure	Generators
2	4	49	cyclic	4	G	(2,38,5,20)(3,46,6,28)(4,12,7,30)(8,37,29, 19)(9,13,33,31)(10,24,34,48)(11,43,35,22) (14,18,32,42)(15,45,36,27)(16,26,40,44) (17,21,41,39)(23,49,47,25)
7	49	1	abelian	G	49	(1,15,29,22,43,8,36)(2,16,30,23,44,9,37) (3,17,31,24,45,10,38)(4,18,32,25,46,11,39) (5,19,33,26,47,12,40)(6,20,34,27,48,13,41) (7,21,35,28,49,14,42),(1,45,19,11,35,37,27) (2,48,15,10,33,39,28)(3,47,18,14,30,41,22) (4,49,16,13,29,38,26)(5,46,21,9,34,36,24) (6,43,17,12,32,42,23)(7,44,20,8,31,40,25)

2. Let G be a primitive group of degree 49 with 3 generators. We have $|G| = 294 = 2\times3\times7^2$.

Generators of G:	a_1 = (2,8)(3,15)(4,22)(5,29)(6,36)(7,43)(10,16)(11,23)(12,30) (13,37)(14,44)(18,24)(19,31)(20,38)(21,45)(26,32)(27,39) (28,46)(34,40)(35,47)(42,48) (order 2)
	a_2 = (2,6,4)(3,7,5)(8,22,36)(9,27,39)(10,28,40)(11,23,41)(12,24,42) (13,25,37)(14,26,38)(15,29,43)(16,34,46)(17,35,47)(18,30,48) (19,31,49)(20,32,44)(21,33,45) (order 3)
	a_3 = (1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31) (4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34) (7,14,28,21,42,49,35) (order 7)

This generating set is not minimal. There is a smaller generating set with 2 elements. G is a solvable group. G acts transitively on the set of 49 elements {1, 2, …, 49}. The center of G is trivial.

The derived subgroup $D = [G, G]$ is a solvable group of order 147, generated by {(2,6,4)(3,7,5)(8,22,36)(9,27,39)(10,28,40)(11,23,41)(12,24,42)(13,25,37)(14,26,38)(15,29,43)(16,34,46)(17,35,47)(18,30,48)(19,31,49)(20,32,44)(21,33,45)(1,30,46,38,20,28,12)(2,32,45,41,21,26,8)(3,34,49,40,15,23,11)(4,31,48,42,19,22,9)(5,29,44,39,17,27,14)(6,35,47,36,16,25,10)(7,33,43,37,18,24,13)} and $G/D \cong C_2$.

Lower central series				
Level	Order	Nature	Quotient	Successive quotient
1	147	solvable	cyclic	C_2

Derived series				
Level	Order	Nature	Quotient	Successive quotient
1	147	solvable	cyclic	C_2
2	49	abelian	dihedral	C_3
3	1	trivial	solvable	C_7^2

Conjugacy classes of subgroups					
Serial	Order	Nature	Conjugacy classes	Maximal subgroup classes	Generators
1	1	trivial	1	--	---
2	2	cyclic	21	1	(2,8)(3,15)(4,22)(5,29)(6,36)(7,43)(10,16) (11,23)(12,30)(13,37)(14,44)(18,24)(19,31) (20,38)(21,45)(26,32)(27,39)(28,46)

					(34,40)(35,47)(42,48)
3	3	cyclic	49	1	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)
4	6	dihedral	49	2,3	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)(2,8)(3,15)(4,22)(5,29)(6,36)(7,43)(10,16)(11,23)(12,30)(13,37)(14,44)(18,24)(19,31)(20,38)(21,45)(26,32)(27,39)(28,46)(34,40)(35,47)(42,48)
5	7	cyclic	2	1	(1,2,4,3,6,7,5)(8,9,11,10,13,14,12)(15,16,18,17,20,21,19)(22,23,25,24,27,28,26)(29,30,32,31,34,35,33)(36,37,39,38,41,42,40)(43,44,46,45,48,49,47)
6	7	cyclic	3	1	(1,9,25,17,41,49,33)(2,11,24,20,42,47,29)(3,13,28,19,36,44,32)(4,10,27,21,40,43,30)(5,8,23,18,38,48,35)(6,14,26,15,37,46,31)(7,12,22,16,39,45,34)
7	7	cyclic	3	1	(1,10,26,18,42,44,34)(2,13,22,17,40,46,35)(3,12,25,21,37,48,29)(4,14,23,20,36,45,33)(5,11,28,16,41,43,31)(6,8,24,19,39,49,30)(7,9,27,15,38,47,32)
8	14	dihedral	3	2,7	(2,22)(3,29)(4,36)(5,43)(6,8)(7,15)(9,27)(10,34)(11,41)(12,48)(14,20)(16,28)(17,35)(18,42)(19,49)(24,30)(25,37)(26,44)(32,38)(33,45)(40,46), (1,10,26,18,42,44,34)(2,13,22,17,40,46,35)(3,12,25,21,37,48,29)(4,14,23,20,36,45,33)(5,11,28,16,41,43,31)(6,8,24,19,39,49,30)(7,9,27,15,38,47,32)
9	14	cyclic	21	2,6	(2,8)(3,15)(4,22)(5,29)(6,36)(7,43)(10,16)(11,23)(12,30)(13,37)(14,44)(18,24)(19,31)(20,38)(21,45)(26,32)(27,39)(28,46)(34,40)(35,47)(42,48),(1,9,25,17,41,49,33)(2,11,24,20,42,47,29)(3,13,28,19,36,44,32)(4,10,27,21,40,43,30)(5,8,23,18,38,48,35)(6,14,26,15,37,46,31)(7,12,22,16,39,45,34)
10	21	solvable	14	3,5	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)(1,2,4,3,6,7,5)(8,9,11,10,13,14,12)(15,16,18,17,20,21,19)(22,23,25,24,27,28,26)(29,30,32,31,34,35,33)(36,37,39,38,41,42,40)(43,44,46,45,48,49,47)
11	49	abelian	1	5,6,7	(1,2,4,3,6,7,5)(8,9,11,10,13,14,12)(15,16,18,17,20,21,19)(22,23,25,24,27,28,26)(29,30,32,31,34,35,33)(36,37,39,38,41,42,40)(43,44,46,45,48,49,47)(1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35)

12	98	solvable	3	8,9,11	(1,30,46,38,20,28,12)(2,32,45,41,21,26,8)(3,34,49,40,15,23,11)(4,31,48,42,19,22,9)(5,29,44,39,17,27,14)(6,35,47,36,16,25,10)(7,33,43,37,18,24,13), (1,41,9,49,25,33,17)(2,48,11,35,24,5,20,8,42,23,47,18,29,38)(3,6,13,14,28,26,19,15,36,37,44,46,32,31)(4,34,10,7,27,12,21,22,40,16,43,39,30,45)
13	147	solvable	1	10,11	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)(1,9,25,17,41,49,33)(2,11,24,20,42,47,29)(3,13,28,19,36,44,32)(4,10,27,21,40,43,30)(5,8,23,18,38,48,35)(6,14,26,15,37,46,31)(7,12,22,16,39,45,34)
14	294	*G*	1	4,12,13	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)(1,7,49,45,17,16,9,12,33,34,41,39,25,22)(2,14,47,31,20,37,11,26,29,6,42,46,24,15)(3,21,44,10,19,30,13,40,32,27,36,4,28,43)(5,35,48,38,18,23,8)

Maximal normal subgroups				
Serial	Order	Nature	Quotient	Generators
1	147	solvable	cyclic	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)(1,39,13,47,23,31,21)(2,38,14,43,25,34,19)(3,42,8,46,27,33,16)(4,41,12,44,24,35,15)(5,37,10,49,22,32,20)

Sylow subgroups						
p	Order	Conjug. classes	Nature	Normalizer	Normal closure	Generators
2	2	21	cyclic	14	*G*	(2,8)(3,15)(4,22)(5,29)(6,36)(7,43)(10,16)(11,23)(12,30)(13,37)(14,44)(18,24)(19,31)(20,38)(21,45)(26,32)(27,39)(28,46)(34,40)(35,47)(42,48)
3	3	49	cyclic	6	147	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)
7	49	1	abelian	*G*	49	(1,7,3,2,5,6,4)(8,14,10,9,12,13,11)(15,21,17,16,19,20,18)(22,28,24,23,26,27,25)(29,35,31,30,33,34,32)(36,42,38,37,40,41,39)(43,49,45,44,47,48,46),(1,43,15,8,29,36,22)(2,44,16,9,30,37,23)(3,45,17,10,31,38,24)(4,46,18,11,32,39,25)(5,47,19,12,33,40,26)(6,48,20,13,34,41,27)(7,49,21,14,35,42,28)

3. Let G be a primitive group of degree 49 with 4 generators. We have $|G| = 392 = 2^3 \times 7^2$.

Generators of G:	a_1 = (2,8)(3,15)(4,22)(5,29)(6,36)(7,43)(10,16)(11,23)(12,30)(13,37)(14,44)(18,24)(19,31)(20,38)(21,45)(26,32)(27,39)(28,46)(34,40)(35,47)(42,48) (order 2)
	a_2 = (8,29)(9,30)(10,31)(11,32)(12,33)(13,34)(14,35)(15,36)(16,37)(17,38)(18,39)(19,40)(20,41)(21,42)(22,43)(23,44)(24,45)(25,46)(26,47)(27,48)(28,49) (order 2)
	a_3 = (2,5)(3,6)(4,7)(9,12)(10,13)(11,14)(16,19)(17,20)(18,21)(23,26)(24,27)(25,28)(30,33)(31,34)(32,35)(37,40)(38,41)(39,42)(44,47)(45,48)(46,49) (order 2)
	a_4 = (1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35) (order 7)

This generating set is not minimal. There is a smaller generating set with 2 elements. G is a solvable group. G acts transitively on the set of 49 elements {1, 2, …, 49}. The center of G is trivial.

The derived subgroup D = [G, G] is a solvable group of order 98, generated by {(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(1,45,19,11,35,37,27)(2,48,15,10,33,39,28)(3,47,18,14,30,41,22)(4,49,16,13,29,38,26)(5,46,21,9,34,36,24)(6,43,17,12,32,42,3)(7,44,20,8,31,40,25)(1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35)} and $G/D \cong C_2^2$.

Lower central Series				
Level	Order	Nature	Quotient	Successive quotient
1	98	solvable	abelian	C_2^2
2	49	abelian	nilpotent	C_2

Lower central Series				
Level	Order	Nature	Quotient	Successive quotient
1	98	solvable	abelian	C_2^2
2	49	abelian	nilpotent	C_2

Derived Series				
Level	Order	Nature	Quotient	Successive quotient
1	98	solvable	abelian	C_2^2
2	49	abelian	nilpotent	C_2
3	1	trivial	solvable	C_7^2

Conjugacy classes of subgroups					
Serial	Order	Nature	Conjugacy classes	Maximal subgroup classes	Generators
1	1	trivial	1	--	---
2	2	cyclic	14	1	(8,29)(9,30)(10,31)(11,32)(12,33)(13,34)(14,35)(15,36)(16,37)(17,38)(18,39)(19,40)(20,41)(21,42)(22,43)(23,44)(24,45)(25,46)(26,47)(27,48)(28,49)
3	2	cyclic	14	1	(2,8)(3,15)(4,22)(5,29)(6,36)(7,43)(10,16)(11,23)(12,30)(13,37)(14,44)(18,24)(19,31)(20,38)(21,45)(26,32)(27,39)(28,46)(34,40)(35,47)(42,48)
					(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,

4	2	cyclic	49	1	40)(17,41)(18,42)(19,37)(20,38)(21, 39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)
5	4	abelian	49	3,4	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11, 35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22, 43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(2,8)(3,15)(4,22)(5,29)(6,36)(7, 43)(10,16)(11,23)(12,30)(13,37)(14,44)(18,24)(19,31)(20,38)(21,45)(26,32)(27, 39)(28,46)(34,40)(35,47)(42,48)
6	4	cyclic	49	4	(2,8,5,29)(3,15,6,36)(4,22,7,43)(9,12,33, 30)(10,19,34,37)(11,26,35,44)(13,40,31, 16)(14,47,32,23)(17,20,41,38)(18,27,42, 45)(21,48,39,24)(25,28,49,46)
7	4	abelian	49	2,4	(8,29)(9,30)(10,31)(11,32)(12,33)(13,34)(14,35)(15,36)(16,37)(17,38)(18,39)(19, 40)(20,41)(21,42)(22,43)(23,44)(24,45)(25,46)(26,47)(27,48)(28,49)(2,5)(3,6)(4,7)(9,12)(10,13)(11,14)(16,19)(17,20)(18,21)(23,26)(24,27)(25,28)(30,33)(31, 34)(32,35)(37,40)(38,41)(39,42)(44,47)(45,48)(46,49)
8	7	cyclic	2	1	(1,2,4,3,6,7,5)(8,9,11,10,13,14,12)(15, 16,18,17,20,21,19)(22,23,25,24,27,28, 26)(29,30,32,31,34,35,33)(36,37,39,38, 41,42,40)(43,44,46,45,48,49,47)
9	7	cyclic	2	1	(1,9,25,17,41,49,33)(2,11,24,20,42,47, 29)(3,13,28,19,36,44,32)(4,10,27,21,40, 43,30)(5,8,23,18,38,48,35)(6,14,26,15, 37,46,31)(7,12,22,16,39,45,34)
10	7	yclic	4	1	(1,10,26,18,42,44,34)(2,13,22,17,40,46, 35)(3,12,25,21,37,48,29)(4,14,23,20,36, 45,33)(5,11,28,16,41,43,31)(6,8,24,19, 39,49,30)(7,9,27,15,38,47,32)
11	8	nilpotent	49	5,6,7	(2,8)(3,15)(4,22)(5,29)(6,36)(7,43)(10, 16)(11,23)(12,30)(13,37)(14,44)(18,24)(19,31)(20,38)(21,45)(26,32)(27,39)(28, 46)(34,40)(35,47)(42,48)(8,29)(9,30)(10, 31)(11,32)(12,33)(13,34)(14,35)(15,36)(16,37)(17,38)(18,39)(19,40)(20,41)(21, 42)(22,43)(23,44)(24,45)(25,46)(26,47)(27,48)(28,49)
12	14	dihedral	2	2,8	(2,5)(3,6)(4,7)(9,12)(10,13)(11,14)(16, 19)(17,20)(18,21)(23,26)(24,27)(25,28)(30,33)(31,34)(32,35)(37,40)(38,41)(39, 42)(44,47)(45,48)(46,49)(1,2,4,3,6,7,5)(8,9,11,10,13,14,12)(15,16,18,17,20,21, 19)(22,23,25,24,27,28,26)(29,30,32,31, 34,35,33)(36,37,39,38,41,42,40)(43,44, 46,45,48,49,47)
					(2,29)(3,36)(4,43)(5,8)(6,15)(7,22)(9, 33)(10,40)(11,47)(13,19)(14,26)(16, 34)(17,41)(18,48)(21,27)(23,35)(24,

13	14	dihedral	2	3,9	42)(25,49)(31,37)(32,44)(39,45)(1,9,25,17,41,49,33)(2,11,24,20,42,47,29)(3,13,28,19,36,44,32)(4,10,27,21,40,43,30)(5,8,23,18,38,48,35)(6,14,26,15,37,46,31)(7,12,22,16,39,45,34)
14	14	cyclic	14	3,9	(2,8)(3,15)(4,22)(5,29)(6,36)(7,43)(10,16)(11,23)(12,30)(13,37)(14,44)(18,24)(19,31)(20,38)(21,45)(26,32)(27,39)(28,46)(34,40)(35,47)(42,48)(1,9,25,17,41,49,33)(2,11,24,20,42,47,29)(3,13,28,19,36,44,32)(4,10,27,21,40,43,30)(5,8,23,18,38,48,35)(6,14,26,15,37,46,31)(7,12,22,16,39,45,34)
15	14	dihedral	14	4,8	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(1,2,4,3,6,7,5)(8,9,11,10,13,14,12)(15,16,18,17,20,21,19)(22,23,25,24,27,28,26)(29,30,32,31,34,35,33)(36,37,39,38,41,42,40)(43,44,46,45,48,49,47)
16	14	dihedral	14	4,9	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(1,9,25,17,41,49,33)(2,11,24,20,42,47,29)(3,13,28,19,36,44,32)(4,10,27,21,40,43,30)(5,8,23,18,38,48,35)(6,14,26,15,37,46,31)(7,12,22,16,39,45,34)
17	14	cyclic	14	2,8	(8,29)(9,30)(10,31)(11,32)(12,33)(13,34)(14,35)(15,36)(16,37)(17,38)(18,39)(19,40)(20,41)(21,42)(22,43)(23,44)(24,45)(25,46)(26,47)(27,48)(28,49)(1,2,4,3,6,7,5)(8,9,11,10,13,14,12)(15,16,18,17,20,21,19)(22,23,25,24,27,28,26)(29,30,32,31,34,35,33)(36,37,39,38,41,42,40)(43,44,46,45,48,49,47)
18	14	dihedral	28	4,10	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46),(1,10,26,18,42,44,34)(2,13,22,17,40,46,35)(3,12,25,21,37,48,29)(4,14,23,20,36,45,33)(5,11,28,16,41,43,31)(6,8,24,19,39,49,30)(7,9,27,15,38,47,32)
19	28	dihedral	14	7,12,15,17	(2,5)(3,6)(4,7)(9,12)(10,13)(11,14)(16,19)(17,20)(18,21)(23,26)(24,27)(25,28)(30,33)(31,34)(32,35)(37,40)(38,41)(39,42)(44,47)(45,48)(46,49)(1,7,3,2,5,6,4)(8,35,10,30,12,34,11,29,

					14,31,9,33,13,32)(15,42,17,37,19,41, 18,36,21,38,16,40,20,39)(22,49,24,44, 26,48,25,43,28,45,23,47,27,46)
20	28	dihedral	14	5,13,14,16	(1,9)(3,44)(4,30)(5,23)(6,37)(7,16)(10, 43)(11,29)(12,22)(13,36)(14,15)(17, 49)(18,35)(19,28)(20,42)(24,47)(25, 33)(27,40)(31,46)(34,39)(38,48)(1,49, 17,9,33,41,25)(2,35,20,23,29,48,24,8, 47,38,11,5,42,18)(3,14,19,37,32,6,28, 15,44,31,13,26,36,46)(4,7,21,16,30,34, 27,22,43,45,10,12,40,39)
21	49	abelian	1	8,9,10	(1,2,4,3,6,7,5)(8,9,11,10,13,14,12)(15, 16,18,17,20,21,19)(22,23,25,24,27,28, 26)(29,30,32,31,34,35,33)(36,37,39, 38,41,42,40)(43,44,46,45,48,49,47)(1, 8,22,15,36,43,29)(2,9,23,16,37,44,30) (3,10,24,17,38,45,31)(4,11,25,18,39,46, 32)(5,12,26,19,40,47,33)(6,13,27,20,41, 48,34)(7,14,28,21,42,49,35)
22	98	solvable	1	15,16,18,21	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11, 35)(12,30)(13,31)(14,32)(15,36)(16, 40)(17,41)(18,42)(19,37)(20,38)(21, 39)(22,43)(23,47)(24,48)(25,49)(26, 44)(27,45)(28,46)(1,8,22,15,36,43,29) (2,9,23,16,37,44,30)(3,10,24,17,38,45, 31)(4,11,25,18,39,46,32)(5,12,26,19, 40,47,33)(6,13,27,20,41,48,34)(7,14, 28,21,42,49,35)(1,2,4,3,6,7,5)(8,9,11, 10,13,14,12)(15,16,18,17,20,21,19) (22,23,25,24,27,28,26)(29,30,32,31, 34,35,33)(36,37,39,38,41,42,40)(43, 44,46,45,48,49,47)
23	98	solvable	2	12,17,21	(1,8,22,15,36,43,29)(2,12,23,19,37,47, 30,5,9,26,16,40,44,33)(3,13,24,20,38, 48,31,6,10,27,17,41,45,34)(4,14,25,21, 39,49,32,7,11,28,18,42,46,35)(1,7,3,2, 5,6,4)(8,14,10,9,12,13,11)(15,21,17,16, 19,20,18)(22,28,24,23,26,27,25)(29,35, 31,30,33,34,32)(36,42,38,37,40,41,39) (43,49,45,44,47,48,46)
24	98	solvable	2	13,14,21	(1,30,46,38,20,28,12)(2,32,45,41,21, 26,8)(3,34,49,40,15,23,11)(4,31,48, 42,19,22,9)(5,29,44,39,17,27,14)(6, 35,47,36,16,25,10)(7,33,43,37,18,24, 13),(1,41,9,49,25,33,17)(2,48,11,35, 24,5,20,8,42,23,47,18,29,38)(3,6,13, 14,28,26,19,15,36,37,44,46,32,31)(4, 34,10,7,27,12,21,22,40,16,43,39,30,45)
25	196	solvable	1	19,22,23	(1,7,3,2,5,6,4)(8,35,10,30,12,34,11, 29,14,31,9,33,13,32)(15,42,17,37, 19,41,18,36,21,38,16,40,20,39)(22, 49,24,44,26,48,25,43,28,45,23,47,27, 46)(1,43,15,8,29,36,22)(2,47,16,12, 30,40,23,5,44,19,9,33,37,26)(3,48,17,

					13,31,41,24,6,45,20,10,34,38,27)(4,49,18,14,32,42,25,7,46,21,11,35,39,28)
26	196	solvable	1	20,22,24	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(1,8,9,23,25,18,17,38,41,48,49,35,33,5)(2,22,11,16,24,39,20,45,42,34,47,7,29,12)(3,36,13,44,28,32,19)(4,15,10,37,27,46,21,31,40,6,43,14,30,26)
27	196	solvable	1	6,22	(2,8,5,29)(3,15,6,36)(4,22,7,43)(9,12,33,30)(10,19,34,37)(11,26,35,44)(13,40,31,16)(14,47,32,23)(17,20,41,38)(18,27,42,45)(21,48,39,24)(25,28,49,46)(1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35)
28	392	*G*	1	11,25,26,27	(8,29)(9,30)(10,31)(11,32)(12,33)(13,34)(14,35)(15,36)(16,37)(17,38)(18,39)(19,40)(20,41)(21,42)(22,43)(23,44)(24,45)(25,46)(26,47)(27,48)(28,49)(1,15,17,31,33,26,25,46,49,14,9,37,41,6)(2,36,20,3,29,19,24,32,47,28,11,44,42,13)(4,43,21,10,30,40,27)(5,22,18,45,35,12,23,39,48,7,8,16,38,34)

Maximal normal subgroups				
Serial	Order	Nature	Quotient	Generators
1	196	solvable	cyclic	(2,29,5,8)(3,36,6,15)(4,43,7,22)(9,30,33,12)(10,37,34,19)(11,44,35,26)(13,16,31,40)(14,23,32,47)(17,38,41,20)(18,45,42,27)(21,24,39,48)(25,46,49,28),(1,29,43,36,15,22,8)(2,30,44,37,16,23,9)(3,31,45,38,17,24,10)(4,32,46,39,18,25,11)(5,33,47,40,19,26,12)(6,34,48,41,20,27,13)(7,35,49,42,21,28,14)
2	196	solvable	cyclic	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46),(1,9,4,10,6,14,5,8,2,11,3,13,7,12)(15,44,18,45,20,49,19,43,16,46,17,48,21,47)(22,30,25,31,27,35,26,29,23,32,24,34,28,33)(36,37,39,38,41,42,40)
3	196	solvable	cyclic	(1,49,17,9,33,41,25)(2,35,20,23,29,48,24,8,47,38,11,5,42,18)(3,14,19,37,32,6,28,15,44,31,13,26,36,46)(4,7,21,16,30,34,27,22,43,45,10,12,40,39)(1,28,38,30,12,20,46)(2,14,41,44,8,27,45,29,26,17,32,5,21,39)(3,35,40,16,11,6,49,36,23,10,34,47,15,25)(4,7,42,37,9,13,48)

Sylow subgroups						
p	Order	Conjug. classes	Nature	Normalizer	Normal closure	Generators
2	8	49	nilpotent	8	G	(2,8)(3,15)(4,22)(5,29)(6,36)(7,43)(10,16)(11,23)(12,30)(13,37)(14,44)(18,24)(19,31)(20,38)(21,45)(26,32)(27,39)(28,46)(34,40)(35,47)(42,48)(8,29)(9,30)(10,31)(11,32)(12,33)(13,34)(14,35)(15,36)(16,37)(17,38)(18,39)(19,40)(20,41)(21,42)(22,43)(23,44)(24,45)(25,46)(26,47)(27,48)(28,49)
7	49	1	abelian	G	49	(1,15,29,22,43,8,36)(2,16,30,23,44,9,37)(3,17,31,24,45,10,38)(4,18,32,25,46,11,39)(5,19,33,26,47,12,40)(6,20,34,27,48,13,41)(7,21,35,28,49,14,42),(1,41,9,49,25,33,17)(2,42,11,47,24,29,20)(3,36,13,44,28,32,19)(4,40,10,43,27,30,21)(5,38,8,48,23,35,18)(6,37,14,46,26,31,15)(7,39,12,45,22,34,16)

4. Let G be a primitive group of degree 49 with 3 generators. We have $|G| = 392 = 2^3 \times 7^2$.

Generators of G:	a_1 = (2,48,5,24)(3,14,6,32)(4,16,7,40)(8,21,29,39)(9,11,33,35)(10,38,34,20)(12,44,30,26)(13,22,31,43)(15,23,36,47)(17,19,41,37)(18,46,42,28)(25,27,49,45) (order 4)
	a_2 = (2,38,5,20)(3,46,6,28)(4,12,7,30)(8,37,29,19)(9,13,33,31)(10,24,34,48)(11,43,35,22)(14,18,32,42)(15,45,36,27)(16,26,40,44)(17,21,41,39)(23,49,47,25) (order 4)
	a_3 = (1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35) (order 7)

This generating set is not minimal. There is a smaller generating set with 2 elements. G is a solvable group. G acts transitively on the set of 49 elements {1, 2, …, 49}. The center of G is trivial.

The derived subgroup $D = [G, G]$ is a solvable group of order 98, generated by {(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(1,13,23,21,39,47,31)(2,14,25,19,38,43,34)(3,8,27,16,42,46,33)(4,12,24,15,41,44,35)(5,10,22,20,37,49,32)(6,9,28,18,40,45,29)(7,11,26,17,36,48,30)(1,24,40,32,14,16,48)(2,27,36,31,12,18,49)(3,26,39,35,9,20,43)(4,28,37,34,8,17,47)(5,25,42,30,13,15,45)(6,22,38,33,11,21,44)(7,23,41,29,10,19,46)} and $G/D \cong C_2^2$.

Lower central Series				
Level	Order	Nature	Quotient	Successive quotient
1	98	solvable	abelian	C_2^2
2	49	abelian	nilpotent	C_2

Derived Series				
Level	Order	Nature	Quotient	Successive quotient
1	98	solvable	abelian	$C_2^{\ 2}$
2	49	abelian	nilpotent	C_2
3	1	trivial	solvable	$C_7^{\ 2}$

Conjugacy classes of subgroups					
Serial	Order	Nature	Conjugacy classes	Maximal subgroup classes	Generators
1	1	trivial	1	--	-
2	2	cyclic	49	1	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12, 30)(13,31)(14,32)(15,36)(16,40)(17,41)(18, 42)(19,37)(20,38)(21,39)(22,43)(23,47)(24, 48)(25,49)(26,44)(27,45)(28,46)
3	4	cyclic	49	2	(2,24,5,48)(3,32,6,14)(4,40,7,16)(8,39,29,21)(9,35,33,11)(10,20,34,38)(12,26,30,44)(13,43, 31,22)(15,47,36,23)(17,37,41,19)(18,28,42, 46)(25,45,49,27)
4	4	cyclic	49	2	(2,10,5,34)(3,18,6,42)(4,26,7,44)(8,41,29,17)(9,43,33,22)(11,31,35,13)(12,16,30,40)(14,28, 32,46)(15,49,36,25)(19,39,37,21)(20,24,38, 48)(23,27,47,45)
5	4	cyclic	49	2	(2,20,5,38)(3,28,6,46)(4,30,7,12)(8,19,29,37)(9,31,33,13)(10,48,34,24)(11,22,35,43)(14,42, 32,18)(15,27,36,45)(16,44,40,26)(17,39,41, 21)(23,25,47,49)
6	7	cyclic	4	1	(1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10, 24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26, 19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21, 42,49,35)
7	7	cyclic	4	1	(1,2,4,3,6,7,5)(8,9,11,10,13,14,12)(15,16,18, 17,20,21,19)(22,23,25,24,27,28,26)(29,30,32, 31,34,35,33)(36,37,39,38,41,42,40)(43,44,46, 45,48,49,47)
8	8	nilpotent	49	3,4,5	(2,20,5,38)(3,28,6,46)(4,30,7,12)(8,19,29,37)(9,31,33,13)(10,48,34,24)(11,22,35,43)(14,42, 32,18)(15,27,36,45)(16,44,40,26)(17,39,41, 21)(23,25,47,49)(2,10,5,34)(3,18,6,42)(4,26,7, 44)(8,41,29,17)(9,43,33,22)(11,31,35,13)(12,1 6,30,40)(14,28,32,46)(15,49,36,25)(19,39,37, 21)(20,24,38,48)(23,27,47,45)
9	14	dihedral	28	2,7	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12, 30)(13,31)(14,32)(15,36)(16,40)(17,41)(18, 42)(19,37)(20,38)(21,39)(22,43)(23,47)(24, 48)(25,49)(26,44)(27,45)(28,46),(1,2,4,3,6, 7,5)(8,9,11,10,13,14,12)(15,16,18,17,20,21, 19)(22,23,25,24,27,28,26)(29,30,32,31,34,35, 33)(36,37,39,38,41,42,40)(43,44,46,45,48,49, 47)
10	14	dihedral	28	2,6	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12, 30)(13,31)(14,32)(15,36)(16,40)(17,41)(18, 42)(19,37)(20,38)(21,39)(22,43)(23,47)(24, 48)(25,49)(26,44)(27,45)(28,46),(1,8,22,15,

					36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35)
11	49	abelian	1	6,7	(1,2,4,3,6,7,5)(8,9,11,10,13,14,12)(15,16,18,17,20,21,19)(22,23,25,24,27,28,26)(29,30,32,31,34,35,33)(36,37,39,38,41,42,40)(43,44,46,45,48,49,47),(1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35)
12	98	solvable	1	9,10,11	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35)(1,2,4,3,6,7,5)(8,9,11,10,13,14,12)(15,16,18,17,20,21,19)(22,23,25,24,27,28,26)(29,30,32,31,34,35,33)(36,37,39,38,41,42,40)(43,44,46,45,48,49,47)
13	196	solvable	1	4,12	(2,10,5,34)(3,18,6,42)(4,26,7,44)(8,41,29,17)(9,43,33,22)(11,31,35,13)(12,16,30,40)(14,28,32,46)(15,49,36,25)(19,39,37,21)(20,24,38,48)(23,27,47,45)(1,10,26,18,42,44,34)(2,13,22,17,40,46,35)(3,12,25,21,37,48,29)(4,14,23,20,36,45,33)(5,11,28,16,41,43,31)(6,8,24,19,39,49,30)(7,9,27,15,38,47,32)
14	196	solvable	1	5,12	(2,20,5,38)(3,28,6,46)(4,30,7,12)(8,19,29,37)(9,31,33,13)(10,48,34,24)(11,22,35,43)(14,42,32,18)(15,27,36,45)(16,44,40,26)(17,39,41,21)(23,25,47,49)(1,20,30,28,46,12,38)(2,21,32,26,45,8,41)(3,15,34,23,49,11,40)(4,19,31,22,48,9,42)(5,17,29,27,44,14,39)(6,16,35,25,47,10,36)(7,18,33,24,43,13,37)
15	196	solvable	1	3,12	(2,24,5,48)(3,32,6,14)(4,40,7,16)(8,39,29,21)(9,35,33,11)(10,20,34,38)(12,26,30,44)(13,43,31,22)(15,47,36,23)(17,37,41,19)(18,28,42,46)(25,45,49,27)(1,24,40,32,14,16,48)(2,27,36,31,12,18,49)(3,26,39,35,9,20,43)(4,28,37,34,8,17,47)(5,25,42,30,13,15,45)(6,22,38,33,11,21,44)(7,23,41,29,10,19,46)
16	392	*G*	1	8,13,14,15	(2,48,5,24)(3,14,6,32)(4,16,7,40)(8,21,29,39)(9,11,33,35)(10,38,34,20)(12,44,30,26)(13,22,31,43)(15,23,36,47)(17,19,41,37)(18,46,42,28)(25,27,49,45)(1,23,5,49)(2,34,7,11)(4,15,6,40)(8,32,33,13)(9,21,35,37)(10,41,31,18)(12,26,29,43)(14,45,30,24)(16,36,42,19)(17,47,38,22)(20,25,39,48)(27,44,46,28)

Proper normal subgroups					
Serial	Order	Nature	Quotient	Subgroups	Generators
1	49	abelian	nilpotent	--	(1,9,25,17,41,49,33)(2,11,24,20,42,47,29)(3,13,28,19,36,44,32)(4,10,27,21,40,43,30)(5,8,23,18,38,48,35)(6,14,26,15,37,46,31)(7,12,22,16,39,45,34),(1,31,47,39,21,23,13)(2,34,43,38,19,25,14)(3,33,46,42,16,27,8)(4,35,44,41,15,24,12)(5,32,49,37,20,22,10)(6,29,45,40,18,28,9)(7,30,48,36,17,26,11)
2	98	solvable	abelian	1	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(1,7,3,2,5,6,4)(8,14,10,9,12,13,11)(15,21,17,16,19,20,18)(22,28,24,23,26,27,25)(29,35,31,30,33,34,32)(36,42,38,37,40,41,39)(43,49,45,44,47,48,46)(1,17,33,25,49,9,41)(2,20,29,24,47,11,42)(3,19,32,28,44,13,36)(4,21,30,27,43,10,40)(5,18,35,23,48,8,38)(6,15,31,26,46,14,37)(7,16,34,22,45,12,39)
3	196	solvable	cyclic	1,2	(2,34,5,10)(3,42,6,18)(4,44,7,26)(8,17,29,41)(9,22,33,43)(11,13,35,31)(12,40,30,16)(14,46,32,28)(15,25,36,49)(19,21,37,39)(20,48,38,24)(23,45,47,27)(1,29,43,36,15,22,8)(2,30,44,37,16,23,9)(3,31,45,38,17,24,10)(4,32,46,39,18,25,11)(5,33,47,40,19,26,12)(6,34,48,41,20,27,13)(7,35,49,42,21,28,14)
4	196	solvable	cyclic	1,2	(2,38,5,20)(3,46,6,28)(4,12,7,30)(8,37,29,19)(9,13,33,31)(10,24,34,48)(11,43,35,22)(14,18,32,42)(15,45,36,27)(16,26,40,44)(17,21,41,39)(23,49,47,25)(1,21,31,23,47,13,39)(2,19,34,25,43,14,38)(3,16,33,27,46,8,42)(4,15,35,24,44,12,41)(5,20,32,22,49,10,37)(6,18,29,28,45,9,40)(7,17,30,26,48,11,36)
5	196	solvable	cyclic	1,2	(2,48,5,24)(3,14,6,32)(4,16,7,40)(8,21,29,39)(9,11,33,35)(10,38,34,20)(12,44,30,26)(13,22,31,43)(15,23,36,47)(17,19,41,37)(18,46,42,28)(25,27,49,45)(1,19,35,27,45,11,37)(2,15,33,28,48,10,39)(3,18,30,22,47,14,41)(4,16,29,26,49,13,38)(5,21,34,24,46,9,36)(6,17,32,23,43,12,42)(7,20,31,25,44,8,40)

Sylow subgroups						
p	Order	Conjug. classes	Nature	Normalizer	Normal closure	Generators
2	8	49	nilpotent	8	*G*	(2,48,5,24)(3,14,6,32)(4,16,7,40)(8,21,29,39)(9,11,33,35)(10,38,34,20)(12,44,30,26)(13,22,31,43)(15,23,36,47)(17,19,41,37)(18,46,42,28)(25,27,49,45)(2,38,5,20)(3,46,6,28)(4,12,7,30)(8,37,29,19)(9,13,33,31)(10,24,34,48)(11,43,35,22)(14,18,32,42)(15,45,36,27)(16,26,40,44)(17,21,41,39)(23,49,47,25)
						(1,15,29,22,43,8,36)(2,16,30,23,44,9,37)(3,17,31,24,45,10,38)(4,18,32,25,46,11,

7	49	1	abelian	G	49	39)(5,19,33,26,47,12,40)(6,20,34,27,48, 13,41)(7,21,35,28,49,14,42)(1,23,39,31, 13,21,47)(2,25,38,34,14,19,43)(3,27,42, 33,8,16,46)(4,24,41,35,12,15,44)(5,22, 37,32,10,20,49)(6,28,40,29,9,18,45)(7, 26,36,30,11,17,48)

5.Let G be a primitive group of degree 49 with 2 generators. We have $|G| = 392 = 2^3 \times 7^2$.

Generators of G:	a_1 = (2,33,38,31,5,9,20,13)(3,41,46,39,6,17, 28,21)(4,49,12,47,7,25,30,23)(8,42,37,14,29, 18,19,32)(10,11,24,43,34,35,48,22)(15,44,45, 16,36,26,27,40) (order 8)
	a_2 = (1,8,22,15,36,43,29)(2,9,23,16,37,44,30) (3,10,24,17,38,45,31)(4,11,25,18,39,46,32) (5,12,26,19,40,47,33)(6,13,27,20,41,48,34) (7,14,28,21,42,49,35) (order 7)

G is a solvable group. G acts transitively on the set of 49 elements {1, 2, …, 49}. The center of G is trivial.

The derived subgroup $D = [G, G]$ is an abelian group of order 49, generated by {(1,3,5,4,7,2,6)(8,10,12,11,14,9,13)(15,17,19,18,21,16,20)(22,24,26,25,28,23,27)(29,31,33,32,35, 30,34)(36,38,40,39,42,37,41)(43,45,47,46,49,44,48)(1,21,31,23,47,13,39)(2,19,34,25,43,14,38)(3, 16,33,27,46,8,42)(4,15,35,24,44,12,41)(5,20,32,22,49,10,37)(6,18,29,28,45,9,40)(7,17,30,26,48, 11,36)} and $G/D \cong C_8$.

Lower central series				
Level	Order	Nature	Quotient	Successive quotient
1	49	abelian	cyclic	C_8

Derived Series				
Level	Order	Nature	Quotient	Successive quotient
1	49	abelian	cyclic	C_8
2	1	trivial	solvable	C_7^2

Conjugacy classes of subgroups					
Serial	Order	Nature	Conjugacy classes	Maximal subgroup classes	Generators
1	1	trivial	1	--	---
2	2	cyclic	49	1	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12, 30)(13,31)(14,32)(15,36)(16,40)(17,41)(18, 42)(19,37)(20,38)(21,39)(22,43)(23,47)(24, 48)(25,49)(26,44)(27,45)(28,46)
3	4	cyclic	49	2	(2,20,5,38)(3,28,6,46)(4,30,7,12)(8,19,29, 37)(9,31,33,13)(10,48,34,24)(11,22,35,43) (14,42,32,18)(15,27,36,45)(16,44,40,26) (17,39,41,21)(23,25,47,49)
4	7	cyclic	4	1	(1,2,4,3,6,7,5)(8,9,11,10,13,14,12)(15,16, 18,17,20,21,19)(22,23,25,24,27,28,26)(29, 30,32,31,34,35,33)(36,37,39,38,41,42,40) (43,44,46,45,48,49,47)
5	7	cyclic	4	1	(1,8,22,15,36,43,29)(2,9,23,16,37,44,30) (3,10,24,17,38,45,31)(4,11,25,18,39,46, 32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,

					34)(7,14,28,21,42,49,35)
6	8	cyclic	49	3	(2,9,38,13,5,33,20,31)(3,17,46,21,6,41,28, 39)(4,25,12,23,7,49,30,47)(8,18,37,32,29, 42,19,14)(10,35,24,22,34,11,48,43)(15,26, 45,40,36,44,27,16)
7	14	dihedral	28	2,4	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35) (12,30)(13,31)(14,32)(15,36)(16,40)(17, 41)(18,42)(19,37)(20,38)(21,39)(22,43) (23,47)(24,48)(25,49)(26,44)(27,45)(28, 46)(1,2,4,3,6,7,5)(8,9,11,10,13,14,12)(15, 16,18,17,20,21,19)(22,23,25,24,27,28,26) (29,30,32,31,34,35,33)(36,37,39,38,41,42, 40)(43,44,46,45,48,49,47)
8	14	dihedral	28	2,5	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35) (12,30)(13,31)(14,32)(15,36)(16,40)(17, 41)(18,42)(19,37)(20,38)(21,39)(22,43) (23,47)(24,48)(25,49)(26,44)(27,45)(28, 46)(1,8,22,15,36,43,29)(2,9,23,16,37,44, 30)(3,10,24,17,38,45,31)(4,11,25,18,39, 46,32)(5,12,26,19,40,47,33)(6,13,27,20, 41,48,34)(7,14,28,21,42,49,35)
9	49	abelian	1	4,5	(1,2,4,3,6,7,5)(8,9,11,10,13,14,12)(15, 16,18,17,20,21,19)(22,23,25,24,27,28, 26)(29,30,32,31,34,35,33)(36,37,39,38, 41,42,40)(43,44,46,45,48,49,47)(1,8,22, 15,36,43,29)(2,9,23,16,37,44,30)(3,10,24, 17,38,45,31)(4,11,25,18,39,46,32)(5,12, 26,19,40,47,33)(6,13,27,20,41,48,34)(7, 14,28,21,42,49,35)
10	98	solvable	1	7,8,9	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11, 35)(12,30)(13,31)(14,32)(15,36)(16,40) (17,41)(18,42)(19,37)(20,38)(21,39)(22, 43)(23,47)(24,48)(25,49)(26,44)(27,45) (28,46)(1,8,22,15,36,43,29)(2,9,23,16,37, 44,30)(3,10,24,17,38,45,31)(4,11,25,18,39, 46,32)(5,12,26,19,40,47,33)(6,13,27,20,41, 48,34)(7,14,28,21,42,49,35)(1,2,4,3,6,7,5) (8,9,11,10,13,14,12)(15,16,18,17,20,21,19) (22,23,25,24,27,28,26)(29,30,32,31,34,35, 33)(36,37,39,38,41,42,40)(43,44,46,45,48, 49,47)
11	196	solvable	1	3,10	(2,20,5,38)(3,28,6,46)(4,30,7,12)(8,19,29, 37)(9,31,33,13)(10,48,34,24)(11,22,35,43) (14,42,32,18)(15,27,36,45)(16,44,40,26) (17,39,41,21)(23,25,47,49)(1,20,30,28,46, 12,38)(2,21,32,26,45,8,41)(3,15,34,23,49, 11,40)(4,19,31,22,48,9,42)(5,17,29,27,44, 14,39)(6,16,35,25,47,10,36)(7,18,33,24, 43,13,37)
12	392	*G*	1	6,11	(2,33,38,31,5,9,20,13)(3,41,46,39,6,17, 28,21)(4,49,12,47,7,25,30,23)(8,42,37, 14,29,18,19,32)(10,11,24,43,34,35,48,22) (15,44,45,16,36,26,27,40)(1,8,22,15,36,43, 29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,

					31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35)

Maximal normal subgroups				
Serial	Order	Nature	Quotient	Generators
1	196	solvable	cyclic	(2,38,5,20)(3,46,6,28)(4,12,7,30)(8,37,29,19)(9,13,33,31)(10,24,34,48)(11,43,35,22)(14,18,32,42)(15,45,36,27)(16,26,40,44)(17,21,41,39)(23,49,47,25)(1,40,14,48,24,32,16)(2,36,12,49,27,31,18)(3,39,9,43,26,35,20)(4,37,8,47,28,34,17)(5,42,13,45,25,30,15)

Sylow subgroups						
p	Order	Conjug. classes	Nature	Normalizer	Normal closure	Generators
2	8	49	cyclic	8	*G*	(2,33,38,31,5,9,20,13)(3,41,46,39,6,17,28,21)(4,49,12,47,7,25,30,23)(8,42,37,14,29,18,19,32)(10,11,24,43,34,35,48,22)(15,44,45,16,36,26,27,40)
7	49	1	abelian	*G*	49	(1,15,29,22,43,8,36)(2,16,30,23,44,9,37)(3,17,31,24,45,10,38)(4,18,32,25,46,11,39)(5,19,33,26,47,12,40)(6,20,34,27,48,13,41)(7,21,35,28,49,14,42)(1,44,18,10,34,42,26)(2,46,17,13,35,40,22)(3,48,21,12,29,37,25)(4,45,20,14,33,36,23)(5,43,16,11,31,41,28)(6,49,19,8,30,39,24)(7,47,15,9,32,38,27)

6. Let *G* be a primitive group of degree 49 with 4 generators. We have $|G| = 588 = 2^2 \times 3 \times 7^2$.

Generators of *G*:	a_1 = (2,29,5,8)(3,36,6,15)(4,43,7,22)(9,30,33,12)(10,37,34,19)(11,44,35,26)(13,16,31,40)(14,23,32,47)(17,38,41,20)(18,45,42,27)(21,24,39,48)(25,46,49,28) (order 4)
	a_2 = (2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46) (order 2)
	a_3 = (2,6,4)(3,7,5)(8,22,36)(9,27,39)(10,28,40)(11,23,41)(12,24,42)(13,25,37)(14,26,38)(15,29,43)(16,34,46)(17,35,47)(18,30,48)(19,31,49)(20,32,44)(21,33,45) (order 3)
	a_4 = (1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35) (order 7)

This generating set is not minimal. There is a smaller generating set with 2 elements. *G* is a solvable group. *G* acts transitively on the set of 49 elements {1, 2, …, 49}. The center of *G* is trivial.

The derived subgroup $D = [G, G]$ is a solvable group of order 147, generated by {(2,6,4)(3,7,5)(8,22,36)(9,27,39)(10,28,40)(11,23,41)(12,24,42)(13,25,37)(14,26,38)(15,29,43)(16,34,46)(17,35,47)(18,30,48)(19,31,49)(20,32,44)(21,33,45)(1,30,46,38,20,28,12)(2,32,45,41,21,26,8)(3,34,49,40,15,23,11)(4,31,48,42,19,22,9)(5,29,44,39,17,27,14)(6,35,47,36,16,25,10)(7,33,43,37,18,24,13)} and $G/D \cong C_4$.

Lower central Series				
Level	Order	Nature	Quotient	Successive quotient
1	147	solvable	cyclic	C_4

Derived Series				
Level	Order	Nature	Quotient	Successive quotient
1	147	solvable	cyclic	C_4
2	49	abelian	solvable	C_3
3	1	trivial	solvable	C_7^2

Conjugacy classes of subgroups					
Serial	Order	Nature	Conjugacy classes	Maximal subgroup classes	Generators
1	1	trivial	1	--	---
2	2	cyclic	49	1	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)
3	3	cyclic	49	1	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)
4	4	cyclic	147	2	(2,15,5,36)(3,22,6,43)(4,29,7,8)(9,18,33,42)(10,25,34,49)(11,32,35,14)(12,39,30,21)(13,46,31,28)(16,19,40,37)(17,26,41,44)(20,47,38,23)(24,27,48,45)
5	6	cyclic	49	2,3	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)
6	7	cyclic	2	1	(1,3,5,4,7,2,6)(8,10,12,11,14,9,13)(15,17,19,18,21,16,20)(22,24,26,25,28,23,27)(29,31,33,32,35,30,34)(36,38,40,39,42,37,41)(43,45,47,46,49,44,48)
7	7	cyclic	6	1	(1,14,24,16,40,48,32)(2,12,27,18,36,49,31)(3,9,26,20,39,43,35)(4,8,28,17,37,47,34)(5,13,25,15,42,45,30)(6,11,22,21,38,44,33)(7,10,23,19,41,46,29)
8	12	solvable	49	4,5	(2,15,5,36)(3,22,6,43)(4,29,7,8)(9,18,33,42)(10,25,34,49)(11,32,35,14)(12,39,30,21)(13,46,31,28)(16,19,40,37)(17,26,41,44)(20,47,38,23)(24,27,48,45)(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)
9	14	dihedral	14	2,6	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,

					46)(1,3,5,4,7,2,6)(8,10,12,11,14,9,13)(15,17,19,18,21,16,20)(22,24,26,25,28,23,27)(29,31,33,32,35,30,34)(36,38,40,39,42,37,41)(43,45,47,46,49,44,48)
10	14	dihedral	42	2,7	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(1,14,24,16,40,48,32)(2,12,27,18,36,49,31)(3,9,26,20,39,43,35)(4,8,28,17,37,47,34)(5,13,25,15,42,45,30)(6,11,22,21,38,44,33)(7,10,23,19,41,46,29)
11	21	solvable	14	3,6	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33), (1,3,5,4,7,2,6)(8,10,12,11,14,9,13)(15,17,19,18,21,16,20)(22,24,26,25,28,23,27)(29,31,33,32,35,30,34)(36,38,40,39,42,37,41)(43,45,47,46,49,44,48)
12	42	solvable	14	5,9,11	(1,3,5,4,7,2,6)(8,10,12,11,14,9,13)(15,17,19,18,21,16,20)(22,24,26,25,28,23,27)(29,31,33,32,35,30,34)(36,38,40,39,42,37,41)(43,45,47,46,49,44,48)(2,7,6,5,4,3)(8,15,22,29,36,43)(9,21,27,33,39,45)(10,16,28,34,40,46)(11,17,23,35,41,47)(12,18,24,30,42,48)(13,19,25,31,37,49)(14,20,26,32,38,44)
13	49	abelian	1	6,7	(1,3,5,4,7,2,6)(8,10,12,11,14,9,13)(15,17,19,18,21,16,20)(22,24,26,25,28,23,27)(29,31,33,32,35,30,34)(36,38,40,39,42,37,41)(43,45,47,46,49,44,48)(1,14,24,16,40,48,32)(2,12,27,18,36,49,31)(3,9,26,20,39,43,35)(4,8,28,17,37,47,34)(5,13,25,15,42,45,30)(6,11,22,21,38,44,33)(7,10,23,19,41,46,29)
14	98	solvable	1	9,10,13	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(1,14,24,16,40,48,32)(2,12,27,18,36,49,31)(3,9,26,20,39,43,35)(4,8,28,17,37,47,34)(5,13,25,15,42,45,30)(6,11,22,21,38,44,33)(7,10,23,19,41,46,29)(1,3,5,4,7,2,6)(8,10,12,11,14,9,13)(15,17,19,18,21,16,20)(22,24,26,25,28,23,27)(29,31,33,32,35,30,34)(36,38,40,39,42,37,41)(43,45,47,46,49,44,48)
15	147	solvable	1	11,13	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)(1,14,24,16,40,48,32)(2,12,27,18,36,49,31)(3,9,26,20,

					39,43,35)(4,8,28,17,37,47,34)(5,13,25,15,42,45,30)(6,11,22,21,38,44,33)(7,10,23,19,41,46,29)
16	196	solvable	3	4,14	(2,15,5,36)(3,22,6,43)(4,29,7,8)(9,18,33,42)(10,25,34,49)(11,32,35,14)(12,39,30,21)(13,46,31,28)(16,19,40,37)(17,26,41,44)(20,47,38,23)(24,27,48,45)(1,22,36,29,8,15,43)(2,23,37,30,9,16,44)(3,24,38,31,10,17,45)(4,25,39,32,11,18,46)(5,26,40,33,12,19,47)(6,27,41,34,13,20,48)(7,28,42,35,14,21,49)
17	294	solvable	1	12,14,15	(1,14,24,16,40,48,32)(2,12,27,18,36,49,31)(3,9,26,20,39,43,35)(4,8,28,17,37,47,34)(5,13,25,15,42,45,30)(6,11,22,21,38,44,33)(7,10,23,19,41,46,29)(2,7,6,5,4,3)(8,15,22,29,36,43)(9,21,27,33,39,45)(10,16,28,34,40,46)(11,17,23,35,41,47)(12,18,24,30,42,48)(13,19,25,31,37,49)(14,20,26,32,38,44)
18	588	*G*	1	8,16,17	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)(1,48,11,21)(2,41,9,42)(3,27,12,35)(4,20,8,49)(5,34,10,28)(6,13,14,7)(15,43,46,18)(16,36,44,39)(17,22,47,32)(19,29,45,25)(23,40,30,38)(24,26,33,31)

Proper normal subgroups					
Serial	Order	Nature	Quotient	Subgroups	Generators
1	49	abelian	solvable	--	(1,7,3,2,5,6,4)(8,14,10,9,12,13,11)(15,21,17,16,19,20,18)(22,28,24,23,26,27,25)(29,35,31,30,33,34,32)(36,42,38,37,40,41,39)(43,49,45,44,47,48,46)(1,43,15,8,29,36,22)(2,44,16,9,30,37,23)(3,45,17,10,31,38,24)(4,46,18,11,32,39,25)(5,47,19,12,33,40,26)(6,48,20,13,34,41,27)(7,49,21,14,35,42,28)
2	98	solvable	dihedral	1	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(1,43,15,8,29,36,22)(2,44,16,9,30,37,23)(3,45,17,10,31,38,24)(4,46,18,11,32,39,25)(5,47,19,12,33,40,26)(6,48,20,13,34,41,27)(7,49,21,14,35,42,28)(1,7,3,2,5,6,4)(8,14,10,9,12,13,11)(15,21,17,16,19,20,18)(22,28,24,23,26,27,25)(29,35,31,30,33,34,32)(36,42,38,37,40,41,39)(43,49,45,44,47,48,46)
3	147	solvable	cyclic	1	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)(1,39,13,47,23,31,21)(2,38,14,43,25,34,19)(3,42,8,46,27,33,16)(4,41,12,44,24,35,15)(5,37,10,49,22,32,20)(6,40,9,45,28,29,18)(7,36,11,48,26,30,17)
					(2,7,6,5,4,3)(8,15,22,29,36,43)(9,21,27,33,39,

4	294	solvable	cyclic	1,2,3	45)(10,16,28,34,40,46)(11,17,23,35,41,47)(12,18,24,30,42,48)(13,19,25,31,37,49)(14,20,26,32,38,44)(1,16,32,24,48,14,40)(2,18,31,27,49,12,36)(3,20,35,26,43,9,39)(4,17,34,28,47,8,37)(5,15,30,25,45,13,42)(6,21,33,22,44,11,38)(7,19,29,23,46,10,41)

Sylow subgroups						
p	Order	Conjug. classes	Nature	Normalizer	Normal closure	Generators
2	4	147	cyclic	4	*G*	(2,29,5,8)(3,36,6,15)(4,43,7,22)(9,30,33,12)(10,37,34,19)(11,44,35,26)(13,16,31,40)(14,23,32,47)(17,38,41,20)(18,45,42,27)(21,24,39,48)(25,46,49,28)
3	3	49	cyclic	12	147	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)
7	49	1	abelian	*G*	49	(1,36,8,43,22,29,15)(2,37,9,44,23,30,16)(3,38,10,45,24,31,17)(4,39,11,46,25,32,18)(5,40,12,47,26,33,19)(6,41,13,48,27,34,20)(7,42,14,49,28,35,21)(1,48,16,14,32,40,24)(2,49,18,12,31,36,27)(3,43,20,9,35,39,26)(4,47,17,8,34,37,28)(5,45,15,13,30,42,25)(6,44,21,11,33,38,22)(7,46,19,10,29,41,23)

7. Let *G* be a primitive group of degree 49 with 4 generators. We have $|G| = 588 = 2^2 \times 3 \times 7^2$.

Generators of *G*:	a_1 = (2,8)(3,15)(4,22)(5,29)(6,36)(7,43)(10,16)(11,23)(12,30)(13,37)(14,44)(18,24)(19,31)(20,38)(21,45)(26,32)(27,39)(28,46)(34,40)(35,47)(42,48) (order 2)
	a_2 = (2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46) (order 2)
	a_3 = (2,6,4)(3,7,5)(8,22,36)(9,27,39)(10,28,40)(11,23,41)(12,24,42)(13,25,37)(14,26,38)(15,29,43)(16,34,46)(17,35,47)(18,30,48)(19,31,49)(20,32,44)(21,33,45) (order 3)
	a_4 = (1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35) (order 7)

This generating set is not minimal. There is a smaller generating set with 2 elements. *G* is a solvable group. *G* acts transitively on the set of 49 elements {1, 2, …, 49}. The center of *G* is trivial.

The derived subgroup $D = [G, G]$ is a solvable group of order 147, generated by {(2,6,4)(3,7,5)(8,22,36)(9,27,39)(10,28,40)(11,23,41)(12,24,42)(13,25,37)(14,26,38)(15,29,43)(16,34,46)(17,35,47)(18,30,48)(19,31,49)(20,32,44)(21,33,45)(1,30,46,38,20,28,12)(2,32,45,41,21,26,8)(3,34,49,40,15,23,11)(4,31,48,42,19,22,9)(5,29,44,39,17,27,14)(6,35,47,36,16,25,10)(7,33,43,37,18,24,13)} and $G/D \cong C_2^2$.

Lower central Series				
Level	Order	Nature	Quotient	Successive quotient
1	147	solvable	abelian	C_2^2

Derived Series				
Level	Order	Nature	Quotient	Successive quotient
1	147	solvable	abelian	C_2^2
2	49	abelian	dihedral	C_3
3	1	trivial	solvable	C_7^2

Conjugacy classes of subgroups					
Serial	Order	Nature	Conjugacy classes	Maximal subgroup classes	Generators
1	1	trivial	1	--	---
2	2	cyclic	21	1	(2,8)(3,15)(4,22)(5,29)(6,36)(7,43)(10,16)(11,23)(12,30)(13,37)(14,44)(18,24)(19,31)(20,38)(21,45)(26,32)(27,39)(28,46)(34,40)(35,47)(42,48)
3	2	cyclic	21	1	(2,15)(3,22)(4,29)(5,36)(6,43)(7,8)(9,21)(10,28)(11,35)(12,42)(13,49)(17,23)(18,30)(19,37)(20,44)(25,31)(26,38)(27,45)(33,39)(34,46)(41,47)
4	2	cyclic	49	1	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)
5	3	cyclic	49	1	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)
6	4	abelian	147	2,3,4	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46), (2,8)(3,15)(4,22)(5,29)(6,36)(7,43)(10,16)(11,23)(12,30)(13,37)(14,44)(18,24)(19,31)(20,38)(21,45)(26,32)(27,39)(28,46)(34,40)(35,47)(42,48)
7	6	dihedral	49	2,5	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)(2,8)(3,15)(4,22)(5,29)(6,36)(7,43)(10,16)(11,23)(12,30)(13,37)(14,44)(18,24)(19,31)(20,38)(21,45)(26,32)(27,39)(28,46)(34,40)(35,47)(42,48)
8	6	dihedral	49	3,5	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)(2,15)(3,22)(4,29)(5,36)(6,43)(7,8)(9,21)(10,28)(11,35)(12,42)(13,49)(17,23)(18,30)(19,37)(20,44)(25,31)(26,38)(27,45)(33,39)

					(34,46)(41,47)
9	6	cyclic	49	4,5	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)
10	7	cyclic	2	1	(1,7,3,2,5,6,4)(8,14,10,9,12,13,11)(15,21,17,16,19,20,18)(22,28,24,23,26,27,25)(29,35,31,30,33,34,32)(36,42,38,37,40,41,39)(43,49,45,44,47,48,46)
11	7	cyclic	3	1	(1,14,24,16,40,48,32)(2,12,27,18,36,49,31)(3,9,26,20,39,43,35)(4,8,28,17,37,47,34)(5,13,25,15,42,45,30)(6,11,22,21,38,44,33)(7,10,23,19,41,46,29)
12	7	cyclic	3	1	(1,19,35,27,45,11,37)(2,15,33,28,48,10,39)(3,18,30,22,47,14,41)(4,16,29,26,49,13,38)(5,21,34,24,46,9,36)(6,17,32,23,43,12,42)(7,20,31,25,44,8,40)
13	12	dihedral	49	6,7,8,9	(2,8)(3,15)(4,22)(5,29)(6,36)(7,43)(10,16)(11,23)(12,30)(13,37)(14,44)(18,24)(19,31)(20,38)(21,45)(26,32)(27,39)(28,46)(34,40)(35,47)(42,48)(2,7,6,5,4,3)(8,15,22,29,36,43)(9,21,27,33,39,45)(10,16,28,34,40,46)(11,17,23,35,41,47)(12,18,24,30,42,48)(13,19,25,31,37,49)(14,20,26,32,38,44)
14	14	dihedral	3	3,12	(2,15)(3,22)(4,29)(5,36)(6,43)(7,8)(9,21)(10,28)(11,35)(12,42)(13,49)(17,23)(18,30)(19,37)(20,44)(25,31)(26,38)(27,45)(33,39)(34,46)(41,47)(1,19,35,27,45,11,37)(2,15,33,28,48,10,39)(3,18,30,22,47,14,41)(4,16,29,26,49,13,38)(5,21,34,24,46,9,36)(6,17,32,23,43,12,42)(7,20,31,25,44,8,40)
15	14	dihedral	3	2,11	(2,36)(3,43)(4,8)(5,15)(6,22)(7,29)(9,39)(10,46)(12,18)(13,25)(14,32)(16,40)(17,47)(20,26)(21,33)(23,41)(24,48)(28,34)(30,42)(31,49)(38,44)(1,14,24,16,40,48,32)(2,12,27,18,36,49,31)(3,9,26,20,39,43,35)(4,8,28,17,37,47,34)(5,13,25,15,42,45,30)(6,11,22,21,38,44,33)(7,10,23,19,41,46,29)
					(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)

16	14	dihedral	14	4,10	(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(1,7,3,2,5,6,4)(8,14,10,9,12,13,11)(15,21,17,16,19,20,18)(22,28,24,23,26,27,25)(29,35,31,30,33,34,32)(36,42,38,37,40,41,39)(43,49,45,44,47,48,46)
17	14	cyclic	21	2,12	(2,8)(3,15)(4,22)(5,29)(6,36)(7,43)(10,16)(11,23)(12,30)(13,37)(14,44)(18,24)(19,31)(20,38)(21,45)(26,32)(27,39)(28,46)(34,40)(35,47)(42,48)(1,49,17,9,33,41,25)(2,47,20,11,29,42,24)(3,44,19,13,32,36,28)(4,43,21,10,30,40,27)(5,48,18,8,35,38,23)(6,46,15,14,31,37,26)(7,45,16,12,34,39,22)
18	14	cyclic	21	3,11	(2,15)(3,22)(4,29)(5,36)(6,43)(7,8)(9,21)(10,28)(11,35)(12,42)(13,49)(17,23)(18,30)(19,37)(20,44)(25,31)(26,38)(27,45)(33,39)(34,46)(41,47)(1,14,24,16,40,48,32)(2,12,27,18,36,49,31)(3,9,26,20,39,43,35)(4,8,28,17,37,47,34)(5,13,25,15,42,45,30)(6,11,22,21,38,44,33)(7,10,23,19,41,46,29)
19	14	dihedral	21	4,12	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(1,19,35,27,45,11,37)(2,15,33,28,48,10,39)(3,18,30,22,47,14,41)(4,16,29,26,49,13,38)(5,21,34,24,46,9,36)(6,17,32,23,43,12,42)(7,20,31,25,44,8,40)
20	14	dihedral	21	4,11	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(1,14,24,16,40,48,32)(2,12,27,18,36,49,31)(3,9,26,20,39,43,35)(4,8,28,17,37,47,34)(5,13,25,15,42,45,30)(6,11,22,21,38,44,33)(7,10,23,19,41,46,29)
21	21	solvable	14	5,10	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)(1,7,3,2,5,6,4)(8,14,10,9,12,13,11)(15,21,17,16,19,20,18)(22,28,24,23,26,27,25)(29,35,31,30,33,34,32)(36,42,38,37,40,41,39)(43,

					49,45,44,47,48,46)
22	28	dihedral	21	6,14,17,19	(1,9)(3,44)(4,30)(5,23)(6,37)(7, 16)(10,43)(11,29)(12,22)(13,36) (14,15)(17,49)(18,35)(19,28)(20, 42)(24,47)(25,33)(27,40)(31,46) (34,39)(38,48)(1,41,9,49,25,33,17) (2,48,11,35,24,5,20,8,42,23,47,18, 29,38)(3,6,13,14,28,26,19,15,36,37, 44,46,32,31)(4,34,10,7,27,12,21,22, 40,16,43,39,30,45)
23	28	dihedral	21	6,15,18,20	(1,16)(3,9)(4,37)(5,30)(6,44)(7,23) (8,17)(11,38)(12,31)(13,45)(14,24) (18,36)(19,29)(20,43)(21,22)(25, 42)(26,35)(27,49)(32,40)(34,47) (41,46)(1,48,16,14,32,40,24)(2,13, 18,42,31,5,27,15,49,30,12,25,36, 45)(3,6,20,21,35,33,26,22,43,44,9, 11,39,38)(4,41,17,7,34,19,28,29,47, 23,8,46,37,10)
24	42	solvable	14	9,16,21	(1,7,3,2,5,6,4)(8,14,10,9,12,13, 11)(15,2117,16,19,20,18)(22,28, 24,23,26,27,25)(29,35,31,30,33, 34,32)(36,42,38,37,40,41,39)(43, 49,45,44,47,48,46)(2,7,6,5,4,3)(8, 15,22,29,36,43)(9,21,27,33,39,45) (10,16,28,34,40,46)(11,17,23,35,41, 47)(12,18,24,30,42,48)(13,19,25,31, 37,49)(14,20,26,32,38,44)
25	49	abelian	1	10,11,12	(1,7,3,2,5,6,4)(8,14,10,9,12,13,11) (15,2117,16,19,20,18)(22,28,24,23, 26,27,25)(29,35,31,30,33,34,32)(36, 42,38,37,40,41,39)(43,49,45,44,47, 48,46)(1,14,24,16,40,48,32)(2,12,27, 18,36,49,31)(3,9,26,20,39,43,35)(4,8, 28,17,37,47,34)(5,13,25,15,42,45,30) (6,11,22,21,38,44,33)(7,10,23,19,41, 46,29)
26	98	solvable	1	16,19,20,25	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34) (11,35)(12,30)(13,31)(14,32)(15, 36)(16,40)(17,41)(18,42)(19,37) (20,38)(21,39)(22,43)(23,47)(24, 48)(25,49)(26,44)(27,45)(28,46) (1,14,24,16,40,48,32)(2,12,27,18, 36,49,31)(3,9,26,20,39,43,35)(4,8, 28,17,37,47,34)(5,13,25,15,42,45, 30)(6,11,22,21,38,44,33)(7,10,23, 19,41,46,29)(1,7,3,2,5,6,4)(8,14,10, 9,12,13,11)(15,21,17,16,19,20,18) (22,28,24,23,26,27,25)(29,35,31, 30,33,34,32)(36,42,38,37,40,41, 39)(43,49,45,44,47,48,46)
					(1,28,38,30,12,20,46)(2,26,41,32, 8,21,45)(3,23,40,34,11,15,49)(4, 22,42,31,9,19,48)(5,27,39,29,14,

27	98	solvable	3	15,17,25	17,44)(6,25,36,35,10,16,47)(7,24,37,33,13,18,43)(1,33,49,41,17,25,9)(2,5,47,48,20,18,11,8,29,35,42,38,24,23)(3,26,44,6,19,46,13,15,32,14,36,31,28,37)(4,12,43,34,21,39,10,22,30,7,40,45,27,16)
28	98	solvable	3	14,18,25	(1,35,45,37,19,27,11)(2,33,48,39,15,28,10)(3,30,47,41,18,22,14)(4,29,49,38,16,26,13)(5,34,46,36,21,24,9)(6,32,43,42,17,23,12)(7,31,44,40,20,25,8)(1,40,14,48,24,32,16)(2,5,12,13,27,25,18,15,36,42,49,45,31,30)(3,33,9,6,26,11,20,22,39,21,43,38,35,44)(4,19,8,41,28,46,17,29,37,7,47,10,34,23)
29	147	solvable	1	21,25	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)(1,14,24,16,40,48,32)(2,12,27,18,36,49,31)(3,9,26,20,39,43,35)(4,8,28,17,37,47,34)(5,13,25,15,42,45,30)(6,11,22,21,38,44,33)(7,10,23,19,41,46,29)
30	196	solvable	3	22,23,26,27,28	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(1,43,49,21,17,10,9,30,33,40,41,27,25,4)(2,29,47,42,20,24,11)(3,8,44,35,19,38,13,23,32,5,36,48,28,18)(6,22,46,7,15,45,14,16,31,12,37,34,26,39)
31	294	solvable	1	24,26,29	(1,14,24,16,40,48,32)(2,12,27,18,36,49,31)(3,9,26,20,39,43,35)(4,8,28,17,37,47,34)(5,13,25,15,42,45,30)(6,11,22,21,38,44,33)(7,10,23,19,41,46,29)(2,7,6,5,4,3)(8,15,22,29,36,43)(9,21,27,33,39,45)(10,16,28,34,40,46)(11,17,23,35,41,47)(12,18,24,30,42,48)(13,19,25,31,37,49)(14,20,26,32,38,44)
32	294	solvable	1	7,27,29	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)(1,43,49,21,17,10,9,30,33,40,41,27,25,4)(2,29,47,42,20,24,11)(3,8,44,35,19,38,13,23,32,5,36,48,28,18)(6,22,46,7,15,45,14,16,31,12,37,34,26,39)
					(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,

33	294	solvable	1	8,28,29	40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)(1,8,14,28,24,17,16,37,40,47,48,34,32,4)(2,36,12,49,27,31,18)(3,15,9,42,26,45,20,30,39,5,43,13,35,25)(6,29,11,7,22,10,21,23,38,19,44,41,33,46)
34	588	*G*	1	13,30,31,32,33	(2,8)(3,15)(4,22)(5,29)(6,36)(7,43)(10,16)(11,23)(12,30)(13,37)(14,44)(18,24)(19,31)(20,38)(21,45)(26,32)(27,39)(28,46)(34,40)(35,47)(42,48)(1,7,4,2,3,5)(8,21,25,30,38,47)(9,17,26,29,42,46)(10,19,22,35,39,44)(11,16,24,33,36,49)(12,15,28,32,37,45)(13,20,27,34,41,48)(14,18,23,31,40,43)

Proper normal subgroups					
Serial	Order	Nature	Quotient	Subgroups	Generators
1	49	abelian	dihedral	--	(1,7,3,2,5,6,4)(8,14,10,9,12,13,11)(15,21,17,16,19,20,18)(22,28,24,23,26,27,25)(29,35,31,30,33,34,32)(36,42,38,37,40,41,39)(43,49,45,44,47,48,46)(1,43,15,8,29,36,22)(2,44,16,9,30,37,23)(3,45,17,10,31,38,24)(4,46,18,11,32,39,25)(5,47,19,12,33,40,26)(6,48,20,13,34,41,27)(7,49,21,14,35,42,28)
2	98	solvable	dihedral	1	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(1,7,3,2,5,6,4)(8,14,10,9,12,13,11)(15,21,17,16,19,20,18)(22,28,24,23,26,27,25)(29,35,31,30,33,34,32)(36,42,38,37,40,41,39)(43,49,45,44,47,48,46)(1,43,15,8,29,36,22)(2,44,16,9,30,37,23)(3,45,17,10,31,38,24)(4,46,18,11,32,39,25)(5,47,19,12,33,40,26)(6,48,20,13,34,41,27)(7,49,21,14,35,42,28)
3	147	solvable	abelian	1	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)(1,39,13,47,23,31,21)(2,38,14,43,25,34,19)(3,42,8,46,27,33,16)(4,41,12,44,24,35,15)(5,37,10,49,22,32,20)(6,40,9,45,28,29,18)(7,36,11,48,26,30,17)
4	294	solvable	cyclic	1,3	(1,26,20,45,30,8,28,41,46,2,12,21,38,32)(3,33,15,24,34,43,23,13,49,37,11,7,40,18)(4,5,19,17,31,29,22,27,48,44,9,14,42,39)(6,47,16,10,35,36,25)(1,47,11)(2,45,13)(3,48,9)(4,43,12)(5,46,8)(6,44,10)(7,49,14)(15,40,32)(16,38,34)(17,41,30)(18,36,33)(19,39,29)(20,37,31)(21,42,35)(22,26,25)(23,24,27)

5	294	solvable	cyclic	1,2,3	(2,7,6,5,4,3)(8,15,22,29,36,43)(9,21,27,33,39,45)(10,16,28,34,40,46)(11,17,23,35,41,47)(12,18,24,30,42,48)(13,19,25,31,37,49)(14,20,26,32,38,44)(1,39,13,47,23,31,21)(2,38,14,43,25,34,19)(3,42,8,46,27,33,16)(4,41,12,44,24,35,15)(5,37,10,49,22,32,20)(6,40,9,45,28,29,18)(7,36,11,48,26,30,17)
6	294	solvable	cyclic	1,3	(2,6,4)(3,7,5)(8,22,36)(9,27,39)(10,28,40)(11,23,41)(12,24,42)(13,25,37)(14,26,38)(15,29,43)(16,34,46)(17,35,47)(18,30,48)(19,31,49)(20,32,44)(21,33,45)(1,36,41,13,9,44,49,28,25,32,33,19,17,3)(2,43,42,27,11,30,47,21,24,4,29,40,20,10)(5,15,38,6,8,37,48,14,23,46,35,26,18,31)(7,22,39,34,12,16,45)

Sylow subgroups						
p	Order	Conjug. classes	Nature	Normalizer	Normal closure	Generators
2	4	147	abelian	4	*G*	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(2,8)(3,15)(4,22)(5,29)(6,36)(7,43)(10,16)(11,23)(12,30)(13,37)(14,44)(18,24)(19,31)(20,38)(21,45)(26,32)(27,39)(28,46)(34,40)(35,47)(42,48)
3	3	49	cyclic	12	147	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)
7	49	1	abelian	*G*	49	(1,7,3,2,5,6,4)(8,14,10,9,12,13,11)(15,21,17,16,19,20,18)(22,28,24,23,26,27,25)(29,35,31,30,33,34,32)(36,42,38,37,40,41,39)(43,49,45,44,47,48,46)(1,43,15,8,29,36,22)(2,44,16,9,30,37,23)(3,45,17,10,31,38,24)(4,46,18,11,32,39,25)(5,47,19,12,33,40,26)(6,48,20,13,34,41,27)(7,49,21,14,35,42,28)

8. Let G be a primitive group of degree 49 with 2 generators. We have $|G| = 588 = 2^2 \times 3 \times 7^2$.

Generators of *G*:	a_1 = (2,30,3,38,4,46,5,12,6,20,7,28)(8,35,15,37,22,45,29,11,36,19,43,27)(9,47,17,13,25,21,33,23,41,31,49,39)(10,16,18,24,26,32,34,40,42,48,44,14) (order 12)
	a_2 = (1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35) (order 7)

G is a solvable group. G acts transitively on the set of 49 elements {1, 2, …, 49}. The center of G is trivial.

The derived subgroup $D = [G, G]$ is an abelian group of order 49, generated by {(1,39,13,47,23,31,21)(2,38,14,43,25,34,19)(3,42,8,46,27,33,16)(4,41,12,44,24,35,15)(5,37,10,49,22,32,20)(6,40,9,45,28,29,18)(7,36,11,48,26,30,17)(1,49,17,9,33,41,25)(2,47,20,11,29,42,24)(3,44,19,13,32,36,28)(4,43,21,10,30,40,27)(5,48,18,8,35,38,23)(6,46,15,14,31,37,26)(7,45,16,12,34,39,22)} and $G/D \cong C_3 \times C_4$.

Lower central Series				
Level	Order	Nature	Quotient	Successive quotient
1	49	abelian	cyclic	$C_3 \times C_4$

Derived Series				
Level	Order	Nature	Quotient	Successive quotient
1	49	abelian	cyclic	$C_3 \times C_4$
2	1	trivial	solvable	C_7^2

Conjugacy classes of subgroups					
Serial	Order	Nature	Conjugacy classes	Maximal subgroup classes	Generators
1	1	trivial	1	--	---
2	2	cyclic	49	1	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)
3	3	cyclic	49	1	(2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42)(11,27,37)(12,28,38)(13,23,39)(14,24,40)(15,29,43)(16,32,48)(17,33,49)(18,34,44)(19,35,45)(20,30,46)(21,31,47)
4	4	cyclic	49	2	(2,20,5,38)(3,28,6,46)(4,30,7,12)(8,19,29,37)(9,31,33,13)(10,48,34,24)(11,22,35,43)(14,42,32,18)(15,27,36,45)(16,44,40,26)(17,39,41,21)(23,25,47,49)
5	6	cyclic	49	2,3	(2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42)(11,27,37)(12,28,38)(13,23,39)(14,24,40)(15,29,43)(16,32,48)(17,33,49)(18,34,44)(19,35,45)(20,30,46)(21,31,47)(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)
6	7	cyclic	2	1	(1,21,31,23,47,13,39)(2,19,34,25,43,14,38)(3,16,33,27,46,8,42)(4,15,35,24,44,12,41)(5,20,32,22,49,10,37)(6,18,29,28,45,9,40)(7,17,30,26,48,11,36)
7	7	cyclic	2	1	(1,32,48,40,16,24,14)(2,31,49,36,18,27,12)(3,35,43,39,20,26,9)(4,34,47,37,17,28,8)(5,30,45,42,15,25,13)(6,

					33,44,38,21,22,11)(7,29,46,41,19, 23,10)
8	7	cyclic	2	1	(1,4,6,5,2,3,7)(8,11,13,12,9,10,14) (15,18,20,19,16,17,21)(22,25,27,26, 23,24,28)(29,32,34,33,30,31,35)(36, 39,41,40,37,38,42)(43,46,48,47,44, 45,49)
9	7	cyclic	2	1	(1,11,27,19,37,45,35)(2,10,28,15,39, 48,33)(3,14,22,18,41,47,30)(4,13,26, 16,38,49,29)(5,9,24,21,36,46,34)(6, 12,23,17,42,43,32)(7,8,25,20,40,44, 31)
10	12	cyclic	49	4,5	(2,30,3,38,4,46,5,12,6,20,7,28)(8,35, 15,37,22,45,29,11,36,19,43,27)(9,47, 17,13,25,21,33,23,41,31,49,39)(10,16, 18,24,26,32,34,40,42,48,44,14)
11	14	dihedral	14	2,9	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11, 35)(12,30)(13,31)(14,32)(15,36)(16, 40)(17,41)(18,42)(19,37)(20,38)(21, 39)(22,43)(23,47)(24,48)(25,49)(26, 44)(27,45)(28,46)(1,11,27,19,37,45, 35)(2,10,28,15,39,48,33)(3,14,22,18, 41,47,30)(4,13,26,16,38,49,29)(5,9, 24,21,36,46,34)(6,12,23,17,42,43,32) (7,8,25,20,40,44,31)
12	14	dihedral	14	2,6	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34) (11,35)(12,30)(13,31)(14,32)(15, 36)(16,40)(17,41)(18,42)(19,37) (20,38)(21,39)(22,43)(23,47)(24,48) (25,49)(26,44)(27,45)(28,46)(1,21, 31,23,47,13,39)(2,19,34,25,43,14,38) (3,16,33,27,46,8,42)(4,15,35,24,44, 12,41)(5,20,32,22,49,10,37)(6,18,29, 28,45,9,40)(7,17,30,26,48,11,36)
13	14	dihedral	14	2,7	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11, 35)(12,30)(13,31)(14,32)(15,36)(16, 40)(17,41)(18,42)(19,37)(20,38)(21, 39)(22,43)(23,47)(24,48)(25,49)(26, 44)(27,45)(28,46)(1,32,48,40,16,24, 14)(2,31,49,36,18,27,12)(3,35,43,39, 20,26,9)(4,34,47,37,17,28,8)(5,30,45, 42,15,25,13)(6,33,44,38,21,22,11)(7, 29,46,41,19,23,10)
14	14	dihedral	14	2,8	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11, 35)(12,30)(13,31)(14,32)(15,36)(16, 40)(17,41)(18,42)(19,37)(20,38)(21, 39)(22,43)(23,47)(24,48)(25,49)(26, 44)(27,45)(28,46)(1,4,6,5,2,3,7)(8, 11,13,12,9,10,14)(15,18,20,19,16, 17,21)(22,25,27,26,23,24,28)(29, 32,34,33,30,31,35)(36,39,41,40,37, 38,42)(43,46,48,47,44,45,49)
					(2,4,6)(3,5,7)(8,22,36)(9,25,41) (10,26,42)(11,27,37)(12,28,38)(13,

15	21	solvable	14	3,9	23,39)(14,24,40)(15,29,43)(16,32,48)(17,33,49)(18,34,44)(19,35,45)(20,30,46)(21,31,47)(1,11,27,19,37,45,35)(2,10,28,15,39,48,33)(3,14,22,18,41,47,30)(4,13,26,16,38,49,29)(5,9,24,21,36,46,34)(6,12,23,17,42,43,32)(7,8,25,20,40,44,31)
16	21	solvable	14	3,7	(2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42)(11,27,37)(12,28,38)(13,23,39)(14,24,40)(15,29,43)(16,32,48)(17,33,49)(18,34,44)(19,35,45)(20,30,46)(21,31,47)(1,32,48,40,16,24,14)(2,31,49,36,18,27,12)(3,35,43,39,20,26,9)(4,34,47,37,17,28,8)(5,30,45,42,15,25,13)(6,33,44,38,21,22,11)(7,29,46,41,19,23,10)
17	21	solvable	14	3,8	(2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42)(11,27,37)(12,28,38)(13,23,39)(14,24,40)(15,29,43)(16,32,48)(17,33,49)(18,34,44)(19,35,45)(20,30,46)(21,31,47)(1,4,6,5,2,3,7)(8,11,13,12,9,10,14)(15,18,20,19,16,17,21)(22,25,27,26,23,24,28)(29,32,34,33,30,31,35)(36,39,41,40,37,38,42)(43,46,48,47,44,45,49)
18	21	solvable	14	3,6	(2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42)(11,27,37)(12,28,38)(13,23,39)(14,24,40)(15,29,43)(16,32,48)(17,33,49)(18,34,44)(19,35,45)(20,30,46)(21,31,47)(1,21,31,23,47,13,39)(2,19,34,25,43,14,38)(3,16,33,27,46,8,42)(4,15,35,24,44,12,41)(5,20,32,22,49,10,37)(6,18,29,28,45,9,40)(7,17,30,26,48,11,36)
19	42	solvable	14	5,14,17	(1,4,6,5,2,3,7)(8,11,13,12,9,10,14)(15,18,20,19,16,17,21)(22,25,27,26,23,24,28)(29,32,34,33,30,31,35)(36,39,41,40,37,38,42)(43,46,48,47,44,45,49)(2,7,6,5,4,3)(8,43,36,29,22,15)(9,49,41,33,25,17)(10,44,42,34,26,18)(11,45,37,35,27,19)(12,46,38,30,28,20)(13,47,39,31,23,21)(14,48,40,32,24,16)
20	42	solvable	14	5,12,18	(1,21,31,23,47,13,39)(2,19,34,25,43,14,8)(3,16,33,27,46,8,42)(4,15,35,24,44,12,41)(5,20,32,22,49,10,37)(6,18,29,28,45,9,40)(7,17,30,26,48,11,36)(2,7,6,5,4,3)(8,43,36,29,22,15)(9,49,41,33,25,17)(10,44,42,34,26,18)(11,45,37,35,27,19)(12,46,38,30,28,20)(13,47,39,31,23,21)(14,48,40,32,24,16)
					(1,32,48,40,16,24,14)(2,31,49,36,18,

21	42	solvable	14	5,13,16	27,12)(3,35,43,39,20,26,9)(4,34,47, 37,17,28,8)(5,30,45,42,15,25,13)(6, 33,44,38,21,22,11)(7,29,46,41,19,23, 10)(2,7,6,5,4,3)(8,43,36,29,22,15)(9, 49,41,33,25,17)(10,44,42,34,26,18) (11,45,37,35,27,19)(12,46,38,30,28, 20)(13,47,3931,23,21)(14,48,40,32, 24,16)
22	42	solvable	14	5,11,15	(1,11,27,19,37,45,35)(2,10,28,15, 39,48,33)(3,14,22,18,41,47,30)(4, 13,26,16,38,49,29)(5,9,24,21,36, 46,34)(6,12,23,17,42,43,32)(7,8,25, 20,40,44,31)(2,7,6,5,4,3)(8,43,36, 29,22,15)(9,49,41,33,25,17)(10, 44,42,34,26,18)(11,45,37,35,27, 19)(12,46,38,30,28,20)(13,47,39, 31,23,21)(14,48,40,32,24,16)
23	49	abelian	1	6,7,8,9	(1,4,6,5,2,3,7)(8,11,13,12,9,10,14) (15,18,20,19,16,17,21)(22,25,27,26, 23,24,28)(29,32,34,33,30,31,35)(36, 39,41,40,37,38,42)(43,46,48,47,44, 45,49)(1,11,27,19,37,45,35)(2,10,28, 15,39,48,33)(3,14,22,18,41,47,30)(4, 13,26,16,38,49,29)(5,9,24,21,36,46, 34)(6,12,23,17,42,43,32)(7,8,25,20, 40,44,31)
24	98	solvable	1	11,12,13,14,23	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34) (11,35)(12,30)(13,31)(14,32)(15,36) (16,40)(17,41)(18,42)(19,37)(20,38) (21,39)(22,43)(23,47)(24,48)(25,49) (26,44)(27,45)(28,46),(1,11,27,19,37, 45,35)(2,10,28,15,39,48,33)(3,14,22, 18,41,47,30)(4,13,26,16,38,49,29) (5,9,24,21,36,46,34)(6,12,23,17,42, 43,32)(7,8,25,20,40,44,31)(1,4,6,5, 2,3,7)(8,11,13,12,9,10,14)(15,18,20, 19,16,17,21)(22,25,27,26,23,24,28) (29,32,34,33,30,31,35)(36,39,41,40, 37,38,42)(43,46,48,47,44,45,49)
25	147	solvable	1	15,16,17,18,23	(2,4,6)(3,5,7)(8,22,36)(9,25,41)(10, 26,42)(11,27,37)(12,28,38)(13,23, 39)(14,24,40)(15,29,43)(16,32,48) (17,33,49)(18,34,44)(19,35,45)(20, 30,46)(21,31,47)(1,11,27,19,37,45, 35)(2,10,28,15,39,48,33)(3,14,22,18, 41,47,30)(4,13,26,16,38,49,29)(5,9, 24,21,36,46,34)(6,12,23,17,42,43, 32)(7,8,25,20,40,44,31)(1,4,6,5,2,3, 7)(8,11,13,12,9,10,14)(15,18,20,19, 16,17,21)(22,25,27,26,23,24,28)(29, 32,34,33,30,31,35)(36,39,41,40,37, 38,42)(43,46,48,47,44,45,49)
					(2,20,5,38)(3,28,6,46)(4,30,7,12)(8,19, 29,37)(9,31,33,13)(10,48,34,24)(11,

26	196	solvable	1	4,24	22,35,43)(14,42,32,18)(15,27,36,45)(16,44,40,26)(17,39,41,21)(23,25,47,49)(1,30,46,38,20,28,12)(2,32,45,41,21,26,8)(3,34,49,40,15,23,11)(4,31,48,42,19,22,9)(5,29,44,39,17,27,14)(6,35,47,36,16,25,10)(7,33,43,37,18,24,13)
27	294	solvable	1	19,20,21,22,24,25	(1,11,27,19,37,45,35)(2,10,28,15,39,48,33)(3,14,22,18,41,47,30)(4,13,26,16,38,49,29)(5,9,24,21,36,46,34)(6,12,23,17,42,43,32)(7,8,25,20,40,44,31)(1,4,6,5,2,3,7)(8,11,13,12,9,10,14)(15,18,20,19,16,17,21)(22,25,27,26,23,24,28)(29,32,34,33,30,31,35)(36,39,41,40,37,38,42)(43,46,48,47,44,45,49)(2,7,6,5,4,3)(8,43,36,29,22,15)(9,49,41,33,25,17)(10,44,42,34,26,18)(11,45,37,35,27,19)(12,46,38,30,28,20)(13,47,39,31,23,21)(14,48,40,32,24,16)
28	588	*G*	1	10,26,27	(2,30,3,38,4,46,5,12,6,20,7,28)(8,35,15,37,22,45,29,11,36,19,43,27)(9,47,17,13,25,21,33,23,41,31,49,39)(10,16,18,24,26,32,34,40,42,48,44,14)(1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35)

Maximal normal subgroups				
Serial	Order	Nature	Quotient	Generators
1	294	solvable	cyclic	(2,7,6,5,4,3)(8,43,36,29,22,15)(9,49,41,33,25,17)(10,44,42,34,26,18)(11,45,37,35,27,19)(12,46,38,30,28,20)(13,47,39,31,23,21)(14,48,40,32,24,16)(1,9,25,17,41,49,33)(2,11,24,20,42,47,29)(3,13,28,19,36,44,32)(4,10,27,21,40,43,30)(5,8,23,18,38,48,35)(6,14,26,15,37,46,31)(7,12,22,16,39,45,34)(1,26,42,34,10,18,44)(2,22,40,35,13,17,46)(3,25,37,29,12,21,48)(4,23,36,33,14,20,45)(5,28,41,31,11,16,43)(6,24,39,30,8,19,49)(7,27,38,32,9,15,47)
2	196	solvable	cyclic	(1,21,31,23,47,13,39)(2,19,34,25,43,14,38)(3,16,33,27,46,8,42)(4,15,35,24,44,12,41)(5,20,32,22,49,10,37)(6,18,29,28,45,9,40)(7,17,30,26,48,11,36)(1,23,5,49)(2,34,7,11)(4,15,6,40)(8,32,33,13)(9,21,35,37)(10,41,31,18)(12,26,29,43)(14,45,30,24)(16,36,42,19)(17,47,38,22)(20,25,39,48)(27,44,46,28)

Sylow subgroups						
p	Order	Conjug. classes	Nature	Normalizer	Normal closure	Generators
2	4	49	cyclic	12	196	(2,20,5,38)(3,28,6,46)(4,30,7,12)(8,19,29,37)(9,31,33,13)(10,48,34,24)(11,22,35,43)(14,42,32,18)(15,27,36,45)(16,44,40,26)(17,39,41,21)(23,25,47,49)
3	3	49	] cyclic	12	147	(2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42)(11,27,37)(12,28,38)(13,23,39)(14,24,40)(15,29,43)(16,32,48)(17,33,49)(18,34,44)

						(19,35,45)(20,30,46)(21,31,47)
7	49	1	abelian	*G*	49	(1,14,24,16,40,48,32)(2,12,27,18,36,49,31)(3,9,26,20,39,43,35)(4,8,28,17,37,47,34)(5,13,25,15,42,45,30)(6,11,22,21,38,44,33)(7,10,23,19,41,46,29)(1,15,29,22,43,8,36)(2,16,30,23,44,9,37)(3,17,31,24,45,10,38)(4,18,32,25,46,11,39)(5,19,33,26,47,12,40)(6,20,34,27,48,13,41)(7,21,35,28,49,14,42)

9. Let G be a primitive group of degree 49 with 3 generators. We have $|G| = 784 = 2^4\times 7^2$.

Generators of G:	a_1 = (2,33)(3,41)(4,49)(5,9)(6,17)(7,25)(10,48)(11,35)(12,23)(13,38)(14,18)(16,26)(19,37)(20,31)(21,46)(24,34)(27,45)(28,39)(30,47)(32,42)(40,44) (order 2)
	a_2 = (2,33,38,31,5,9,20,13)(3,41,46,39,6,17,28,21)(4,49,12,47,7,25,30,23)(8,42,37,14,29,18,19,32)(10,11,24,43,34,35,48,22)(15,44,45,16,36,26,27,40) (order 8)
	a_3 = (1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35) (order 7)

This generating set is not minimal. There is a smaller set with 2 elements. G is a solvable group. G acts transitively on the set of 49 elements {1, 2, …, 49}. The center of G is trivial.

The derived subgroup $D = [G, G]$ is a solvable group of order 196, generated by {(2,20,5,38)(3,28,6,46)(4,30,7,12)(8,19,29,37)(9,31,33,13)(10,48,34,24)(11,22,35,43)(14,42,32,18)(15,27,36,45)(16,44,40,26)(17,39,41,21)(23,25,47,49)(1,21,31,23,47,13,39)(2,19,34,25,43,14,38)(3,16,33,27,46,8,42)(4,15,35,24,44,12,41)(5,20,32,22,49,10,37)(6,18,29,28,45,9,40)(7,17,30,26,48,11,36)} and $G/D \cong C_2^2$.

Lower central Series				
Level	Order	Nature	Quotient	Successive quotient
1	196	solvable	abelian	C_2^2
2	98	solvable	nilpotent	C_2
3	49	abelian	nilpotent	C_2

Derived Series				
Level	Order	Nature	Quotient	Successive quotient
1	196	solvable	abelian	C_2^2
2	49	abelian	nilpotent	C_4
3	1	trivial	solvable	C_7^2

Conjugacy classes of subgroups					
Serial	Order	Nature	Conjugacy classes	Maximal subgroup classes	Generators
1	1	trivial	1	--	---
2	2	cyclic	28	1	(8,32)(9,31)(10,35)(11,34)(12,30)(13,33)(14,29)(15,40)(16,36)(17,39)(18,37)(19,42)(20,38)(21,41)(22,48)(23,49)(24,43)(25,47)(26,45)(27,44)(28,46)
3	2	cyclic	28	1	(2,9)(3,17)(4,25)(5,33)(6,41)(7,49)(8,29)(10,24)(12,47)(13,20)(14,42)(15,36)(16,44)(18,32)(21,28)(22,43)(23,30)(26,40)(31,38)(34,48)(39,46)
					(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,

4	2	cyclic	49	1	30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)
5	4	cyclic	49	4	(2,20,5,38)(3,28,6,46)(4,30,7,12)(8,19,29,37)(9,31,33,13)(10,48,34,24)(11,22,35,43)(14,42,32,18)(15,27,36,45)(16,44,40,26)(17,39,41,21)(23,25,47,49)
6	4	abelian	98	3,4	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(2,9)(3,17)(4,25)(5,33)(6,41)(7,49)(8,29)(10,24)(12,47)(13,20)(14,42)(15,36)(16,44)(18,32)(21,28)(22,43)(23,30)(26,40)(31,38)(34,48)(39,46)
7	4	abelian	98	2,4	(8,32)(9,31)(10,35)(11,34)(12,30)(13,33)(14,29)(15,40)(16,36)(17,39)(18,37)(19,42)(20,38)(21,41)(22,48)(23,49)(24,43)(25,47)(26,45)(27,44)(28,46)(2,5)(3,6)(4,7)(8,14)(9,13)(10,11)(15,16)(17,21)(18,19)(22,24)(23,25)(26,27)(29,32)(31,33)(34,35)(36,40)(37,42)(39,41)(43,48)(44,45)(47,49)
8	7	cyclic	4	1	(1,5,7,6,3,4,2)(8,12,14,13,10,11,9)(15,19,21,20,17,18,16)(22,26,28,27,24,25,23)(29,33,35,34,31,32,30)(36,40,42,41,38,39,37)(43,47,49,48,45,46,44)
9	7	cyclic	4	1	(1,15,29,22,43,8,36)(2,16,30,23,44,9,37)(3,17,31,24,45,10,38)(4,18,32,25,46,11,39)(5,19,33,26,47,12,40)(6,20,34,27,48,13,41)(7,21,35,28,49,14,42)
10	8	cyclic	49	5	(2,33,38,31,5,9,20,13)(3,41,46,39,6,17,28,21)(4,49,12,47,7,25,30,23)(8,42,37,14,29,18,19,32)(10,11,24,43,34,35,48,22)(15,44,45,16,36,26,27,40)
11	8	nilpotent	49	5,6	(2,13)(3,21)(4,23)(5,31)(6,39)(7,47)(8,19)(9,38)(10,34)(11,43)(12,25)(15,27)(17,46)(18,42)(20,33)(22,35)(26,44)(28,41)(29,37)(30,49)(36,45),(2,9)(3,17)(4,25)(5,33)(6,41)(7,49)(8,29)(10,24)(12,47)(13,20)(14,42)(15,36)(16,44)(18,32)(21,28)(22,43)(23,30)(26,40)(31,38)(34,48)(39,46)
12	8	nilpotent	49	5,7	(2,20)(3,28)(4,30)(5,38)(6,46)(7,12)(8,18)(9,33)(10,43)(11,24)(14,37)(15,26)(16,45)(17,41)(19,32)(22,34)(25,49)(27,40)(29,42)(35,48)(36,44)(8,32)(9,31)(10,35)(11,34)(12,30)(13,33)(14,29)(15,40)(16,36)(17,39)(18,37)(19,42)(20,38)(21,41)(22,48)(23,49)(24,43)(25,47)(26,45)(27,44)(28,46)
13	14	dihedral	4	2,8	(2,5)(3,6)(4,7)(8,14)(9,13)(10,11)(15,16)(17,21)(18,19)(22,24)(23,25)(26,27)(29,32)(31,33)(34,35)(36,40)(37,42)(39,41)(43,48)(44,45)(47,49)(1,5,7,6,3,4,2)(8,12,14,13,10,11,9)(15,19,21,20,17,18,16)(22,26,28,27,24,25,23)(29,33,35,34,31,32,30)(36,40,42,41,38,39,37)(43,47,49,48,45,46,44)

14	14	dihedral	4	3,9	(2,9)(3,17)(4,25)(5,33)(6,41)(7,49)(8,29)(10,24)(12,47)(13,20)(14,42)(15,36)(16,44)(18,32)(21,28)(22,43)(23,30)(26,40)(31,38)(34,48)(39,46)(1,15,29,22,43,8,36)(2,16,30,23,44,9,37)(3,17,31,24,45,10,38)(4,18,32,25,46,11,39)(5,19,33,26,47,12,40)(6,20,34,27,48,13,41)(7,21,35,28,49,14,42)
15	14	cyclic	28	3,9	(2,9)(3,17)(4,25)(5,33)(6,41)(7,49)(8,29)(10,24)(12,47)(13,20)(14,42)(15,36)(16,44)(18,32)(21,28)(22,43)(23,30)(26,40)(31,38)(34,48)(39,46)(1,45,19,11,35,37,27)(2,48,15,10,33,39,28)(3,47,18,14,30,41,22)(4,49,16,13,29,38,26)(5,46,21,9,34,36,24)(6,43,17,12,32,42,23)(7,44,20,8,31,40,25)
16	14	dihedral	28	4,8	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(1,5,7,6,3,4,2)(8,12,14,13,10,11,9)(15,19,21,20,17,18,16)(22,26,28,27,24,25,23)(29,33,35,34,31,32,30)(36,40,42,41,38,39,37)(43,47,49,48,45,46,44)
17	14	dihedral	28	4,9	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(1,15,29,22,43,8,36)(2,16,30,23,44,9,37)(3,17,31,24,45,10,38)(4,18,32,25,46,11,39)(5,19,33,26,47,12,40)(6,20,34,27,48,13,41)(7,21,35,28,49,14,42)
18	14	cyclic	28	2,8	(8,32)(9,31)(10,35)(11,34)(12,30)(13,33)(14,29)(15,40)(16,36)(17,39)(18,37)(19,42)(20,38)(21,41)(22,48)(23,49)(24,43)(25,47)(26,45)(27,44)(28,46),(1,5,7,6,3,4,2)(8,12,14,13,10,11,9)(15,19,21,20,17,18,16)(22,26,28,27,24,25,23)(29,33,35,34,31,32,30)(36,40,42,41,38,39,37)(43,47,49,48,45,46,44)
19	16	nilpotent	49	10,11,12	(2,9)(3,17)(4,25)(5,33)(6,41)(7,49)(8,29)(10,24)(12,47)(13,20)(14,42)(15,36)(16,44)(18,32)(21,28)(22,43)(23,30)(26,40)(31,38)(34,48)(39,46)(8,32)(9,31)(10,35)(11,34)(12,30)(13,33)(14,29)(15,40)(16,36)(17,39)(18,37)(19,42)(20,38)(21,41)(22,48)(23,49)(24,43)(25,47)(26,45)(27,44)(28,46)
20	28	dihedral	28	7,13,16,18	(1,4)(3,5)(6,7)(9,12)(10,13)(11,14)(15,17)(16,18)(19,20)(22,28)(23,27)(24,25)(29,34)(30,31)(33,35)(36,37)(38,42)(39,40)(43,47)(44,49)(46,48)(1,4,6,5,2,3,7)(8,34,13,30,9,35,14,32,11,33,12,31,10,29)(15,37,20,42,16,39,21,40,18,38,19,36,17,41)(22,47,27,45,23,43,28,48,25,44,26,49,24,46)
21	28	dihedral	28	6,14,15,17	(1,11)(3,47)(4,29)(5,24)(6,42)(7,20)(8,25)(10,33)(12,17)(13,49)(14,41)(15,39)(18,22)(19,45)(21,34)(26,38)(27,35)(28,48)(31,40)(32,43)(36,46)(1,37,11,45,27,35,19)(2,46,10,

					34,28,5,15,9,39,24,48,21,33,36)(3,6,14,12,22, 23,18,17,41,42,47,43,30,32)(4,31,13,7,26,8, 16,25,38,20,49,40,29,44)
22	49	abelian	1	8,9	(1,5,7,6,3,4,2)(8,12,14,13,10,11,9)(15,19,21, 20,17,18,16)(22,26,28,27,24,25,23)(29,33,35, 34,31,32,30)(36,40,42,41,38,39,37)(43,47,49, 48,45,46,44)(1,9,25,17,41,49,33)(2,11,24,20, 42,47,29)(3,13,28,19,36,44,32)(4,10,27,21,40, 43,30)(5,8,23,18,38,48,35)(6,14,26,15,37,46, 31)(7,12,22,16,39,45,34)
23	98	solvable	1	16,17,22	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12, 30)(13,31)(14,32)(15,36)(16,40)(17,41)(18, 42)(19,37)(20,38)(21,39)(22,43)(23,47)(24, 48)(25,49)(26,44)(27,45)(28,46)(1,9,25,17, 41,49,33)(2,11,24,20,42,47,29)(3,13,28,19,36, 44,32)(4,10,27,21,40,43,30)(5,8,23,18,38,48, 35)(6,14,26,15,37,46,31)(7,12,22,16,39,45,34) (1,5,7,6,3,4,2)(8,12,14,13,10,11,9)(15,19,21, 20,17,18,16)(22,26,28,27,24,25,23)(29,33,35, 34,31,32,30)(36,40,42,41,38,39,37)(43,47,49, 48,45,46,44)
24	98	solvable	4	13,18,22	(1,6,2,7,4,5,3)(8,13,9,14,11,12,10)(15,20,16, 21,18,19,17)(22,27,23,28,25,26,24)(29,34,30, 35,32,33,31)(36,41,37,42,39,40,38)(43,48,44, 49,46,47,45)(1,12,28,20,38,46,30)(2,14,26,17, 41,44,32,5,8,27,21,39,45,29)(3,10,23,16,40,47, 34,6,11,25,15,36,49,35)(4,13,22,18,42,43,31,7, 9,24,19,37,48,33)
25	98	solvable	4	14,15,22	(1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10, 24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26, 19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21, 42,49,35)(1,19,35,27,45,11,37)(2,36,33,21,48, 24,39,9,15,5,28,34,10,46)(3,32,30,43,47,42,41, 17,18,23,22,12,14,6)(4,44,29,40,49,20,38,25,1 6,8,26,7,13,31)
26	196	solvable	1	5,23	(2,20,5,38)(3,28,6,46)(4,30,7,12)(8,19,29,37) (9,31,33,13)(10,48,34,24)(11,22,35,43)(14,42, 32,18)(15,27,36,45)(16,44,40,26)(17,39,41,21) (23,25,47,49)(1,38,12,46,28,30,20)(2,41,8,45, 26,32,21)(3,40,11,49,23,34,15)(4,42,9,48,22, 31,19)(5,39,14,44,27,29,17)(6,36,10,47,25,35, 16)(7,37,13,43,24,33,18)
27	196	solvable	2	20,23,24	(1,4,6,5,2,3,7)(8,34,13,30,9,35,14,32,11,33, 12,31,10,29)(15,37,20,42,16,39,21,40,18,38, 19,36,17,41)(22,47,27,45,23,43,28,48,25,44, 26,49,24,46)(1,46,20,12,30,38,28)(2,44,21, 14,32,39,26,5,45,17,8,29,41,27)(3,47,15,10, 34,36,23,6,49,16,11,35,40,25)(4,43,19,13,31, 37,22,7,48,18,9,33,42,24)
28	196	solvable	2	21,23,25	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12, 30)(13,31)(14,32)(15,36)(16,40)(17,41)(18, 42)(19,37)(20,38)(21,39)(22,43)(23,47)(24, 48)(25,49)(26,44)(27,45)(28,46)(1,33,35,48, 45,39,37,15,19,28,27,10,11,2)(3,25,30,8,47,7,

					41,31,18,44,22,40,14,20)(4,9,29,5,49,34,38,46,16,36,26,21,13,24)(6,17,32,23,43,12,42)
29	392	solvable	1	12,26,27	(2,20)(3,28)(4,30)(5,38)(6,46)(7,12)(8,18)(9,33)(10,43)(11,24)(14,37)(15,26)(16,45)(17,41)(19,32)(22,34)(25,49)(27,40)(29,42)(35,48)(36,44)(1,17,30,27,46,14,38,5,20,29,28,44,12,39)(2,18,32,24,45,13,41,7,21,33,26,43,8,37)(3,15,34,23,49,11,40)(4,16,31,25,48,10,42,6,19,35,22,47,9,36)
30	392	solvable	1	11,26,28	(2,9)(3,17)(4,25)(5,33)(6,41)(7,49)(8,29)(10,24)(12,47)(13,20)(14,42)(15,36)(16,44)(18,32)(21,28)(22,43)(23,30)(26,40)(31,38)(34,48)(39,46)(1,31,32,49,48,36,40,18,16,27,24,12,14,2)(3,23,35,10,43,7,39,29,20,46,26,41,9,19)(4,13,34,5,47,30,37,45,17,42,28,15,8,25)(6,21,33,22,44,11,38)
31	392	solvable	1	10,26	(2,33,38,31,5,9,20,13)(3,41,46,39,6,17,28,21)(4,49,12,47,7,25,30,23)(8,42,37,14,29,18,19,32)(10,11,24,43,34,35,48,22)(15,44,45,16,36,26,27,40)(1,9,25,17,41,49,33)(2,11,24,20,42,47,29)(3,13,28,19,36,44,32)(4,10,27,21,40,43,30)(5,8,23,18,38,48,35)(6,14,26,15,37,46,31)(7,12,22,16,39,45,34)
32	784	*G*	1	19,29,30,31	(8,32)(9,31)(10,35)(11,34)(12,30)(13,33)(14,29)(15,40)(16,36)(17,39)(18,37)(19,42)(20,38)(21,41)(22,48)(23,49)(24,43)(25,47)(26,45)(27,44)(28,46),(1,38,43,24,15,3,8,45,29,17,36,10,22,31)(2,18,44,11,16,32,9,39,30,25,37,4,23,46)(5,48,47,20,19,13,12,34,33,41,40,27,26,6)(7,35,49,42,21,28,14)

Proper normal subgroups					
Serial	Order	Nature	Quotient	Subgroups	Generators
1	49	abelian	nilpotent	--	(1,3,5,4,7,2,6)(8,10,12,11,14,9,13)(15,17,19,18,21,16,20)(22,24,26,25,28,23,27)(29,31,33,32,35,30,34)(36,38,40,39,42,37,41)(43,45,47,46,49,44,48)(1,15,29,22,43,8,36)(2,16,30,23,44,9,37)(3,17,31,24,45,10,38)(4,18,32,25,46,11,39)(5,19,33,26,47,12,40)(6,20,34,27,48,13,41)(7,21,35,28,49,14,42)
2	98	solvable	nilpotent	1	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(1,15,29,22,43,8,36)(2,16,30,23,44,9,37)(3,17,31,24,45,10,38)(4,18,32,25,46,11,39)(5,19,33,26,47,12,40)(6,20,34,27,48,13,41)(7,21,35,28,49,14,42)(1,3,5,4,7,2,6)(8,10,12,11,14,9,13)(15,17,19,18,21,16,20)(22,24,26,25,28,23,27)(29,31,33,32,35,30,34)(36,38,40,39,42,37,41)(43,45,47,46,49,44,48)
					(2,38,5,20)(3,46,6,28)(4,12,7,30)(8,37,29,19)(9,13,33,31)(10,24,34,48)(11,43,35,22)(14,18,32,42)(15,45,36,27)(16,26,40,44)(17,21,41,39)

3	196	solvable	abelian	1,2	(23,49,47,25)(1,46,20,12,30,38,28)(2,45,21,8,32,41,26)(3,49,15,11,34,40,23)(4,48,19,9,31,42,22)(5,44,17,14,29,39,27)(6,47,16,10,35,36,25)(7,43,18,13,33,37,24)
4	392	solvable	cyclic	1,2,3	(2,31,20,33,5,13,38,9)(3,39,28,41,6,21,46,17)(4,47,30,49,7,23,12,25)(8,14,19,42,29,32,37,18)(10,43,48,11,34,22,24,35)(15,16,27,44,36,40,45,26)(1,39,13,47,23,31,21)(2,38,14,43,25,34,19)(3,42,8,46,27,33,16)(4,41,12,44,24,35,15)(5,37,10,49,22,32,20)(6,40,9,45,28,29,18)(7,36,11,48,26,30,17)
5	392	solvable	cyclic	1,2,3	(2,38,5,20)(3,46,6,28)(4,12,7,30)(8,37,29,19)(9,13,33,31)(10,24,34,48)(11,43,35,22)(14,18,32,42)(15,45,36,27)(16,26,40,44)(17,21,41,39)(23,49,47,25)(1,46)(2,45)(3,49)(4,48)(5,44)(6,47)(7,43)(8,41)(9,42)(10,36)(11,40)(12,38)(13,37)(14,39)(15,23)(16,25)(17,27)(18,24)(19,22)(20,28)(21,26)
6	392	solvable	cyclic	1,2,3	(2,33)(3,41)(4,49)(5,9)(6,17)(7,25)(10,48)(11,35)(12,23)(13,38)(14,18)(16,26)(19,37)(20,31)(21,46)(24,34)(27,45)(28,39)(30,47)(32,42)(40,44)(1,11,18,41,34,5,26,16,44,31,10,28,42,43)(2,7,17,15,35,32,22,27,46,47,13,9,40,38)(3,48,21,12,29,37,25)(4,30,20,24,33,49,23,8,45,39,14,6,36,19)

Sylow subgroups						
p	Order	Conjug. classes	Nature	Normalizer	Normal closure	Generators
2	16	49	nilpotent	16	*G*	(2,33)(3,41)(4,49)(5,9)(6,17)(7,25)(10,48)(11,35)(12,23)(13,38)(14,18)(16,26)(19,37)(20,31)(21,46)(24,34)(27,45)(28,39)(30,47)(32,42)(40,44)(8,32)(9,31)(10,35)(11,34)(12,30)(13,33)(14,29)(15,40)(16,36)(17,39)(18,37)(19,42)(20,38)(21,41)(22,48)(23,49)(24,43)(25,47)(26,45)(27,44)(28,46)
7	49	1	abelian	*G*	49	(1,22,36,29,8,15,43)(2,23,37,30,9,16,44)(3,24,38,31,10,17,45)(4,25,39,32,11,18,46)(5,26,40,33,12,19,47)(6,27,41,34,13,20,48)(7,28,42,35,14,21,49)(1,38,12,46,28,30,20)(2,41,8,45,26,32,21)(3,40,11,49,23,34,15)(4,42,9,48,22,31,19)(5,39,14,44,27,29,17)(6,36,10,47,25,35,16)(7,37,13,43,24,33,18)

10. Let G be a primitive group of degree 49 with 2 generators. We have $|G| = 784 = 2^4 \times 7^2$.

Generators of G:	a_1 = (2,35,33,48,38,22,31,10,5,11,9,24,20,43,13,34)(3,37,41,14,46,29,39,18,6,19,17,32,28,8,21,42)(4,45,49,16,12,36,47,26,7,27,25,40,30,15,23,44) (order 16)
	a_2 = (1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35) (order 7)

G is a solvable group. G acts transitively on the set of 49 elements {1, 2, …, 49}. The center of G is trivial. The derived subgroup $D = [G, G]$ is an abelian group of order 49, generated by

{(1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19, 40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35)(1,28,38,30,12,20,46)(2,26,41,32,8,21,45)(3, 23,40,34,11,15,49)(4,22,42,31,9,19,48)(5,27,39,29,14,17,44)(6,25,36,35,10,16,47)(7,24,37,33,13, 18,43)} and $G/D \cong C_{16}$.

Lower central Series				
Level	Order	Nature	Quotient	Successive quotient
1	49	abelian	cyclic	C_{16}

Derived Series				
Level	Order	Nature	Quotient	Successive quotient
1	49	abelian	cyclic	C_{16}
2	1	trivial	solvable	C_7^2

Conjugacy classes of subgroups					
Serial	Order	Nature	Conjugacy classes	Maximal subgroup classes	Generators
1	1	trivial	1	--	---
2	2	cyclic	49	1	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)
3	4	cyclic	49	2	(2,38,5,20)(3,46,6,28)(4,12,7,30)(8,37,29,19)(9,13,33,31)(10,24,34,48)(11,43,35,22)(14,18,32,42)(15,45,36,27)(16,26,40,44)(17,21,41,39)(23,49,47,25)
4	7	cyclic	8	1	(1,4,6,5,2,3,7)(8,11,13,12,9,10,14)(15,18,20,19,16,17,21)(22,25,27,26,23,24,28)(29,32,34,33,30,31,35)(36,39,41,40,37,38,42)(43,46,48,47,44,45,49)
5	8	cyclic	49	3	(2,33,38,31,5,9,20,13)(3,41,46,39,6,17,28,21)(4,49,12,47,7,25,30,23)(8,42,37,14,29,18,19,32)(10,11,24,43,34,35,48,22)(15,44,45,16,36,26,27,40)
6	14	dihedral	56	2,4	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(1,4,6,5,2,3,7)(8,11,13,12,9,10,14)(15,18,20,19,16,17,21)(22,25,27,26,23,24,28)(29,32,34,33,30,31,35)(36,39,41,40,37,38,42)(43,46,48,47,44,45,49)
7	16	cyclic	49	5	(2,35,33,48,38,22,31,10,5,11,9,24,20,43,13,34)(3,37,41,14,46,29,39,18,6,19,17,32,28,8,21,42)(4,45,49,16,12,36,47,26,7,27,25,40,30,15,23,44)
8	49	abelian	1	4	(1,4,6,5,2,3,7)(8,11,13,12,9,10,14)(15,18,20,19,16,17,21)(22,25,27,26,23,24,28)(29,32,34,33,30,31,35)(36,39,41,40,37,38,42)(43,46,48,47,44,45,49)(1,12,28,20,38,46,30)(2,8,26,21,41,45,32)(3,11,23,15,40,49,34)(4,9,22,19,42,48,31)(5,14,27,17,39,44,29)(6,10,25,16,36,47,35)(7,13,24,18,37,43,33)

9	98	solvable	1	6,8	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(1,12,28,20,38,46,30)(2,8,26,21,41,45,32)(3,11,23,15,40,49,34)(4,9,22,19,42,48,31)(5,14,27,17,39,44,29)(6,10,25,16,36,47,35)(7,13,24,18,37,43,33)(1,4,6,5,2,3,7)(8,11,13,12,9,10,14)(15,18,20,19,16,17,21)(22,25,27,26,23,24,28)(29,32,34,33,30,31,35)(36,39,41,40,37,38,42)(43,46,48,47,44,45,49)
10	196	solvable	1	3,9	(2,38,5,20)(3,46,6,28)(4,12,7,30)(8,37,29,19)(9,13,33,31)(10,24,34,48)(11,43,35,22)(14,18,32,42)(15,45,36,27)(16,26,40,44)(17,21,41,39)(23,49,47,25)(1,12,28,20,38,46,30)(2,8,26,21,41,45,32)(3,11,23,15,40,49,34)(4,9,22,19,42,48,31)(5,14,27,17,39,44,29)(6,10,25,16,36,47,35)(7,13,24,18,37,43,33)
11	392	solvable	1	5,10	(2,33,38,31,5,9,20,13)(3,41,46,39,6,17,28,21)(4,49,12,47,7,25,30,23)(8,42,37,14,29,18,19,32)(10,11,24,43,34,35,48,22)(15,44,45,16,36,26,27,40)(1,12,28,20,38,46,30)(2,8,26,21,41,45,32)(3,11,23,15,40,49,34)(4,9,22,19,42,48,31)(5,14,27,17,39,44,29)(6,10,25,16,36,47,35)(7,13,24,18,37,43,33)
12	784	*G*	1	7,11	(2,35,33,48,38,22,31,10,5,11,9,24,20,43,13,34)(3,37,41,14,46,29,39,18,6,19,17,32,28,8,21,42)(4,45,49,16,12,36,47,26,7,27,25,40,30,15,23,44)(1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35)

Maximal normal subgroups				
Serial	Order	Nature	Quotient	Generators
1	392	solvable	cyclic	(2,33,38,31,5,9,20,13)(3,41,46,39,6,17,28,21)(4,49,12,47,7,25,30,23)(8,42,37,14,29,18,19,32)(10,11,24,43,34,35,48,22)(15,44,45,16,36,26,27,40)(1,7,3,2,5,6,4)(8,14,10,9,12,13,11)(15,21,17,16,19,20,18)(22,28,24,23,26,27,25)(29,35,31,30,33,34,32)(36,42,38,37,40,41,39)

Sylow subgroups						
p	Order	Conjug. classes	Nature	Normalizer	Normal closure	Generators
2	16	49	cyclic	16	*G*	(2,35,33,48,38,22,31,10,5,11,9,24,20,43,13,34)(3,37,41,14,46,29,39,18,6,19,17,32,28,8,21,42)(4,45,49,16,12,36,47,26,7,27,25,40,30,15,23,44)
7	49	1	abelian	*G*	49	(1,15,29,22,43,8,36)(2,16,30,23,44,9,37)(3,17,31,24,45,10,38)(4,18,32,25,46,11,39)(5,19,33,26,47,12,40)(6,20,34,27,48,13,41)(7,21,35,28,49,14,42)(1,23,39,31,13,21,47)

						(2,25,38,34,14,19,43)(3,27,42,33,8,16,46)(4,24,41,35,12,15,44)(5,22,37,32,10,20,49)(6,28,40,29,9,18,45)(7,26,36,30,11,17,48)

11. Let G be a primitive group of degree 49 with 3 generators. We have $|G| = 784 = 2^4 \times 7^2$.

Generators of G:	a_1 = (2,48,5,24)(3,14,6,32)(4,16,7,40)(8,21,29,39)(9,11,33,35)(10,38,34,20)(12,44,30,26)(13,22,31,43)(15,23,36,47)(17,19,41,37)(18,46,42,28)(25,27,49,45) (order 4)
	a_2 = (2,33,38,31,5,9,20,13)(3,41,46,39,6,17,28,21)(4,49,12,47,7,25,30,23)(8,42,37,14,29,18,19,32)(10,11,24,43,34,35,48,22)(15,44,45,16,36,26,27,40) (order 8)
	a_3 = (1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35) (order 7)

This generating set is not minimal. There is a smaller generating set with 2 elements. G is a solvable group. G acts transitively on the set of 49 elements {1, 2, …, 49}. The center of G is trivial.

The derived subgroup $D = [G, G]$ is a solvable group of order 196, generated by {(2,20,5,38)(3,28,6,46)(4,30,7,12)(8,19,29,37)(9,31,33,13)(10,48,34,24)(11,22,35,43)(14,42,32,18)(15,27,36,45)(16,44,40,26)(17,39,41,21)(23,25,47,49)(1,28,38,30,12,20,46)(2,26,41,32,8,21,45)(3,23,40,34,11,15,49)(4,22,42,31,9,19,48)(5,27,39,29,14,17,44)(6,25,36,35,10,16,47)(7,24,37,33,13,18,43)} and $G/D \cong C_2^2$.

Lower central Series				
Level	Order	Nature	Quotient	Successive quotient
1	196	solvable	abelian	C_2^2
2	98	solvable	nilpotent	C_2
3	49	abelian	nilpotent	C_2

Derived Series				
Level	Order	Nature	Quotient	Successive quotient
1	196	solvable	abelian	C_2^2
2	49	abelian	nilpotent	C_4
3	1	trivial	solvable	C_7^2

Conjugacy classes of subgroups					
Serial	Order	Nature	Conjugacy classes	Maximal subgroup classes	Generators
1	1	trivial	1	--	---
2	2	cyclic	49	1	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)
3	4	cyclic	49	2	(2,38,5,20)(3,46,6,28)(4,12,7,30)(8,37,29,19)(9,13,33,31)(10,24,34,48)(11,43,35,22)(14,18,32,42)(15,45,36,27)(16,26,40,44)(17,21,41,39)(23,49,47,25)
4	4	cyclic	98	2	(2,10,5,34)(3,18,6,42)(4,26,7,44)(8,41,29,17)(9,43,33,22)(11,31,35,13)(12,16,30,40)(14,28,32,46)(15,49,36,25)(19,39,37,21)(20,24,38,48)(23,27,47,45)

5	4	cyclic	98	2	(2,22,5,43)(3,29,6,8)(4,36,7,15)(9,24,33,48)(10,31,34,13)(11,38,35,20)(12,45,30,27)(14,17,32,41)(16,25,40,49)(18,39,42,21)(19,46,37,28)(23,26,47,44)
6	7	cyclic	8	1	(1,4,6,5,2,3,7)(8,11,13,12,9,10,14)(15,18,20,19,16,17,21)(22,25,27,26,23,24,28)(29,32,34,33,30,31,35)(36,39,41,40,37,38,42)(43,46,48,47,44,45,49)
7	8	cyclic	49	3	(2,33,38,31,5,9,20,13)(3,41,46,39,6,17,28,21)(4,49,12,47,7,25,30,23)(8,42,37,14,29,18,19,32)(10,11,24,43,34,35,48,22)(15,44,45,6,36,26,27,40)
8	8	nilpotent	49	3,4	(2,38,5,20)(3,46,6,28)(4,12,7,30)(8,37,29,19)(9,13,33,31)(10,24,34,48)(11,43,35,22)(14,18,32,42)(15,45,36,27)(16,26,40,44)(17,21,41,39)(23,49,47,25)(2,10,5,34)(3,18,6,42)(4,26,7,44)(8,41,29,17)(9,43,33,22)(11,31,35,13)(12,16,30,40)(14,28,32,46)(15,49,36,25)(19,39,37,21)(20,24,38,48)(23,27,47,45)
9	8	nilpotent	49	3,5	(2,35,5,11)(3,37,6,19)(4,45,7,27)(8,28,29,46)(9,10,33,34)(12,15,30,36)(13,48,31,24)(14,39,32,21)(16,47,40,23)(17,18,41,42)(20,22,38,43)(25,26,49,44)(2,22,5,43)(3,29,6,8)(4,36,7,15)(9,24,33,48)(10,31,34,13)(11,38,35,20)(12,45,30,27)(14,17,32,41)(16,25,40,49)(18,39,42,21)(19,46,37,28)(23,26,47,44)
10	14	dihedral	56	2,6	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(1,4,6,5,2,3,7)(8,11,13,12,9,10,14)(15,18,20,19,16,17,21)(22,25,27,26,23,24,28)(29,32,34,33,30,31,35)(36,39,41,40,37,38,42)(43,46,48,47,44,45,49)
11	16	nilpotent	49	7,8,9	(2,22,5,43)(3,29,6,8)(4,36,7,15)(9,24,33,48)(10,31,34,13)(11,38,35,20)(12,45,30,27)(14,17,32,41)(16,25,40,49)(18,39,42,21)(19,46,37,28)(23,26,47,44)(2,10,5,34)(3,18,6,42)(4,26,7,44)(8,41,29,17)(9,43,33,22)(11,31,35,13)(12,16,30,40)(14,28,32,46)(15,49,36,25)(19,39,37,21)(20,24,38,48)(23,27,47,45)
12	49	abelian	1	6	(1,4,6,5,2,3,7)(8,11,13,12,9,10,14)(15,18,20,19,16,17,21)(22,25,27,26,23,24,28)(29,32,34,33,30,31,35)(36,39,41,40,37,38,42)(43,46,48,47,44,45,49)(1,12,28,20,38,46,30)(2,8,26,21,41,45,32)(3,11,23,15,40,49,34)(4,9,22,19,42,48,31)(5,14,27,17,39,44,29)(6,10,25,16,36,47,35)(7,13,24,18,37,43,33)
13	98	solvable	1	10,12	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(1,12,28,20,38,46,30)(2,8,26,21,41,45,32)(3,11,23,15,40,49,34)(4,9,22,19,42,48,31)(5,14,27,17,39,44,

					29)(6,10,25,16,36,47,35)(7,13,24,18,37,43, 33)(1,4,6,5,2,3,7)(8,11,13,12,9,10,14)(15,18, 20,19,16,17,21)(22,25,27,26,23,24,28)(29, 32,34,33,30,31,35)(36,39,41,40,37,38,42) (43,46,48,47,44,45,49)
14	196	solvable	1	3,13	(2,38,5,20)(3,46,6,28)(4,12,7,30)(8,37,29,19) (9,13,33,31)(10,24,34,48)(11,43,35,22)(14, 18,32,42)(15,45,36,27)(16,26,40,44)(17,21, 41,39)(23,49,47,25)(1,12,28,20,38,46,30)(2, 8,26,21,41,45,32)(3,11,23,15,40,49,34)(4,9, 22,19,42,48,31)(5,14,27,17,39,44,29)(6,10, 25,16,36,47,35)(7,13,24,18,37,43,33)
15	196	solvable	2	4,13	(2,10,5,34)(3,18,6,42)(4,26,7,44)(8,41,29,17) (9,43,33,22)(11,31,35,13)(12,16,30,40)(14, 28,32,46)(15,49,36,25)(19,39,37,21)(20,24, 38,48)(23,27,47,45)(1,26,42,34,10,18,44)(2, 22,40,35,13,17,46)(3,25,37,29,12,21,48)(4, 23,36,33,14,20,45)(5,28,41,31,11,16,43)(6, 24,39,30,8,19,49)(7,27,38,32,9,15,47)
16	196	solvable	2	5,13	(2,22,5,43)(3,29,6,8)(4,36,7,15)(9,24,33,48) (10,31,34,13)(11,38,35,20)(12,45,30,27)(14, 17,32,41)(16,25,40,49)(18,39,42,21)(19,46, 37,28)(23,26,47,44)(1,36,8,43,22,29,15)(2, 37,9,44,23,30,16)(3,38,10,45,24,31,17)(4,39, 11,46,25,32,18)(5,40,12,47,26,33,19)(6,41, 13,48,27,34,20)(7,42,14,49,28,35,21)
17	392	solvable	1	8,14,15	(2,10,5,34)(3,18,6,42)(4,26,7,44)(8,41,29,17) (9,43,33,22)(11,31,35,13)(12,16,30,40)(14, 28,32,46)(15,49,36,25)(19,39,37,21)(20,24, 38,48)(23,27,47,45),(1,12,13,7)(2,46,10,18) (3,30,9,24)(4,28,11,33)(5,38,14,37)(6,20,8, 43)(15,32,48,25)(16,21,45,47)(17,41,44,36) (19,26,49,35)(22,23,34,31)(27,40,29,42)
18	392	solvable	1	9,14,16	(2,22,5,43)(3,29,6,8)(4,36,7,15)(9,24,33,48) (10,31,34,13)(11,38,35,20)(12,45,30,27)(14, 17,32,41)(16,25,40,49)(18,39,42,21)(19,46, 37,28)(23,26,47,44)(1,36,47,12)(2,21,49,23) (3,9,45,42)(4,24,48,17)(5,46,43,6)(7,34,44, 32)(8,35,40,30)(10,19,38,22)(11,37,41,14) (13,27,39,18)(15,16,26,28)(20,29,25,33)
19	392	solvable	1	7,14	(2,33,38,31,5,9,20,13)(3,41,46,39,6,17,28 21)(4,49,12,47,7,25,30,23)(8,42,37,14,29,18, 19,32)(10,11,24,43,34,35,48,22)(15,44,45, 16,36,26,27,40)(1,12,28,20,38,46,30)(2,8,26, 21,41,45,32)(3,11,23,15,40,49,34)(4,9,22,19, 42,48,31)(5,14,27,17,39,44,29)(6,10,25,16, 36,47,35)(7,13,24,18,37,43,33)
20	784	*G*	1	11,17,18,19	(2,48,5,24)(3,14,6,32)(4,16,7,40)(8,21,29,39) (9,11,33,35)(10,38,34,20)(12,44,30,26)(13, 22,31,43)(15,23,36,47)(17,19,41,37)(18,46, 42,28)(25,27,49,45)(1,44,20,35,21,48,2,36) (3,28,12,15,18,8,26,7)(4,19,38,41,17,5,32, 30)(6,13,46,24,16,23,45,11)(9,14,37,33,27, 22,34,40)(10,31,43,42,25,39,49,29)

Maximal normal subgroups				
Serial	Order	Nature	Quotient	Generators
1	32	solvable	cyclic	(2,20,5,38)(3,28,6,46)(4,30,7,12)(8,19,29,37)(9,31,33,13)(10,48,34,24)(11,22,35,43)(14,42,32,18)(15,27,36,45)(16,44,40,26)(17,39,41,21)(23,25,47,49)(1,28,48,19)(2,42,45,12)(3,14,44,40)(4,35,46,33)(5,7,49,47)(6,21,43,26)(8,23,41,17)(9,37,38,10)(11,30,39,31)(13,16,36,24)(15,22,27,20)(18,29,25,34)
2	392	solvable	cyclic	(2,33,38,31,5,9,20,13)(3,41,46,39,6,17,28,21)(4,49,12,47,7,25,30,23)(8,42,37,14,29,18,19,32)(10,11,24,43,34,35,48,22)(15,44,45,16,36,26,27,40)(1,28,38,30,12,20,46)(2,26,41,32,8,21,45)(3,23,40,34,11,15,49)(4,22,42,31,9,19,48)(5,27,39,29,14,17,44)(6,25,36,35,10,16,47)(7,24,37,33,13,18,43)
3	392	solvable	cyclic	(2,10,5,34)(3,18,6,42)(4,26,7,44)(8,41,29,17)(9,43,33,22)(11,31,35,13)(12,16,30,40)(14,28,32,46)(15,49,36,25)(19,39,37,21)(20,24,38,48)(23,27,47,45)(1,9,24,11)(2,35,25,21)(3,26,22,6)(4,39,23,44)(5,20,27,33)(7,43,28,38)(8,40,10,48)(12,32,13,16)(15,18,31,30)(17,36,29,45)(19,49,34,42)(37,41,46,47)

Sylow subgroups						
p	Order	Conjug. classes	Nature	Normalizer	Normal closure	Generators
2	16	49	nilpotent	16	*G*	(2,48,5,24)(3,14,6,32)(4,16,7,40)(8,21,29,39)(9,11,33,35)(10,38,34,20)(12,44,30,26)(13,22,31,43)(15,23,36,47)(17,19,41,37)(18,46,42,28)(25,27,49,45)(2,33,38,31,5,9,20,13)(3,41,46,39,6,17,28,21)(4,49,12,47,7,25,30,23)(8,42,37,14,29,18,19,32)(10,11,24,43,34,35,48,22)(15,44,45,16,36,26,27,40)
7	49	1	abelian	*G*	49	(1,43,15,8,29,36,22)(2,44,16,9,30,37,23)(3,45,17,10,31,38,24)(4,46,18,11,32,39,25)(5,47,19,12,33,40,26)(6,48,20,13,34,41,27)(7,49,21,14,35,42,28)(1,44,18,10,34,42,26)(2,46,17,13,35,40,22)(3,48,21,12,29,37,25)(4,45,20,14,33,36,23)(5,43,16,11,31,41,28)(6,49,19,8,30,39,24)(7,47,15,9,32,38,27)

12. Let G be a primitive group of degree 49 with 4 generators. We have $|G| = 882 = 2\times3^2\times7^2$.

Generators of *G*:	a_1 = (2,8)(3,15)(4,22)(5,29)(6,36)(7,43)(10,16)(11,23)(12,30)(13,37)(14,44)(18,24)(19,31)(20,38)(21,45)(26,32)(27,39)(28,46)(34,40)(35,47)(42,48) (order 2)
	a_2 = (2,6,4)(3,7,5)(8,22,36)(9,27,39)(10,28,40)(11,23,41)(12,24,42)(13,25,37)(14,26,38)(15,29,43)(16,34,46)(17,35,47)(18,30,48)(19,31,49)(20,32,44)(21,33,45) (order 3)
	a_3 = (2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42)(11,27,37)(12,28,38)(13,23,39)(14,24,40)(15,29,43)(16,32,48)(17,33,49)(18,34,44)(19,35,45)(20,30,46)(21,31,47) (order 3)

	a_4 = (1,8,22,15,36,43,29)(2,9,23,16,37,44,30) (3,10,24,17,38,45,31)(4,11,25,18,39,46,32) (5,12,26,19,40,47,33)(6,13,27,20,41,48,34) (7,14,28,21,42,49,35) (order 7)

This generating set is not minimal. There is a smaller generating set with 2 elements. G is a solvable group. G acts transitively on the set of 49 elements {1, 2, …, 49}. The center of G is trivial.

The derived subgroup $D = [G, G]$ is a solvable group of order 147, generated by {(2,6,4)(3,7,5)(8,22,36)(9,27,39)(10,28,40)(11,23,41)(12,24,42)(13,25,37)(14,26,38)(15,29,43)(16,34,46)(17,35,47)(18,30,48)(19,31,49)(20,32,44)(21,33,45)(1,30,46,38,20,28,12)(2,32,45,41,21,26,8)(3,34,49,40,15,23,11)(4,31,48,42,19,22,9)(5,29,44,39,17,27,14)(6,35,47,36,16,25,10)(7,33,43,37,18,24,13)} and $G/D \cong C_2 \times C_3$.

Lower central Series				
Level	Order	Nature	Quotient	Successive quotient
1	147	solvable	cyclic	$C_2 \times C_3$

Derived Series				
Level	Order	Nature	Quotient	Successive quotient
1	147	solvable	cyclic	$C_2 \times C_3$
2	49	abelian	solvable	C_3
3	1	trivial	solvable	C_7^2

Conjugacy classes of subgroups					
Serial	Order	Nature	Conjugacy classes	Maximal subgroup classes	Generators
1	1	trivial	1	--	---
2	2	cyclic	21	1	(2,8)(3,15)(4,22)(5,29)(6,36)(7,43)(10,16)(11,23)(12,30)(13,37)(14,44)(18,24)(19,31)(20,38)(21,45)(26,32)(27,39)(28,46)(34,40)(35,47)(42,48)
3	3	cyclic	14	1	(8,36,22)(9,37,23)(10,38,24)(11,39,25)(12,40,26)(13,41,27)(14,42,28)(15,43,29)(16,44,30)(17,45,31)(18,46,32)(19,47,33)(20,48,34)(21,49,35)
4	3	cyclic	49	1	(2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42)(11,27,37)(12,28,38)(13,23,39)(14,24,40)(15,29,43)(16,32,48)(17,33,49)(18,34,44)(19,35,45)(20,30,46)(21,31,47)
5	3	cyclic	49	1	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)
6	6	dihedral	49	2,5	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)(2,8)(3,15)(4,22)(5,29)(6,36)(7,43)(10,16)(11,23)(12,30)(13,37)(14,44)(18,24)(19,31)(20,38)(21,45)(26,32)(27,39)(28,46)(34,40)(35,47)(42,48)
					(2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42)(11,27,37)(12,28,38)(13,23,39)(14,24,40)(15,29,43)(16,32,48)(17,33,49)(18,34,44)(19,35,45)

7	6	cyclic	147	2,4	(20,30,46)(21,31,47)(2,8)(3,15)(4,22)(5,29)(6,36)(7,43)(10,16)(11,23)(12,30)(13,37)(14,44)(18,24)(19,31)(20,38)(21,45)(26,32)(27,39)(28,46)(34,40)(35,47)(42,48)
8	7	cyclic	2	1	(1,7,3,2,5,6,4)(8,14,10,9,12,13,11)(15,21,17,16,19,20,18)(22,28,24,23,26,27,25)(29,35,31,30,33,34,32)(36,42,38,37,40,41,39)(43,49,45,44,47,48,46)
9	7	cyclic	3	1	(1,14,24,16,40,48,32)(2,12,27,18,36,49,31)(3,9,26,20,39,43,35)(4,8,28,17,37,47,34)(5,13,25,15,42,45,30)(6,11,22,21,38,44,33)(7,10,23,19,41,46,29)
10	7	cyclic	3	1	(1,19,35,27,45,11,37)(2,15,33,28,48,10,39)(3,18,30,22,47,14,41)(4,16,29,26,49,13,38)(5,21,34,24,46,9,36)(6,17,32,23,43,12,42)(7,20,31,25,44,8,40)
11	9	abelian	49	3,4,5	(8,36,22)(9,37,23)(10,38,24)(11,39,25)(12,40,26)(13,41,27)(14,42,28)(15,43,29)(16,44,30)(17,45,31)(18,46,32)(19,47,33)(20,48,34)(21,49,35)(2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42)(11,27,37)(12,28,38)(13,23,39)(14,24,40)(15,29,43)(16,32,48)(17,33,49)(18,34,44)(19,35,45)(20,30,46)(21,31,47)
12	14	dihedral	3	2,9	(2,36)(3,43)(4,8)(5,15)(6,22)(7,29)(9,39)(10,46)(12,18)(13,25)(14,32)(16,40)(17,47)(20,26)(21,33)(23,41)(24,48)(28,34)(30,42)(31,49)(38,44)(1,14,24,16,40,48,32)(2,12,27,18,36,49,31)(3,9,26,20,39,43,35)(4,8,28,17,37,47,34)(5,13,25,15,42,45,30)(6,11,22,21,38,44,33)(7,10,23,19,41,46,29)
13	14	cyclic	21	2,10	(2,8)(3,15)(4,22)(5,29)(6,36)(7,43)(10,16)(11,23)(12,30)(13,37)(14,44)(18,24)(19,31)(20,38)(21,45)(26,32)(27,39)(28,46)(34,40)(35,47)(42,48)(1,49,17,9,33,41,25)(2,47,20,11,29,42,24)(3,44,19,13,32,36,28)(4,43,21,10,30,40,27)(5,48,18,8,35,38,23)(6,46,15,14,31,37,26)(7,45,16,12,34,39,22)
14	18	solvable	49	6,7,11	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)(2,22,6,8,4,36)(3,29,7,15,5,43)(9,25,41)(10,32,42,16,26,48)(11,39,37,23,27,13)(12,46,38,30,28,20)(14,18,40,44,24,34)(17,33,49)(19,47,45,31,35,21)
15	21	solvable	2	3,8	(2,6,4)(3,7,5)(9,13,11)(10,14,12)(16,20,18)(17,21,19)(23,27,25)(24,28,26)(30,34,32)(31,35,33)(37,41,39)(38,42,40)(44,48,46)(45,49,47)(1,7,3,2,5,6,4)(8,14,10,9,12,13,11)(15,21,17,16,19,20,18)(22,28,24,23,26,27,25)(29,35,31,30,33,34,32)(36,42,38,37,40,41,39)(43,49,45,44,47,48,46)
					(8,36,22)(9,37,23)(10,38,24)(11,39,25)(12,40,26)(13,41,27)(14,42,28)(15,43,29)(16,44,30)(17,45,31)(18,46,32)(19,47,33)(20,48,34)

16	21	cyclic	14	3,8	(21,49,35)(1,7,3,2,5,6,4)(8,14,10,9,12,13,11)(15,21,17,16,19,20,18)(22,28,24,23,26,27,25)(29,35,31,30,33,34,32)(36,42,38,37,40,41,39)(43,49,45,44,47,48,46)
17	21	solvable	14	4,8	(2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42)(11,27,37)(12,28,38)(13,23,39)(14,24,40)(15,29,43)(16,32,48)(17,33,49)(18,34,44)(19,35,45)(20,30,46)(21,31,47)(1,7,3,2,5,6,4)(8,14,10,9,12,13,11)(15,21,17,16,19,20,18)(22,28,24,23,26,27,25)(29,35,31,30,33,34,32)(36,42,38,37,40,41,39)(43,49,45,44,47,48,46)
18	21	solvable	14	5,8	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)(1,7,3,2,5,6,4)(8,14,10,9,12,13,11)(15,21,17,16,19,20,18)(22,28,24,23,26,27,25)(29,35,31,30,33,34,32)(36,42,38,37,40,41,39)(43,49,45,44,47,48,46)
19	21	solvable	21	4,9	(2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42)(11,27,37)(12,28,38)(13,23,39)(14,24,40)(15,29,43)(16,32,48)(17,33,49)(18,34,44)(19,35,45)(20,30,46)(21,31,47)(1,14,24,16,40,48,32)(2,12,27,18,36,49,31)(3,9,26,20,39,43,35)(4,8,28,17,37,47,34)(5,13,25,15,42,45,30)(6,11,22,21,38,44,33)(7,10,23,19,41,46,29)
20	21	solvable	21	4,10	(2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42)(11,27,37)(12,28,38)(13,23,39)(14,24,40)(15,29,43)(16,32,48)(17,33,49)(18,34,44)(19,35,45)(20,30,46)(21,31,47)(1,19,35,27,45,11,37)(2,15,33,28,48,10,39)(3,18,30,22,47,14,41)(4,16,29,26,49,13,38)(5,21,34,24,46,9,36)(6,17,32,23,43,12,42)(7,20,31,25,44,8,40)
21	42	solvable	21	7,13,20	(1,49,17,9,33,41,25)(2,47,20,11,29,42,24)(3,44,19,13,32,36,28)(4,43,21,10,30,40,27)(5,48,18,8,35,38,23)(6,46,15,14,31,37,26)(7,45,16,12,34,39,22)(2,22,6,8,4,36)(3,29,7,15,5,43)(9,25,41)(10,32,42,16,26,48)(11,39,37,23,27,13)(12,46,38,30,28,20)(14,18,40,44,24,34)(17,33,49)(19,47,45,31,35,21)
22	42	solvable	21	7,12,19	(1,14,24,16,40,48,32)(2,12,27,18,36,49,31)(3,9,26,20,39,43,35)(4,8,28,17,37,47,34)(5,13,25,15,42,45,30)(6,11,22,21,38,44,33)(7,10,23,19,41,46,29)(2,8,6,36,4,22)(3,15,7,43,5,29)(9,13,41,39,25,23)(10,20,42,46,26,30)(11,27,37)(12,34,38,18,28,44)(14,48,40,32,24,16)(17,21,49,47,33,31)(19,35,45)
23	49	abelian	1	8,9,10	(1,7,3,2,5,6,4)(8,14,10,9,12,13,11)(15,21,17,16,19,20,18)(22,28,24,23,26,27,25)(29,35,31,30,33,34,32)(36,42,38,37,40,41,39)(43,49,45,44,47,48,46)(1,14,24,16,40,48,32)(2,12,27,18,36,49,31)(3,9,26,20,39,43,35)(4,8,28,17,37,47,34)(5,13,25,15,42,45,30)(6,11,22,21,38,44,33)(7,10,23,19,41,46,29)
					(2,4,6)(3,5,7)(9,11,13)(10,12,14)(16,18,20)

24	63	solvable	14	11,15,16,17,18	(17,19,21)(23,25,27)(24,26,28)(30,32,34)(31,33,35)(37,39,41)(38,40,42)(44,46,48)(45,47,49)(1,7,3,2,5,6,4)(8,42,24,9,40,27,11,36,28,10,37,26,13,39,22,14,38,23,12,41,25)(15,49,31,16,47,34,18,43,35,17,44,33,20,46,29,21,45,30,19,48,32)
25	98	solvable	3	12,13,23	(1,28,38,30,12,20,46)(2,26,41,32,8,21,45)(3,23,40,34,11,15,49)(4,22,42,31,9,19,48)(5,27,39,29,14,17,44)(6,25,36,35,10,16,47)(7,24,37,33,13,18,43)(1,33,49,41,17,25,9)(2,5,47,48,20,18,11,8,29,35,42,38,24,23)(3,26,44,6,19,46,13,15,32,14,36,31,28,37)(4,12,43,34,21,39,10,22,30,7,40,45,27,16)
26	147	solvable	1	17,19,20,23	(2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42)(11,27,37)(12,28,38)(13,23,39)(14,24,40)(15,29,43)(16,32,48)(17,33,49)(18,34,44)(19,35,45)(20,30,46)(21,31,47)(1,14,24,16,40,48,32)(2,12,27,18,36,49,31)(3,9,26,20,39,43,35)(4,8,28,17,37,47,34)(5,13,25,15,42,45,30)(6,11,22,21,38,44,33)(7,10,23,19,41,46,29)(1,7,3,2,5,6,4)(8,14,10,9,12,13,11)(15,21,17,16,19,20,18)(22,28,24,23,26,27,25)(29,35,31,30,33,34,32)(36,42,38,37,40,41,39)(43,49,45,44,47,48,46)
27	147	solvable	1	18,23	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)(1,14,24,16,40,48,32)(2,12,27,18,36,49,31)(3,9,26,20,39,43,35)(4,8,28,17,37,47,34)(5,13,25,15,42,45,30)(6,11,22,21,38,44,33)(7,10,23,19,41,46,29)
28	147	solvable	2	15,16,23	(1,5,7,6,3,4,2)(8,12,14,13,10,11,9)(15,19,21,20,17,18,16)(22,26,28,27,24,25,23)(29,33,35,34,31,32,30)(36,40,42,41,38,39,37)(43,47,49,48,45,46,44)(1,8,22,15,36,43,29)(2,13,25,16,41,46,30,6,11,23,20,39,44,34,4,9,27,18,37,48,32)(3,14,26,17,42,47,31,7,12,24,21,40,45,35,5,10,28,19,38,49,33)
29	294	solvable	1	6,25,27	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)(1,43,49,21,17,10,9,30,33,40,41,27,25,4)(2,29,47,42,20,24,11)(3,8,44,35,19,38,13,23,32,5,36,48,28,18)(6,22,46,7,15,45,14,16,31,12,37,34,26,39)
30	294	solvable	3	21,22,25,26	(1,43,15,8,29,36,22)(2,44,16,9,30,37,23)(3,45,17,10,31,38,24)(4,46,18,11,32,39,25)(5,47,19,12,33,40,26)(6,48,20,13,34,41,27)(7,49,21,14,35,42,28)(2,22,6,8,4,36)(3,29,7,15,5,43)(9,25,41)(10,32,42,16,26,48)(11,39,37,23,27,13)(12,46,38,30,28,20)(14,18,40,44,24,34)(17,33,49)(19,47,45,31,35,21)
					(1,2,4,3,6,7,5)(8,37,25,10,41,28,12,36,23,11,

31	441	solvable	1	24,26,27,28	38,27,14,40,22,9,39,24,13,42,26)(15,44,32,17,48,35,19,43,30,18,45,34,21,47,29,16,46,31,20,49,33)(1,15,29,22,43,8,36)(2,20,32,23,48,11,37,6,18,30,27,46,9,41,4,16,34,25,44,13,39)(3,21,33,24,49,12,38,7,19,31,28,47,10,42,5,17,35,26,45,14,40)
32	882	*G*	1	14,29,30,31	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)(1,11,48,28,40,31)(2,18,44,21,37,17)(3,4,46,49,42,38)(5,32,43,14,41,24)(6,25,47,35,36,10)(7,39,45)(8,13,27,26,33,29)(9,20,23,19,30,15)(12,34,22)

Proper normal subgroups					
Serial	Order	Nature	Quotient	Subgroups	Generators
1	49	abelian	solvable	--	(1,7,3,2,5,6,4)(8,14,10,9,12,13,11)(15,21,17,16,19,20,18)(22,28,24,23,26,27,25)(29,35,31,30,33,34,32)(36,42,38,37,40,41,39)(43,49,45,44,47,48,46)(1,43,15,8,29,36,22)(2,44,16,9,30,37,23)(3,45,17,10,31,38,24)(4,46,18,11,32,39,25)(5,47,19,12,33,40,26)(6,48,20,13,34,41,27)(7,49,21,14,35,42,28)
2	147	solvable	dihedral	1	(2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42)(11,27,37)(12,28,38)(13,23,39)(14,24,40)(15,29,43)(16,32,48)(17,33,49)(18,34,44)(19,35,45)(20,30,46)(21,31,47)(1,43,15,8,29,36,22)(2,44,16,9,30,37,23)(3,45,17,10,31,38,24)(4,46,18,11,32,39,25)(5,47,19,12,33,40,26)(6,48,20,13,34,41,27)(7,49,21,14,35,42,28)(1,7,3,2,5,6,4)(8,14,10,9,12,13,11)(15,21,17,16,19,20,18)(22,28,24,23,26,27,25)(29,35,31,30,33,34,32)(36,42,38,37,40,41,39)(43,49,45,44,47,48,46)
3	147	solvable	cyclic	1	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)(1,39,13,47,23,31,21)(2,38,14,43,25,34,19)(3,42,8,46,27,33,16)(4,41,12,44,24,35,15)(5,37,10,49,22,32,20)(6,40,9,45,28,29,18)(7,36,11,48,26,30,17)
4	441	solvable	cyclic	1,2,3	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)(1,39,34,5,37,31,7,36,32,6,40,30,3,42,29,4,41,33,2,38,35)(8,11,13,12,9,10,14)(15,25,48,19,23,45,21,22,46,20,26,44,17,28,43,18,27,47,16,24,49)
5	294	solvable	cyclic	1,3	(2,6,4)(3,7,5)(8,22,36)(9,27,39)(10,28,40)(11,23,41)(12,24,42)(13,25,37)(14,26,38)(15,29,43)(16,34,46)(17,35,47)(18,30,48)(19,31,49)(20,32,44)(21,33,45)(1,36,41,13,9,44,49,28,25,32,33,19,17,3)(2,43,42,27,11,

					30,47,21,24,4,29,40,20,10)(5,15,38,6,8,37,48,14,23,46,35,26,18,31)(7,22,39,34,12,16,45)

Sylow subgroups						
p	Order	Conjug. classes	Nature	Normalizer	Normal closure	Generators
2	2	21	cyclic	42	294	(2,8)(3,15)(4,22)(5,29)(6,36)(7,43)(10,16)(11,23)(12,30)(13,37)(14,44)(18,24)(19,31)(20,38)(21,45)(26,32)(27,39)(28,46)(34,40)(35,47)(42,48)
3	9	49	abelian	18	441	(2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42)(11,27,37)(12,28,38)(13,23,39)(14,24,40)(15,29,43)(16,32,48)(17,33,49)(18,34,44)(19,35,45)(20,30,46)(21,31,47)(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)
7	49	1	abelian	*G*	49	(1,11,27,19,37,45,35)(2,10,28,15,39,48,33)(3,14,22,18,41,47,30)(4,13,26,16,38,49,29)(5,9,24,21,36,46,34)(6,12,23,17,42,43,32)(7,8,25,20,40,44,31)(1,43,15,8,29,36,22)(2,44,16,9,30,37,23)(3,45,17,10,31,38,24)(4,46,18,11,32,39,25)(5,47,19,12,33,40,26)(6,48,20,13,34,41,27)(7,49,21,14,35,42,28)

13. Let G be a primitive group of degree 49 with 5 generators. We have $|G| = 1176 = 2^3 \times 3 \times 7^2$.

Generators of G:	a_1 = (2,8)(3,15)(4,22)(5,29)(6,36)(7,43)(10,16)(11,23)(12,30)(13,37)(14,44)(18,24)(19,31)(20,38)(21,45)(26,32)(27,39)(28,46)(34,40)(35,47)(42,48) (order 2)
	a_2 = (8,29)(9,30)(10,31)(11,32)(12,33)(13,34)(14,35)(15,36)(16,37)(17,38)(18,39)(19,40)(20,41)(21,42)(22,43)(23,44)(24,45)(25,46)(26,47)(27,48)(28,49) (order 2)
	a_3 = (2,5)(3,6)(4,7)(9,12)(10,13)(11,14)(16,19)(17,20)(18,21)(23,26)(24,27)(25,28)(30,33)(31,34)(32,35)(37,40)(38,41)(39,42)(44,47)(45,48)(46,49) (order 2)
	a_4 = (2,6,4)(3,7,5)(8,22,36)(9,27,39)(10,28,40)(11,23,41)(12,24,42)(13,25,37)(14,26,38)(15,29,43)(16,34,46)(17,35,47)(18,30,48)(19,31,49)(20,32,44)(21,33,45) (order 3)
	a_5 = (1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35) (order 7)

This generating set is not minimal. There is a smaller generating set with 2 elements. G is a solvable group. G acts transitively on the set of 49 elements {1, 2, …, 49}. The center of G is trivial.

The derived subgroup $D = [G, G]$ is a solvable group of order 294, generated by {(1,23,39,31,13,21,47)(2,25,38,34,14,19,43)(3,27,42,33,8,16,46)(4,24,41,35,12,15,44)(5,22,37,32,10,20,49)(6,28,40,29,9,18,45)(7,26,36,30,11,17,48)(2,7,6,5,4,3)(8,15,22,29,36,43)(9,21,27,33,39,45)(10,16,28,34,40,46)(11,17,23,35,41,47)(12,18,24,30,42,48)(13,19,25,31,37,49)(14,20,26,32,38,44)} and $G/D \cong C_2^2$.

Lower central Series				
Level	Order	Nature	Quotient	Successive quotient
1	294	solvable	abelian	C_2^2
2	147	solvable	nilpotent	C_2

Derived Series				
Level	Order	Nature	Quotient	Successive quotient
1	294	solvable	abelian	C_2^2
2	49	abelian	solvable	$C_2 \times C_3$
3	1	trivial	solvable	C_7^2

Conjugacy classes of maximal subgroups				
Serial	Order	Nature	Conjugacy classes	Generators
1	588	solvable	1	(1,47,7,48,3,46,2,43,5,49,6,45,4,44)(8,40,14,41,10,39,9,36,12,42,13,38,11,37)(15,26,21,27,17,25,16,22,19,28,20,24,18,23)(29,33,35,34,31,32,30)(1,47,37,6,49,38)(2,48,42,3,43,40)(4,46,39)(5,44,41,7,45,36)(8,26,30,13,28,31)(9,27,35,10,22,33)(11,25,32)(12,23,34,14,24,29)(15,19,16,20,21,17)
2	588	solvable	1	(2,8)(3,15)(4,22)(5,29)(6,36)(7,43)(10,16)(11,23)(12,30)(13,37)(14,44)(18,24)(19,31)(20,38)(21,45)(26,32)(27,39)(28,46)(34,40)(35,47)(42,48)(1,16,41,26,35,46)(2,20,40,28,32,43)(3,17,38,24,31,45)(4,15,37,27,33,49)(5,21,39,22,30,48)(6,19,42,25,29,44)(7,18,36,23,34,47)(8,9,13,12,14,11)
3	588	solvable	1	(2,6,4)(3,7,5)(8,22,36)(9,27,39)(10,28,40)(11,23,41)(12,24,42)(13,25,37)(14,26,38)(15,29,43)(16,34,46)(17,35,47)(18,30,48)(19,31,49)(20,32,44)(21,33,45)(1,13,45,40)(2,27,46,19)(3,41,43,12)(4,20,44,26)(5,6,48,47)(7,34,49,33)(8,10,38,36)(9,24,39,15)(11,17,37,22)(14,31,42,29)(16,23,25,18)(21,30,28,32)
4	392	solvable	3	(2,8)(3,15)(4,22)(5,29)(6,36)(7,43)(10,16)(11,23)(12,30)(13,37)(14,44)(18,24)(19,31)(20,38)(21,45)(26,32)(27,39)(28,46)(34,40)(35,47)(42,48)(1,12,7,13,3,11,2,8,5,14,6,10,4,9)(15,47,21,48,17,46,16,43,19,49,20,45,18,44)(22,33,28,34,24,32,23,29,26,35,27,31,25,30)(36,40,42,41,38,39,37)
5	24	solvable	49	(2,8)(3,15)(4,22)(5,29)(6,36)(7,43)(10,16)(11,23)(12,30)(13,37)(14,44)(18,24)(19,31)(20,38)(21,45)(26,32)(27,39)(28,46)(34,40)(35,47)(42,48)(2,6,4)(3,7,5)(8,43,36,29,22,15)(9,48,39,30,27,18)(10,49,40,31,28,19)(11,44,41,32,23,20)(12,45,42,33,24,21)(13,46,37,34,25,

Proper normal subgroups					
Serial	Order	Nature	Quotient	Subgroups	Generators
1	49	abelian	solvable	--	(1,5,7,6,3,4,2)(8,12,14,13,10,11,9)(15,19,21,20,17,18,16)(22,26,28,27,24,25,23)(29,33,35,34,31,32,30)(36,40,42,41,38,39,37)(43,47,49,48,45,46,44)(1,43,15,8,29,36,22)(2,44,16,9,30,37,23)(3,45,17,10,31,38,24)(4,46,18,11,32,39,25)(5,47,19,12,33,40,26)(6,48,20,13,34,41,

					27)(7,49,21,14,35,42,28)
2	98	solvable	dihedral	1	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(1,5,7,6,3,4,2)(8,12,14,13,10,11,9)(15,19,21,20,17,18,16)(22,26,28,27,24,25,23)(29,33,35,34,31,32,30)(36,40,42,41,38,39,37)(43,47,49,48,45,46,44)(1,43,15,8,29,36,22)(2,44,16,9,30,37,23)(3,45,17,10,31,38,24)(4,46,18,11,32,39,25)(5,47,19,12,33,40,26)(6,48,20,13,34,41,27)(7,49,21,14,35,42,28)
3	196	solvable	dihedral	1,2	(1,5)(2,7)(4,6)(8,33)(9,35)(10,31)(11,34)(12,29)(13,32)(14,30)(15,40)(16,42)(17,38)(18,41)(19,36)(20,39)(21,37)(22,47)(23,49)(24,45)(25,48)(26,43)(27,46)(28,44)(1,28,36,35,8,21,43,7,22,42,29,14,15,49)(2,27,37,34,9,20,44,6,23,41,30,13,16,48)(3,25,38,32,10,18,45,4,24,39,31,11,17,46)(5,26,40,33,12,19,47)
4	147	solvable	nilpotent	1	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)(1,47,21,13,31,39,23)(2,43,19,14,34,38,25)(3,46,16,8,33,42,27)(4,44,15,12,35,41,24)(5,49,20,10,32,37,22)(6,45,18,9,29,40,28)(7,48,17,11,30,36,26)
5	294	solvable	abelian	1,2,4	(2,7,6,5,4,3)(8,15,22,29,36,43)(9,21,27,33,39,45)(10,16,28,34,40,46)(11,17,23,35,41,47)(12,18,24,30,42,48)(13,19,25,31,37,49)(14,20,26,32,38,44)(1,47,21,13,31,39,23)(2,43,19,14,34,38,25)(3,46,16,8,33,42,27)(4,44,15,12,35,41,24)(5,49,20,10,32,37,22)(6,45,18,9,29,40,28)(7,48,17,11,30,36,26)
6	588	solvable	cyclic	1,2,4,5	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)(1,36,37,2)(3,29,42,44)(4,15,40,9)(5,8,39,16)(6,22,41,23)(7,43,38,30)(10,32,21,47)(11,18,19,12)(13,25,20,26)(14,46,17,33)(24,34,28,48)(31,35,49,45)
7	588	solvable	cyclic	1,2,3,4,5	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(1,47,34,22,40,20)(2,44,30,23,37,16)(3,49,32,24,42,18)(4,45,35,25,38,21)(5,48,29,26,41,15)(6,43,33,27,36,19)(7,46,31,28,39,17)(8,12,13)(10,14,11)
8	588	solvable	cyclic	1,2,4,5	(2,29)(3,36)(4,43)(5,8)(6,15)(7,22)(9,33)(10,40)(11,47)(13,19)(14,26)(16,34)(17,41)(18,48)(21,27)(23,35)(24,42)(25,49)(31,37)(32,44)(39,45)(1,15,21,35,31,24,23,44,47,12,13,41,39,4)(2,43,19,14,34,38,25)(3,22,16,49,33,10,27,37,46,5,8,20,42,32)(6,36,18,7,29,17,28,30,45,26,9,48,40,11)

Sylow subgroups						
p	Order	Conjug. classes	Nature	Normalizer	Normal closure	Generators
2	8	147	nilpotent	8	G	(2,8)(3,15)(4,22)(5,29)(6,36)(7,43)(10,16)(11,23)(12,30)(13,37)(14,44)(18,24)(19,31)(20,38)(21,45)(26,32)(27,39)(28,46)(34,40)(35,47)(42,48)(8,29)(9,30)(10,31)(11,32)(12,33)(13,34)(14,35)(15,36)(16,37)(17,38)(18,39)(19,40)(20,41)(21,42)(22,43)(23,44)(24,45)(25,46)(26,47)(27,48)(28,49)
3	3	49	cyclic	24	147	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)
7	49	1	abelian	G	49	(1,22,36,29,8,15,43)(2,23,37,30,9,16,44)(3,24,38,31,10,17,45)(4,25,39,32,11,18,46)(5,26,40,33,12,19,47)(6,27,41,34,13,20,48)(7,28,42,35,14,21,49)(1,32,48,40,16,24,14)(2,31,49,36,18,27,12)(3,35,43,39,20,26,9)(4,34,47,37,17,28,8)(5,30,45,42,15,25,13)(6,33,44,38,21,22,11)(7,29,46,41,19,23,10)

14. Let G be a primitive group of degree 49 with 5 generators. We have $|G| = 1176 = 2^3\times3\times7^2$.

Generators of G:	a_1 = (2,8)(3,15)(4,22)(5,29)(6,36)(7,43)(10,16)(11,23)(12,30)(13,37)(14,44)(18,24)(19,31)(20,38)(21,45)(26,32)(27,39)(28,46)(34,40)(35,47)(42,48) (order 2)
	a_2 = (8,29)(9,30)(10,31)(11,32)(12,33)(13,34)(14,35)(15,36)(16,37)(17,38)(18,39)(19,40)(20,41)(21,42)(22,43)(23,44)(24,45)(25,46)(26,47)(27,48)(28,49) (order 2)
	a_3 = (2,5)(3,6)(4,7)(9,12)(10,13)(11,14)(16,19)(17,20)(18,21)(23,26)(24,27)(25,28)(30,33)(31,34)(32,35)(37,40)(38,41)(39,42)(44,47)(45,48)(46,49) (order 2)
	a_4 = (2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42)(11,27,37)(12,28,38)(13,23,39)(14,24,40)(15,29,43)(16,32,48)(17,33,49)(18,34,44)(19,35,45)(20,30,46)(21,31,47) (order 3)
	a_5 = (1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35) (order 7)

This generating set is not minimal. There is a smaller generating set with 2 elements. G is a solvable group. G acts transitively on the set of 49 elements {1, 2, …, 49}. The center of G is trivial.

The derived subgroup D = [G, G] is a solvable group of order 98, generated by {(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(1,5,7,6,3,4,2)(8,12,14,13,10,11,9)(15,19,21,20,17,18,16)(22,26,28,27,24,25,23)(29,33,35,34,31,32,30)(36,40,42,41,38,39,37)(43,47,49,48,45,46,44)(1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35)} and $G/D \cong C_2^2\times C_3$.

Lower central Series				
Level	Order	Nature	Quotient	Successive quotient
1	98	solvable	abelian	$C_2^2\times C_3$
2	49	abelian	nilpotent	C_2

Derived Series				
Level	Order	Nature	Quotient	Successive quotient
1	98	solvable	abelian	$C_2^2 \times C_3$
2	49	abelian	nilpotent	C_2
3	1	trivial	solvable	C_7^2

Conjugacy classes of maximal subgroups				
Serial	Order	Nature	Conjugacy classes	Generators
1	588	solvable	1	(1,8,15)(2,14,20,5,11,17)(3,9,21,6,12,18)(4,10,16,7,13,19)(22,43,36)(23,49,41,26,46,38)(24,44,42,27,47,39)(25,45,37,28,48,40)(30,35,34,33,32,31)(1,3,5,4,7,2,6)(8,31,12,32,14,30,13,29,10,33,11,35,9,34)(15,38,19,39,21,37,20,36,17,40,18,42,16,41)(22,45,26,46,28,44,27,43,24,47,25,49,23,48)
2	588	solvable	1	(1,17,33,25,49,9,41)(2,38,29,18,47,23,42,8,20,5,24,35,11,48)(3,31,32,46,44,37,36,15,19,26,28,14,13,6)(4,45,30,39,43,16,40,22,21,12,27,7,10,34)(1,28,46,30,38,12)(2,24,47,29,42,11)(3,26,43,35,39,9)(4,23,45,33,36,14)(5,22,49,32,37,10)(6,27,48,34,41,13)(7,25,44,31,40,8)(15,21,18,16,17,19)
3	588	solvable	1	(1,12,28,20,38,46,30)(2,8,26,21,41,45,32)(3,11,23,15,40,49,34)(4,9,22,19,42,48,31)(5,14,27,17,39,44,29)(6,10,25,16,36,47,35)(7,13,24,18,37,43,33)(2,22,3,29,4,36,5,43,6,8,7,15)(9,28,17,30,25,38,33,46,41,12,49,20)(10,35,18,37,26,45,34,11,42,19,44,27)(13,14,21,16,23,24,31,32,39,40,47,48)
4	392	solvable	1	(2,8)(3,15)(4,22)(5,29)(6,36)(7,43)(10,16)(11,23)(12,30)(13,37)(14,44)(18,24)(19,31)(20,38)(21,45)(26,32)(27,39)(28,46)(34,40)(35,47)(42,48)(1,30,4,31,6,35,5,29,2,32,3,34,7,33)(8,44,11,45,13,49,12,43,9,46,10,48,14,47)(15,16,18,17,20,21,19)(22,37,25,38,27,42,26,36,23,39,24,41,28,40)
5	24	nilpotent	49	(8,29)(9,30)(10,31)(11,32)(12,33)(13,34)(14,35)(15,36)(16,37)(17,38)(18,39)(19,40)(20,41)(21,42)(22,43)(23,44)(24,45)(25,46)(26,47)(27,48)(28,49)(2,22,6,8,4,36)(3,29,7,15,5,43)(9,25,41)(10,32,42,16,26,48)(11,39,37,23,27,13)(12,46,38,30,28,20)(14,18,40,44,24,34)(17,33,49)

Proper normal subgroups					
Serial	Order	Nature	Quotient	Subgroups	Generators
1	49	abelian	nilpotent	--	(1,2,4,3,6,7,5)(8,9,11,10,13,14,12)(15,16,18,17,20,21,19)(22,23,25,24,27,28,26)(29,30,32,31,34,35,33)(36,37,39,38,41,42,40)(43,44,46,45,48,49,47)(1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35)

2	147	solvable	nilpotent	1	(2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42)(11,27,37)(12,28,38)(13,23,39)(14,24,40)(15,29,43)(16,32,48)(17,33,49)(18,34,44)(19,35,45)(20,30,46)(21,31,47)(1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35)(1,2,4,3,6,7,5)(8,9,11,10,13,14,12)(15,16,18,17,20,21,19)(22,23,25,24,27,28,26)(29,30,32,31,34,35,33)(36,37,39,38,41,42,40)(43,44,46,45,48,49,47)
3	98	solvable	abelian	1	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35)(1,2,4,3,6,7,5)(8,9,11,10,13,14,12)(15,16,18,17,20,21,19)(22,23,25,24,27,28,26)(29,30,32,31,34,35,33)(36,37,39,38,41,42,40)(43,44,46,45,48,49,47)
4	294	solvable	abelian	1,2,3	(1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35)(1,2,4,3,6,7,5)(8,9,11,10,13,14,12)(15,16,18,17,20,21,19)(22,23,25,24,27,28,26)(29,30,32,31,34,35,33)(36,37,39,38,41,42,40)(43,44,46,45,48,49,47)(2,7,6,5,4,3)(8,43,36,29,22,15)(9,49,41,33,25,17)(10,44,42,34,26,18)(11,45,37,35,27,19)(12,46,38,30,28,20)(13,47,39,31,23,21)(14,48,40,32,24,16)
5	196	solvable	cyclic	1,3	(2,29,5,8)(3,36,6,15)(4,43,7,22)(9,30,33,12)(10,37,34,19)(11,44,35,26)(13,16,31,40)(14,23,32,47)(17,38,41,20)(18,45,42,27)(21,24,39,48)(25,46,49,28)(1,29,43,36,15,22,8)(2,30,44,37,16,23,9)(3,31,45,38,17,24,10)(4,32,46,39,18,25,11)(5,33,47,40,19,26,12)(6,34,48,41,20,27,13)(7,35,49,42,21,28,14)
6	196	solvable	cyclic	1,3	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(1,9,4,10,6,14,5,8,2,11,3,13,7,12)(15,44,18,45,20,49,19,43,16,46,17,48,21,47)(22,30,25,31,27,35,26,29,23,32,24,34,28,33)(36,37,39,38,41,42,40)
					(1,29,43,36,15,22,8)(2,30,44,37,16,23,9)(3,31,45,38,17,24,10)(4,32,46,39,18,25,11)(5,33,47,40,19,26,12)(6,34,48,41,

7	588	solvable	cyclic	1,2,3,4,5	20,27,13)(7,35,49,42,21,28,14)(2,43,3,8,4,15,5,22,6,29,7,36)(9,46,17,12,25,20,33,28,41,30,49,38)(10,11,18,19,26,27,34,35,42,37,44,45)(13,32,21,40,23,48,31,14,39,16,47,24)
8	588	solvable	cyclic	1,2,3,4,6	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(1,44,24,8,16,31)(2,45,22,9,17,29)(3,43,23,10,15,30)(4,49,27,11,21,34)(5,47,26,12,19,33)(6,46,28,13,18,35)(7,48,25,14,20,32)(36,37,38)(39,42,41)
9	196	solvable	cyclic	1,3	(1,49,17,9,33,41,25)(2,35,20,23,29,48,24,8,47,38,11,5,42,18)(3,14,19,37,32,6,28,15,44,31,13,26,36,46)(4,7,21,16,30,34,27,22,43,45,10,12,40,39)(1,28,38,30,12,20,46)(2,14,41,44,8,27,45,29,26,17,32,5,21,39)(3,35,40,16,11,6,49,36,23,10,34,47,15,25)(4,7,42,37,9,13,48,43,22,24,31,33,19,18)
10	588	solvable	cyclic	1,2,3,4,9	(1,30,46,38,20,28,12)(2,44,45,17,21,14,8,29,32,39,41,27,26,5)(3,16,49,10,15,35,11,36,34,25,40,6,23,47)(4,37,48,24,19,7,9,43,31,18,42,13,22,33)(1,9,17)(2,16,15,8,10,3)(4,44,20,22,14,38)(5,30,19,29,12,31)(6,23,21,36,11,45)(7,37,18,43,13,24)(25,49,41)(26,35,40,32,47,34)(27,28,42,39,46,48)
11	392	solvable	cyclic	1,3,5,6,9	(2,29)(3,36)(4,43)(5,8)(6,15)(7,22)(9,33)(10,40)(11,47)(13,19)(14,26)(16,34)(17,41)(18,48)(21,27)(23,35)(24,42)(25,49)(31,37)(32,44)(39,45)(1,33,7,34,3,32,2,29,5,35,6,31,4,30)(8,47,14,48,10,46,9,43,12,49,13,45,11,44)(15,19,21,20,17,18,16)(22,40,28,41,24,39,23,36,26,42,27,38,25,37)

Sylow subgroups						
p	Order	Conjug. classes	Nature	Normalizer	Normal closure	Generators
2	8	49	nilpotent	24	392	(2,8)(3,15)(4,22)(5,29)(6,36)(7,43)(10,16)(11,23)(12,30)(13,37)(14,44)(18,24)(19,31)(20,38)(21,45)(26,32)(27,39)(28,46)(34,40)(35,47)(42,48)(8,29)(9,30)(10,31)(11,32)(12,33)(13,34)(14,35)(15,36)(16,37)(17,38)(18,39)(19,40)(20,41)(21,42)(22,43)(23,44)(24,45)(25,46)(26,47)(27,48)(28,49)
3	3	49	cyclic	24	147	(2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42)(11,27,37)(12,28,38)(13,23,39)(14,24,40)(15,29,43)(16,32,48)(17,33,49)(18,34,44)(19,35,45)(20,30,46)(21,31,47)
7	49	1	abelian	*G*	49	(1,28,38,30,12,20,46)(2,26,41,32,8,21,45)(3,23,40,34,11,15,49)(4,22,42,31,9,19,48)(5,27,39,29,14,17,44)(6,25,36,35,10,16,47)(7,24,37,33,13,18,43)(1,49,17,9,33,41,25)

						(2,47,20,11,29,42,24)(3,44,19,13,32,36,28) (4,43,21,10,30,40,27)(5,48,18,8,35,38,23) (6,46,15,14,31,37,26)(7,45,16,12,34,39,22)

15. Let G be a primitive group of degree 49 with 2 generators. We have $|G| = 1176 = 2^3\times3\times7^2$.

Generators of G:	a_1 = (2,21,30,33,3,23,38,41,4,31,46,49,5,39,12,9,6,47,20,17,7,13,28,25) (8,40,35,42,15,48,37,44,22,14,45,10,29,16,11,18,36,24, 19,26,43,32,27,34) (order 24)
	a_2 = (1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31) (4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34) (7,14,28,21,42,49,35) (order 7)

G is a solvable group. G acts transitively on the set of 49 elements {1, 2, …, 49}. The center of G is trivial.

The derived subgroup $D = [G, G]$ is an abelian group of order 49, generated by {(1,19,35,27,45,11,37)(2,15,33,28,48,10,39)(3,18,30,22,47,14,41)(4,16,29,26,49,13,38)(5,21,34, 24,46,9,36)(6,17,32,23,43,12,42)(7,20,31,25,44,8,40)(1,26,42,34,10,18,44)(2,22,40,35,13,17,46) (3,25,37,29,12,21,48)(4,23,36,33,14,20,45)(5,28,41,31,11,16,43)(6,24,39,30,8,19,49)(7,27,38,32, 9,15,47)} and $G/D \cong C_3\times C_8$.

Lower central Series				
Level	Order	Nature	Quotient	Successive quotient
1	49	abelian	cyclic	$C_3\times C_8$

Derived Series				
Level	Order	Nature	Quotient	Successive quotient
1	49	abelian	cyclic	$C_3\times C_8$
2	1	trivial	solvable	C_7^2

Conjugacy classes of maximal subgroups				
Serial	Order	Nature	Conjugacy classes	Generators
1	588	solvable	1	(2,30,3,38,4,46,5,12,6,20,7,28)(8,35,15,37,22,45,29,11, 36,19,43,27)(9,47,17,13,25,21,33,23,41,31,49,39)(10,16, 18,24,26,32,34,40,42,48,44,14)(1,46,20,12,30,38,28)(2, 45,21,8,32,41,26)(3,49,15,11,34,40,23)(4,48,19,9,31,42, 22)(5,44,17,14,29,39,27)(6,47,16,10,35,36,25)(7,43,18, 13,33,37,24)
2	392	solvable	1	(2,31,20,33,5,13,38,9)(3,39,28,41,6,21,46,17)(4,47,30, 49,7,23,12,25)(8,14,19,42,29,32,37,18)(10,43,48,11,34, 22,24,35)(15,16,27,44,36,40,45,26)(1,16,32,24,48,14, 40)(2,18,31,27,49,12,36)(3,20,35,26,43,9,39)(4,17,34, 28,47,8,37)(5,15,30,25,45,13,42)(6,21,33,22,44,11,38) (7,19,29,23,46,10,41)
3	24	cyclic	49	(2,47,46,33,7,39,38,25,6,31,30,17,5,23,28,9,4,21,20,49, 3,13,12,41)(8,24,45,42,43,16,37,34,36,14,35,26,29,48, 27,18,22,40,19,10,15,32,11,44)

Proper normal subgroups					
Serial	Order	Nature	Quotient	Subgroups	Generators
1	49	abelian	cyclic	--	(1,8,22,15,36,43,29)(2,9,23,16,37,44,30) (3,10,24,17,38,45,31)(4,11,25,18,39,46,32) (5,12,26,19,40,47,33)(6,13,27,20,41,48,34) (7,14,28,21,42,49,35)(1,40,14,48,24,32,16)

					(2,36,12,49,27,31,18)(3,39,9,43,26,35,20) (4,37,8,47,28,34,17)(5,42,13,45,25,30,15) (6,38,11,44,22,33,21)(7,41,10,46,23,29,19)
2	98	solvable	cyclic	1	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35) (12,30)(13,31)(14,32)(15,36)(16,40) (17,41)(18,42)(19,37)(20,38)(21,39) (22,43)(23,47)(24,48)(25,49)(26,44) (27,45)(28,46)(1,8,22,15,36,43,29) (2,9,23,16,37,44,30)(3,10,24,17,38,45,31) (4,11,25,18,39,46,32)(5,12,26,19,40,47,33) (6,13,27,20,41,48,34)(7,14,28,21,42,49,35) (1,40,14,48,24,32,16)(2,36,12,49,27,31,18) (3,39,9,43,26,35,20)(4,37,8,47,28,34,17) (5,42,13,45,25,30,15)(6,38,11,44,22,33,21) (7,41,10,46,23,29,19)
3	196	solvable	cyclic	1,2	(2,20,5,38)(3,28,6,46)(4,30,7,12)(8,19,29,37) (9,31,33,13)(10,48,34,24)(11,22,35,43) (14,42,32,18)(15,27,36,45)(16,44,40,26) (17,39,41,21)(23,25,47,49) (1,26,42,34,10,18,44)(2,22,40,35,13,17,46) (3,25,37,29,12,21,48)(4,23,36,33,14,20,45) (5,28,41,31,11,16,43)(6,24,39,30,8,19,49) (7,27,38,32,9,15,47)
4	392	solvable	cyclic	1,2,3	(1,3,5,4,7,2,6)(8,10,12,11,14,9,13) (15,17,19,18,21,16,20)(22,24,26,25,28,23, 27)(29,31,33,32,35,30,34)(36,38,40,39,42, 37,41)(43,45,47,46,49,44,48)(1,4,24,13,19, 20,10,25)(2,12,43,28,21,22,47,9)(3,15,37, 36,17,5,35,33)(6,41,7,44,18,32,16,49)(8,34, 46,40,26,39,48,29)(11,14,38,30,27,23,31,42)
5	147	solvable	cyclic	1	(2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42) (11,27,37)(12,28,38)(13,23,39)(14,24,40) (15,29,43)(16,32,48)(17,33,49)(18,34,44) (19,35,45)(20,30,46)(21,31,47) (1,23,39,31,13,21,47)(2,25,38,34,14,19,43) (3,27,42,33,8,16,46)(4,24,41,35,12,15,44) (5,22,37,32,10,20,49)(6,28,40,29,9,18,45) (7,26,36,30,11,17,48)(1,16,32,24,48,14,40) (2,18,31,27,49,12,36)(3,20,35,26,43,9,39) (4,17,34,28,47,8,37)(5,15,30,25,45,13,42) (6,21,33,22,44,11,38)(7,19,29,23,46,10,41)
6	294	solvable	cyclic	1,2,5	(2,7,6,5,4,3)(8,43,36,29,22,15)(9,49,41,33, 25,17)(10,44,42,34,26,18)(11,45,37,35,27, 19)(12,46,38,30,28,20)(13,47,39,31,23,21) (14,48,40,32,24,16)(1,21,31,23,47,13,39) (2,19,34,25,43,14,38)(3,16,33,27,46,8,42) (4,15,35,24,44,12,41)(5,20,32,22,49,10,37) (6,18,29,28,45,9,40)(7,17,30,26,48,11,36) (1,14,24,16,40,48,32)(2,12,27,18,36,49,31) (3,9,26,20,39,43,35)(4,8,28,17,37,47,34) (5,13,25,15,42,45,30)(6,11,22,21,38,44,33) (7,10,23,19,41,46,29)
					(2,30,3,38,4,46,5,12,6,20,7,28) (8,35,15,37,22,45,29,11,36,19,43,27)

7	588	solvable	cyclic	1,2,3,5,6	(9,47,17,13,25,21,33,23,41,31,49,39) (10,16,18,24,26,32,34,40,42,48,44,14) (1,45,19,11,35,37,27)(2,48,15,10,33,39,28) (3,47,18,14,30,41,22)(4,49,16,13,29,38,26) (5,46,21,9,34,36,24)(6,43,17,12,32,42,23) (7,44,20,8,31,40,25)

Sylow subgroups						
p	Order	Conjug. classes	Nature	Normalizer	Normal closure	Generators
2	8	49	cyclic	24	392	(2,31,20,33,5,13,38,9)(3,39,28,41,6,21, 46,17)(4,47,30,49,7,23,12,25)(8,14,19, 42,29,32,37,18)(10,43,48,11,34,22,24,35) (15,16,27,44,36,40,45,26)
3	3	49	cyclic	24	147	(2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42) (11,27,37)(12,28,38)(13,23,39)(14,24,40) (15,29,43)(16,32,48)(17,33,49)(18,34,44) (19,35,45)(20,30,46)(21,31,47)
7	49	1	abelian	*G*	49	(1,8,22,15,36,43,29)(2,9,23,16,37,44,30) (3,10,24,17,38,45,31)(4,11,25,18,39,46,32) (5,12,26,19,40,47,33)(6,13,27,20,41,48,34) (7,14,28,21,42,49,35)(1,31,47,39,21,23,13) (2,34,43,38,19,25,14)(3,33,46,42,16,27,8) (4,35,44,41,15,24,12)(5,32,49,37,20,22,10) (6,29,45,40,18,28,9)(7,30,48,36,17,26,11)

16. Let G be a primitive group of degree 49 with 3 generators. We have $|G| = 1176 = 2^3\times3\times7^2$.

Generators of G:	a_1 = (2,48,5,24)(3,14,6,32)(4,16,7,40)(8,21,29,39) (9,11,33,35)(10,38,34,20)(12,44,30,26)(13,22,31,43) (15,23,36,47)(17,19,41,37)(18,46,42,28)(25,27,49,45) (order 4)
	a_2 = (2,30,3,38,4,46,5,12,6,20,7,28) (8,35,15,37,22,45,29,11,36,19,43,27) (9,47,17,13,25,21,33,23,41,31,49,39) (10,16,18,24,26,32,34,40,42,48,44,14) (order 12)
	a_3 = (1,8,22,15,36,43,29)(2,9,23,16,37,44,30) (3,10,24,17,38,45,31)(4,11,25,18,39,46,32) (5,12,26,19,40,47,33)(6,13,27,20,41,48,34) (7,14,28,21,42,49,35) (order 7)

This generating set is not minimal. There is a smaller generating set with 2 elements. G is a solvable group. G acts transitively on the set of 49 elements {1, 2, …, 49}. The center of G is trivial.

The derived subgroup $D = [G, G]$ is a solvable group of order 98, generated by {(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(1,4,6,5,2,3,7)(8,11,13,12,9,10,14)(15,18,20,19,16,17,21)(22,25,27,26,23,24,28)(29,32,34,33,30,31,35)(36,39,41,40,37,38,42)(43,46,48,47,44,45,49)(1,9,25,17,41,49,33)(2,11,24,20,42,47,29)(3,13,28,19,36,44,32)(4,10,27,21,40,43,30)(5,8,23,18,38,48,35)(6,14,26,15,37,46,31)(7,12,22,16,39,45,34)} and $G/D \cong C_2^2\times C_3$.

Lower central Series				
Level	Order	Nature	Quotient	Successive quotient
1	98	solvable	abelian	$C_2^2\times C_3$
2	49	abelian	nilpotent	C_2

Derived Series				
Level	Order	Nature	Quotient	Successive quotient
1	98	solvable	abelian	$C_2^2 \times C_3$
2	49	abelian	nilpotent	C_2
3	1	trivial	solvable	C_7^2

Conjugacy classes of maximal subgroups				
Serial	Order	Nature	Conjugacy classes	Generators
1	588	solvable	1	(1,28,38,30,12,20,46)(2,26,41,32,8,21,45) (3,23,40,34,11,15,49)(4,22,42,31,9,19,48) (5,27,39,29,14,17,44)(6,25,36,35,10,16,47) (7,24,37,33,13,18,43)(2,30,3,38,4,46,5,12,6, 20,7,28)(8,35,15,37,22,45,29,11,36,19,43, 27)(9,47,17,13,25,21,33,23,41,31,49,39) (10,16,18,24,26,32,34,40,42,48,44,14)
2	588	solvable	1	(1,14,24,16,40,48,32)(2,12,27,18,36,49,31) (3,9,26,20,39,43,35)(4,8,28,17,37,47,34) (5,13,25,15,42,45,30)(6,11,22,21,38,44,33) (7,10,23,19,41,46,29) (2,16,3,24,4,32,5,40,6,48,7,14) (8,31,15,39,22,47,29,13,36,21,43,23) (9,27,17,35,25,37,33,45,41,11,49,19) (10,12,18,20,26,28,34,30,42,38,44,46)
3	588	solvable	1	(1,18,34,26,44,10,42)(2,17,35,22,46,13,40) (3,21,29,25,48,12,37)(4,20,33,23,45,14,36) (5,16,31,28,43,11,41)(6,19,30,24,49,8,39) (7,15,32,27,47,9,38) (2,26,3,34,4,42,5,44,6,10,7,18) (8,9,15,17,22,25,29,33,36,41,43,49) (11,47,19,13,27,21,35,23,37,31,45,39) (12,32,20,40,28,48,30,14,38,16,46,24)
4	392	solvable	1	(2,20,5,38)(3,28,6,46)(4,30,7,12)(8,19,29,37) (9,31,33,13)(10,48,34,24)(11,22,35,43) (14,42,32,18)(15,27,36,45)(16,44,40,26) (17,39,41,21)(23,25,47,49)(1,14,11,6) (2,32,9,22)(3,24,12,33)(4,40,8,38) (5,16,10,44)(7,48,13,21)(15,19,46,45) (17,41,47,42)(18,29,43,25)(20,23,49,30) (26,27,31,35)(28,36,34,39)
5	24	nilpotent	49	(2,20,5,38)(3,28,6,46)(4,30,7,12)(8,19,29,37) (9,31,33,13)(10,48,34,24)(11,22,35,43) (14,42,32,18)(15,27,36,45)(16,44,40,26) (17,39,41,21)(23,25,47,49) (2,16,3,24,4,32,5,40,6,48,7,14) (8,31,15,39,22,47,29,13,36,21,43,23) (9,27,17,35,25,37,33,45,41,11,49,19)

Proper normal subgroups					
Serial	Order	Nature	Quotient	Subgroups	Generators
1	49	abelian	nilpotent	--	(1,31,47,39,21,23,13)(2,34,43,38,19,25,14) (3,33,46,42,16,27,8)(4,35,44,41,15,24,12) (5,32,49,37,20,22,10)(6,29,45,40,18,28,9) (7,30,48,36,17,26,11)(1,43,15,8,29,36,22)

					(2,44,16,9,30,37,23)(3,45,17,10,31,38,24) (4,46,18,11,32,39,25)(5,47,19,12,33,40,26) (6,48,20,13,34,41,27)(7,49,21,14,35,42,28)
2	147	solvable	nilpotent	1	(2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42) (11,27,37)(12,28,38)(13,23,39)(14,24,40) (15,29,43)(16,32,48)(17,33,49)(18,34,44) (19,35,45)(20,30,46)(21,31,47) (1,31,47,39,21,23,13)(2,34,43,38,19,25,14) (3,33,46,42,16,27,8)(4,35,44,41,15,24,12) (5,32,49,37,20,22,10)(6,29,45,40,18,28,9) (7,30,48,36,17,26,11)(1,43,15,8,29,36,22) (2,44,16,9,30,37,23)(3,45,17,10,31,38,24) (4,46,18,11,32,39,25)(5,47,19,12,33,40,26) (6,48,20,13,34,41,27)(7,49,21,14,35,42,28)
3	98	solvable	abelian	1	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35) (12,30)(13,31)(14,32)(15,36)(16,40) (17,41)(18,42)(19,37)(20,38)(21,39) (22,43)(23,47)(24,48)(25,49)(26,44) (27,45)(28,46)(1,46,20,12,30,38,28) (2,45,21,8,32,41,26)(3,49,15,11,34,40,23) (4,48,19,9,31,42,22)(5,44,17,14,29,39,27) (6,47,16,10,35,36,25)(7,43,18,13,33,37,24) (1,8,22,15,36,43,29)(2,9,23,16,37,44,30) (3,10,24,17,38,45,31)(4,11,25,18,39,46,32) (5,12,26,19,40,47,33)(6,13,27,20,41,48,34) (7,14,28,21,42,49,35)
4	196	solvable	cyclic	1,3	(2,10,5,34)(3,18,6,42)(4,26,7,44)(8,41, 29,17)(9,43,33,22)(11,31,35,13)(12,16, 30,40)(14,28,32,46)(15,49,36,25)(19, 39,37,21)(20,24,38,48)(23,27,47,45) (1,41,9,49,25,33,17)(2,42,11,47,24,29,20) (3,36,13,44,28,32,19)(4,40,10,43,27,30,21) (5,38,8,48,23,35,18)(6,37,14,46,26,31,15) (7,39,12,45,22,34,16)
5	196	solvable	cyclic	1,3	(2,20,5,38)(3,28,6,46)(4,30,7,12)(8,19, 29,37)(9,31,33,13)(10,48,34,24)(11,22, 35,43)(14,42,32,18)(15,27,36,45)(16, 44,40,26)(17,39,41,21)(23,25,47,49) (1,19,35,27,45,11,37)(2,15,33,28,48,10,39) (3,18,30,22,47,14,41)(4,16,29,26,49,13,38) (5,21,34,24,46,9,36)(6,17,32,23,43,12,42) (7,20,31,25,44,8,40)
6	294	solvable	abelian	1,2,3	(1,46,20,12,30,38,28)(2,45,21,8,32,41,26) (3,49,15,11,34,40,23)(4,48,19,9,31,42,22) (5,44,17,14,29,39,27)(6,47,16,10,35,36,25) (7,43,18,13,33,37,24) (1,8,22,15,36,43,29)(2,9,23,16,37,44,30) (3,10,24,17,38,45,31)(4,11,25,18,39,46,32) (5,12,26,19,40,47,33)(6,13,27,20,41,48,34) (7,14,28,21,42,49,35)(2,7,6,5,4,3) (8,43,36,29,22,15)(9,49,41,33,25,17) (10,44,42,34,26,18)(11,45,37,35,27,19) (12,46,38,30,28,20)(13,47,39,31,23,21) (14,48,40,32,24,16)

7	588	solvable	cyclic	1,2,3,4,6	(2,26,3,34,4,42,5,44,6,10,7,18) (8,9,15,17,22,25,29,33,36,41,43,49) (11,47,19,13,27,21,35,23,37,31,45,39) (12,32,20,40,28,48,30,14,38,16,46,24) (1,40,14,48,24,32,16)(2,36,12,49,27,31,18) (3,39,9,43,26,35,20)(4,37,8,47,28,34,17) (5,42,13,45,25,30,15)(6,38,11,44,22,33,21) (7,41,10,46,23,29,19)
8	588	solvable	cyclic	1,2,3,5,6	(2,30,3,38,4,46,5,12,6,20,7,28) (8,35,15,37,22,45,29,11,36,19,43,27) (9,47,17,13,25,21,33,23,41,31,49,39) (10,16,18,24,26,32,34,40,42,48,44,14) (1,32,48,40,16,24,14)(2,31,49,36,18,27,12) (3,35,43,39,20,26,9)(4,34,47,37,17,28,8) (5,30,45,42,15,25,13)(6,33,44,38,21,22,11) (7,29,46,41,19,23,10)
9	196	solvable	cyclic	1,3	(2,48,5,24)(3,14,6,32)(4,16,7,40)(8,21,29, 39)(9,11,33,35)(10,38,34,20)(12,44,30,26) (13,22,31,43)(15,23,36,47)(17,19,41,37) (18,46,42,28)(25,27,49,45) (1,21,31,23,47,13,39)(2,19,34,25,43,14,38) (3,16,33,27,46,8,42)(4,15,35,24,44,12,41) (5,20,32,22,49,10,37)(6,18,29,28,45,9,40) (7,17,30,26,48,11,36)
10	392	solvable	cyclic	1,3,4,5,9	(2,48,5,24)(3,14,6,32)(4,16,7,40)(8,21,29, 39)(9,11,33,35)(10,38,34,20)(12,44,30,26) (13,22,31,43)(15,23,36,47)(17,19,41,37) (18,46,42,28)(25,27,49,45) (1,19,26,29)(2,31,28,17)(3,48,24,39) (4,22,27,5)(6,9,25,14)(7,42,23,44) (8,35,12,16)(11,47,13,36)(15,45,33,38) (18,41,34,46)(20,21,32,30)(37,40,49,43)
11	588	solvable	cyclic	1,2,3,6,9	(2,16,3,24,4,32,5,40,6,48,7,14) (8,31,15,39,22,47,29,13,36,21,43,23) (9,27,17,35,25,37,33,45,41,11,49,19) (10,12,18,20,26,28,34,30,42,38,44,46) (1,38,12,46,28,30,20)(2,41,8,45,26,32,21) (3,40,11,49,23,34,15)(4,42,9,48,22,31,19) (5,39,14,44,27,29,17)(6,36,10,47,25,35,16) (7,37,13,43,24,33,18)

Sylow subgroups						
p	Order	Conjug. classes	Nature	Normalizer	Normal closure	Generators
2	8	49	nilpotent	24	392	(2,48,5,24)(3,14,6,32)(4,16,7,40)(8,21,29,39) (9,11,33,35)(10,38,34,20)(12,44,30,26) (13,22,31,43)(15,23,36,47)(17,19,41,37) (18,46,42,28)(25,27,49,45)(2,20,5,38) (3,28,6,46)(4,30,7,12)(8,19,29,37) (9,31,33,13)(10,48,34,24)(11,22,35,43) (14,42,32,18)(15,27,36,45)(16,44,40,26) (17,39,41,21)(23,25,47,49)
3	3	49	cyclic	24	147	(2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42) (11,27,37)(12,28,38)(13,23,39)(14,24,40) (15,29,43)(16,32,48)(17,33,49)(18,34,44)

						(19,35,45)(20,30,46)(21,31,47)
7	49	1	abelian	G	49	(1,2,4,3,6,7,5)(8,9,11,10,13,14,12)(15,16,18,17,20,21,19)(22,23,25,24,27,28,26)(29,30,32,31,34,35,33)(36,37,39,38,41,42,40)(43,44,46,45,48,49,47)(1,29,43,36,15,22,8)(2,30,44,37,16,23,9)(3,31,45,38,17,24,10)(4,32,46,39,18,25,11)(5,33,47,40,19,26,12)(6,34,48,41,20,27,13)(7,35,49,42,21,28,14)

17. Let G be a primitive group of degree 49 with 4 generators. We have $|G| = 1176 = 2^3\times3\times7^2$.

Generators of G:	a_1 = (2,29,24)(3,36,32)(4,43,40)(5,8,48)(6,15,14)(7,22,16)(9,41,25)(10,27,30)(11,20,42)(12,34,45)(17,49,33)(18,35,38)(19,28,44)(26,37,46) (order 3)
	a_2 = (2,8,5,29)(3,15,6,36)(4,22,7,43)(9,12,33,30)(10,19,34,37)(11,26,35,44)(13,40,31,16)(14,47,32,23)(17,20,41,38)(18,27,42,45)(21,48,39,24)(25,28,49,46) (order 4)
	a_3 = (2,21,5,39)(3,23,6,47)(4,31,7,13)(8,24,29,48)(9,44,33,26)(10,41,34,17)(11,12,35,30)(14,15,32,36)(16,22,40,43)(18,49,42,25)(19,20,37,38)(27,28,45,46) (order 4)
	a_4 = (1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35) (order 7)

This generating set is not minimal. There is a smaller generating set with 2 elements. G is a solvable group. G acts transitively on the set of 49 elements {1, 2, …, 49}. The center of G is trivial.

The derived subgroup $D = [G, G]$ is a solvable group of order 392, generated by {(2,48,5,24)(3,14,6,32)(4,16,7,40)(8,21,29,39)(9,11,33,35)(10,38,34,20)(12,44,30,26)(13,22,31,43)(15,23,36,47)(17,19,41,37)(18,46,42,28)(25,27,49,45)(1,28,36,25)(2,14,40,18)(3,35,41,46)(4,7,42,39)(5,21,37,11)(6,49,38,32)(8,26,15,23)(9,12,19,16)(10,33,20,44)(13,47,17,30)(24,29,27,43)(31,34,48,45)} and $G/D \cong C_3$.

Lower central Series				
Level	Order	Nature	Quotient	Successive quotient
1	392	solvable	cyclic	C_3

Derived Series				
Level	Order	Nature	Quotient	Successive quotient
1	392	solvable	cyclic	C_3
2	98	solvable	solvable	C_2^2
3	49	abelian	solvable	C_2
4	1	trivial	solvable	C_7^2

Conjugacy classes of maximal subgroups				
Serial	Order	Nature	Conjugacy classes	Generators
1	392	solvable	1	(2,29,5,8)(3,36,6,15)(4,43,7,22)(9,30,33,12)(10,37,34,19)(11,44,35,26)(13,16,31,40)(14,23,32,47)(17,38,41,20)(18,45,42,27)(21,24,39,48)(25,46,49,28)(1,22,10,31)(2,6,11,12)(3,21,8,49)(4,44,9,18)(5,38,13,36)(7,33,14,27)(15,37,45,39)(16,28,46,35)(17,47,43,20)(19,34,48,26)(23,29,32,24)(25,41,30,40)
				(2,5)(3,6)(4,7)(8,21,24,29,39,48)(9,20,27,33,38,45)

2	294	solvable	4	(10,18,26,34,42,44)(11,17,28,35,41,46)(12,19,25,30,37,49)(13,16,22,31,40,43)(14,15,23,32,36,47)(1,49,17,9,33,41,25)(2,47,20,11,29,42,24)(3,44,19,13,32,36,28)(4,43,21,10,30,40,27)(5,48,18,8,35,38,23)(6,46,15,14,31,37,26)(7,45,16,12,34,39,22)
3	24	solvable	49	(8,39,24)(9,38,27)(10,42,26)(11,41,28)(12,37,25)(13,40,22)(14,36,23)(15,47,32)(16,43,31)(17,46,35)(18,44,34)(19,49,30)(20,45,33)(21,48,29)(2,39,5,21)(3,47,6,23)(4,13,7,31)(8,48,29,24)(9,26,33,44)(10,17,34,41)(11,30,35,12)(14,36,32,15)(16,43,40,22)(18,25,42,49)

Maximal normal subgroups				
Serial	Order	Nature	Quotient	Generators
1	392	solvable	cyclic	(2,29,5,8)(3,36,6,15)(4,43,7,22)(9,30,33,12)(10,37,34,19)(11,44,35,26)(13,16,31,40)(14,23,32,47)(17,38,41,20)(18,45,42,27)(21,24,39,48)(25,46,49,28)(1,22,10,31)(2,6,11,12)(3,21,8,49)(4,44,9,18)(5,38,13,36)(7,33,14,27)(15,37,45,39)(16,28,46,35)(17,47,43,20)(19,34,48,26)(23,29,32,24)(25,41,30,40)

Sylow subgroups						
p	Order	Conjug. classes	Nature	Normalizer	Normal closure	Generators
2	8	49	nilpotent	24	392	(2,48,5,24)(3,14,6,32)(4,16,7,40)(8,21,29,39)(9,11,33,35)(10,38,34,20)(12,44,30,26)(13,22,31,43)(15,23,36,47)(17,19,41,37)(18,46,42,28)(25,27,49,45)(2,29,5,8)(3,36,6,15)(4,43,7,22)(9,30,33,12)(10,37,34,19)(11,44,35,26)(13,16,31,40)(14,23,32,47)(17,38,41,20)(18,45,42,27)(21,24,39,48)(25,46,49,28)
3	3	28	cyclic	42	*G*	(8,39,24)(9,38,27)(10,42,26)(11,41,28)(12,37,25)(13,40,22)(14,36,23)(15,47,32)(16,43,31)(17,46,35)(18,44,34)(19,49,30)(20,45,33)(21,48,29)
7	49	1	abelian	*G*	49	(1,6,2,7,4,5,3)(8,13,9,14,11,12,10)(15,20,16,21,18,19,17)(22,27,23,28,25,26,24)(29,34,30,35,32,33,31)(36,41,37,42,39,40,38)(43,48,44,49,46,47,45)(1,15,29,22,43,8,36)(2,16,30,23,44,9,37)(3,17,31,24,45,10,38)(4,18,32,25,46,11,39)(5,19,33,26,47,12,40)(6,20,34,27,48,13,41)(7,21,35,28,49,14,42)

18. Let G be a primitive group of degree 49 with 4 generators. We have $|G| = 1176 = 2^3\times3\times7^2$.

	a_1 = (2,15,40)(3,22,48)(4,29,14)(5,36,16)(6,43,24)(7,8,32)(9,25,41)(10,11,46)(12,18,19)(13,39,23)(17,33,49)(20,26,27)(21,47,31)(28,34,35)(30,42,37)(38,44,45) (order 3)
	a_2 = (2,8,5,29)(3,15,6,36)(4,22,7,43)(9,12,33,30)(10,19,34,37)(11,26,35,44)(13,40,31,16)(14,47,32,23)(17,20,41,38)(18,27,42,45)(21,48,39,24)(25,28,49,46) (order 4)
	a_3 = (2,21,5,39)(3,23,6,47)(4,31,7,13)(8,24,29,48)(9,44,33,26)

Generators of G:	(10,41,34,17)(11,12,35,30)(14,15,32,36)(16,22,40,43)(18,49,42,25)(19,20,37,38)(27,28,45,46) (order 4)
	a_4 = (1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35) (order 7)

This generating set is not minimal. There is a smaller generating set with 2 elements. G is a solvable group. G acts transitively on the set of 49 elements {1, 2, …, 49}. The center of G is trivial.

The derived subgroup $D = [G, G]$ is a solvable group of order 392, generated by {(2,48,5,24)(3,14,6,32)(4,16,7,40)(8,21,29,39)(9,11,33,35)(10,38,34,20)(12,44,30,26)(13,22,31,43)(15,23,36,47)(17,19,41,37)(18,46,42,28)(25,27,49,45)(1,9,4,30)(3,44,5,23)(6,37,7,16)(8,11,32,29)(10,46,33,22)(12,25,31,43)(13,39,35,15)(14,18,34,36)(17,48,40,28)(19,27,38,49)(20,41,42,21)(24,45,47,26)} and $G/D \cong C_3$.

Lower central Series				
Level	Order	Nature	Quotient	Successive quotient
1	392	solvable	cyclic	C_3

Derived Series				
Level	Order	Nature	Quotient	Successive quotient
1	392	solvable	cyclic	C_3
2	98	solvable	solvable	C_2^2
3	49	abelian	solvable	C_2
4	1	trivial	solvable	C_7^2

Conjugacy classes of maximal subgroups				
Serial	Order	Nature	Conjugacy classes	Generators
1	392	solvable	1	(2,29,5,8)(3,36,6,15)(4,43,7,22)(9,30,33,12)(10,37,34,19)(11,44,35,26)(14,23,32,47)(17,38,41,20)(18,45,42,27)(21,24,39,48)(25,46,49,28)(1,22,10,31)(2,6,11,12)(3,21,8,49)(4,44,9,18)(5,38,13,36)(7,33,14,27)(15,37,45,39)(16,28,46,35)(17,47,43,20)(19,34,48,26)(23,29,32,24)(25,41,30,40)
2	294	solvable	4	(1,32,48,40,16,24,14)(2,31,49,36,18,27,12)(3,35,43,39,20,26,9)(4,34,47,37,17,28,8)(5,30,45,42,15,25,13)(6,33,44,38,21,22,11)(7,29,46,41,19,23,10)(2,36,40,5,15,16)(3,43,48,6,22,24)(4,8,14,7,29,32)(9,49,41,33,25,17)(10,35,46,34,11,28)(12,42,19,30,18,37)(13,21,23,31,39,47)(20,44,27,38,26,45)
3	24	solvable	49	(2,15,40)(3,22,48)(4,29,14)(5,36,16)(6,43,24)(7,8,32)(9,25,41)(10,11,46)(12,18,19)(13,39,23)(17,33,49)(20,26,27)(21,47,31)(28,34,35)(30,42,37)(38,44,45)(2,39,5,21)(3,47,6,23)(4,13,7,31)(8,48,29,24)(9,26,33,44)(10,17,34,41)(11,30,35,12)(14,36,32,15)(16,43,40,22)

Proper normal subgroups					
Serial	Order	Nature	Quotient	Subgroups	Generators
1	49	abelian	solvable	--	(1,3,5,4,7,2,6)(8,10,12,11,14,9,13)(15,17,19,18,21,16,20)(22,24,26,25,28,23,27)(29,31,33,32,35,30,34)(36,38,40,39,42,37,41)(43,45,47,46,49,44,48)(1,15,29,22,43,8,36)(2,16,30,23,44,9,37)(3,17,31,24,45,10,38)(4,18,32,25,46,11,39)(5,19,33,26,47,12,40)(6,20,34,27,48,13,41)(7,21,35,28,49,14,42)
2	98	solvable	solvable	1	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(1,15,29,22,43,8,36)(2,16,30,23,44,9,37)(3,17,31,24,45,10,38)(4,18,32,25,46,11,39)(5,19,33,26,47,12,40)(6,20,34,27,48,13,41)(7,21,35,28,49,14,42)(1,3,5,4,7,2,6)(8,10,12,11,14,9,13)(15,17,19,18,21,16,20)(22,24,26,25,28,23,27)(29,31,33,32,35,30,34)(36,38,40,39,42,37,41)(43,45,47,46,49,44,48)
3	392	solvable	cyclic	1,2	(2,29,5,8)(3,36,6,15)(4,43,7,22)(9,30,33,12)(10,37,34,19)(11,44,35,26)(13,16,31,40)(14,23,32,47)(17,38,41,20)(18,45,42,27)(21,24,39,48)(25,46,49,28)(1,22,10,31)(2,6,11,12)(3,21,8,49)(4,44,9,18)(5,38,13,36)(7,33,14,27)(15,37,45,39)(16,28,46,35)(17,47,43,20)(19,34,48,26)(23,29,32,24)(25,41,30,40)

Sylow subgroups						
p	Order	Conjug. classes	Nature	Normalizer	Normal closure	Generators
2	8	49	nilpotent	24	] 392	(2,48,5,24)(3,14,6,32)(4,16,7,40)(8,21,29,39)(9,11,33,35)(10,38,34,20)(12,44,30,26)(13,22,31,43)(15,23,36,47)(17,19,41,37)(18,46,42,28)(25,27,49,45)(2,29,5,8)(3,36,6,15)(4,43,7,22)(9,30,33,12)(10,37,34,19)(11,44,35,26)(13,16,31,40)(14,23,32,47)(17,38,41,20)(18,45,42,27)(21,24,39,48)(25,46,49,28)
3	3	196	cyclic	6	*G*	(2,15,40)(3,22,48)(4,29,14)(5,36,16)(6,43,24)(7,8,32)(9,25,41)(10,11,46)(12,18,19)(13,39,23)(17,33,49)(20,26,27)(21,47,31)(28,34,35)(30,42,37)(38,44,45)
7	49	1	abelian	*G*	49	(1,6,2,7,4,5,3)(8,13,9,14,11,12,10)(15,20,16,21,18,19,17)(22,27,23,28,25,26,24)(29,34,30,35,32,33,31)(36,41,37,42,39,40,38)(43,48,44,49,46,47,45)(1,15,29,22,43,8,36)(2,16,30,23,44,9,37)(3,17,31,24,45,10,38)(4,18,32,25,46,11,39)(5,19,33,26,47,12,40)(6,20,34,27,48,13,41)(7,21,35,28,49,14,42)

19. Let G be a primitive group of degree 49 with 3 generators. We have $|G| = 1568 = 2^5\times 7^2$.

Generators of G:	a_1 = (2,33)(3,41)(4,49)(5,9)(6,17)(7,25)(10,48)(11,35)(12,23)(13,38)(14,18)(16,26)(19,37)(20,31)(21,46)(24,34)(27,45)(28,39)(30,47)(32,42)(40,44) (order 2)
	a_2 = (2,35,33,48,38,22,31,10,5,11,9,24,20,43,13,34)(3,37,41,14,46,29,39,18,6,19,17,32,28,8,21,42)(4,45,49,16,12,36,47,26,7,27,25,40,30,15,23,44) (order 16)
	a_3 = (1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35) (order 7)

This generating set is not minimal. There is a smaller generating set with 2 elements. G is a solvable group. G acts transitively on the set of 49 elements {1, 2, …, 49}. The center of G is trivial.

The derived subgroup $D = [G, G]$ is a solvable group of order 392, generated by {(2,31,20,33,5,13,38,9)(3,39,28,41,6,21,46,17)(4,47,30,49,7,23,12,25)(8,14,19,42,29,32,37,18)(10,43,48,11,34,22,24,35)(15,16,27,44,36,40,45,26)(1,28,38,30,12,20,46)(2,26,41,32,8,21,45)(3,23,40,34,11,15,49)(4,22,42,31,9,19,48)(5,27,39,29,14,17,44)(6,25,36,35,10,16,47)(7,24,37,33,13,18,43)} and $G/D \cong C_2^2$.

Lower central Series				
Level	Order	Nature	Quotient	Successive quotient
1	392	solvable	abelian	C_2^2
2	196	solvable	nilpotent	C_2
3	98	solvable	nilpotent	C_2
4	49	abelian	nilpotent	C_2

Derived Series				
Level	Order	Nature	Quotient	Successive quotient
1	392	solvable	abelian	C_2^2
2	49	abelian	nilpotent	C_8
3	1	trivial	solvable	C_7^2

Conjugacy classes of maximal subgroups				
Serial	Order	Nature	Conjugacy classes	Generators
1	784	solvable	1	(2,35,33,48,38,22,31,10,5,11,9,24,20,43,13,34)(3,37,41,14,46,29,39,18,6,19,17,32,28,8,21,42)(4,45,49,16,12,36,47,26,7,27,25,40,30,15,23,44)(1,10,26,18,42,44,34)(2,13,22,17,40,46,35)(3,12,25,21,37,48,29)(4,14,23,20,36,45,33)(5,11,28,16,41,43,31)(6,8,24,19,39,49,30)(7,9,27,15,38,47,32)
2	784	solvable	1	(8,32)(9,31)(10,35)(11,34)(12,30)(13,33)(14,29)(15,40)(16,36)(17,39)(18,37)(19,42)(20,38)(21,41)(22,48)(23,49)(24,43)(25,47)(26,45)(27,44)(28,46)(1,46,8,32,22,4,15,11,36,25,43,18,29,39)(2,37,9,44,23,30,16)(3,26,10,19,24,40,17,47,38,33,45,5,31,12)(6,14,13,28,27,21,20,42,41,49,48,35,34,7)
3	784	solvable	1	(2,11,5,35)(3,19,6,37)(4,27,7,45)(8,46,29,28)(9,34,33,10)(12,36,30,15)(13,24,31,48)(14,21,32,39)(16,23,40,47)(17,42,41,18)(20,43,38,22)(25,44,49,26)(1,5,8,41,44,46,37,13)(2,35,45,15,43,31,7,23)(3,38,32,12,49,14,33,39)(4,48,26,24,47,6,18,21)(9,17,27,42,36,28,20,10)(11,25,29,16,40,19,30,22)

4	32	nilpotent	49	(8,32)(9,31)(10,35)(11,34)(12,30)(13,33)(14,29)(15,40)(16,36)(17,39)(18,37)(19,42)(20,38)(21,41)(22,48)(23,49)(24,43)(25,47)(26,45)(27,44)(28,46)(2,35,33,48,38,22,31,10,5,11,9,24,20,43,13,34)(3,37,41,14,46,29,39,18,6,19,17,32,28,8,21,42)(4,45,49,16,12,36,47,26,

Proper normal subgroups					
Serial	Order	Nature	Quotient	Subgroups	Generators
1	49	abelian	nilpotent	--	(1,3,5,4,7,2,6)(8,10,12,11,14,9,13)(15,17,19,18,21,16,20)(22,24,26,25,28,23,27)(29,31,33,32,35,30,34)(36,38,40,39,42,37,41)(43,45,47,46,49,44,48)(1,46,20,12,30,38,28)(2,45,21,8,32,41,26)(3,49,15,11,34,40,23)(4,48,19,9,31,42,22)(5,44,17,14,29,39,27)(6,47,16,10,35,36,25)(7,43,18,13,33,37,24)
2	98	solvable	nilpotent	1	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(1,9,25,17,41,49,33)(2,11,24,20,42,47,29)(3,13,28,19,36,44,32)(4,10,27,21,40,43,30)(5,8,23,18,38,48,35)(6,14,26,15,37,46,31)(7,12,22,16,39,45,34)(1,4,6,5,2,3,7)(8,11,13,12,9,10,14)(15,18,20,19,16,17,21)(22,25,27,26,23,24,28)(29,32,34,33,30,31,35)(36,39,41,40,37,38,42)(43,46,48,47,44,45,49)
3	196	solvable	nilpotent	1,2	(2,38,5,20)(3,46,6,28)(4,12,7,30)(8,37,29,19)(9,13,33,31)(10,24,34,48)(11,43,35,22)(14,18,32,42)(15,45,36,27)(16,26,40,44)(17,21,41,39)(23,49,47,25)(1,46,20,12,30,38,28)(2,45,21,8,32,41,26)(3,49,15,11,34,40,23)(4,48,19,9,31,42,22)(5,44,17,14,29,39,27)(6,47,16,10,35,36,25)(7,43,18,13,33,37,24)
4	392	solvable	abelian	1,2,3	(2,13,20,9,5,31,38,33)(3,21,28,17,6,39,46,41)(4,23,30,25,7,47,12,49)(8,32,19,18,29,14,37,42)(10,22,48,35,34,43,24,11)(15,40,27,26,36,16,45,44)(1,21,31,23,47,13,39)(2,19,34,25,43,14,38)(3,16,33,27,46,8,42)(4,15,35,24,44,12,41)(5,20,32,22,49,10,37)(6,18,29,28,45,9,40)(7,17,30,26,48,11,36)
5	784	solvable	cyclic	1,2,3,4	(2,13,20,9,5,31,38,33)(3,21,28,17,6,39,46,41)(4,23,30,25,7,47,12,49)(8,32,19,18,29,14,37,42)(10,22,48,35,34,43,24,11)(15,40,27,26,36,16,45,44)(1,14,11,6)(2,32,9,22)(3,24,12,33)(4,40,8,38)(5,16,10,44)(7,48,13,21)(15,19,46,45)(17,41,47,42)(18,29,43,25)(20,23,49,30)(26,27,31,35)(28,36,34,39)
6	784	solvable	cyclic	1,2,3,4	(2,35,33,48,38,22,31,10,5,11,9,24,20,43,13,34)(3,37,41,14,46,29,39,18,6,19,17,32,28,8,21,42)(4,45,49,16,12,36,47,26,7,27,25,40,30,15,23,44)(1,21,31,23,47,13,39)(2,19,34,25,43,14,38)(3,16,33,27,46,8,42)(4,15,35,24,44,12,41)(5,20,32,22,49,10,37)(6,18,29,28,45,9,40)(7,17,30,26,48,11,36)
					(8,32)(9,31)(10,35)(11,34)(12,30)(13,33)(14,29)

7	784	solvable	cyclic	1,2,3,4	(15,40)(16,36)(17,39)(18,37)(19,42)(20,38)(21,41)(22,48)(23,49)(24,43)(25,47)(26,45)(27,44)(28,46)(1,25,22,39,36,32,29,11,8,18,15,46,43,4)(2,9,23,16,37,44,30)(3,33,24,12,38,19,31,47,10,5,17,26,45,40)(6,49,27,7,41,28,34,42,13,35,20,14,48,21)

Sylow subgroups						
p	Order	Conjug. classes	Nature	Normalizer	Normal closure	Generators
2	32	49	nilpotent	32	G	(8,32)(9,31)(10,35)(11,34)(12,30)(13,33)(14,29)(15,40)(16,36)(17,39)(18,37)(19,42)(20,38)(21,41)(22,48)(23,49)(24,43)(25,47)(26,45)(27,44)(28,46)(2,35,33,48,38,22,31,10,5,11,9,24,20,43,13,34)(3,37,41,14,46,29,39,18,6,19,17,32,28,8,21,42)(4,45,49,16,12,36,47,26,7,27,25,40,30,15,23,44)
7	49	1	abelian	G	49	(1,15,29,22,43,8,36)(2,16,30,23,44,9,37)(3,17,31,24,45,10,38)(4,18,32,25,46,11,39)(5,19,33,26,47,12,40)(6,20,34,27,48,13,41)(7,21,35,28,49,14,42)(1,20,30,28,46,12,38)(2,21,32,26,45,8,41)(3,15,34,23,49,11,40)(4,19,31,22,48,9,42)(5,17,29,27,44,14,39)(6,16,35,25,47,10,36)(7,18,33,24,43,13,37)

20. Let G be a primitive group of degree 49 with 5 generators. We have $|G| = 1764 = 2^2\times3^2\times7^2$.

Generators of G:	a_1 = (2,29,5,8)(3,36,6,15)(4,43,7,22)(9,30,33,12)(10,37,34,19)(11,44,35,26)(13,16,31,40)(14,23,32,47)(17,38,41,20)(18,45,42,27)(21,24,39,48)(25,46,49,28) (order 4)
	a_2 = (2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46) (order 2)
	a_3 = (2,6,4)(3,7,5)(8,22,36)(9,27,39)(10,28,40)(11,23,41)(12,24,42)(13,25,37)(14,26,38)(15,29,43)(16,34,46)(17,35,47)(18,30,48)(19,31,49)(20,32,44)(21,33,45) (order 3)
	a_4 = (2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42)(11,27,37)(12,28,38)(13,23,39)(14,24,40)(15,29,43)(16,32,48)(17,33,49)(18,34,44)(19,35,45)(20,30,46)(21,31,47) (order 3)
	a_5 = (1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35) (order 7)

This generating set is not minimal. There is a smaller generating set with 2 elements. G is a solvable group. G acts transitively on the set of 49 elements {1, 2, …, 49}. The center of G is trivial.

The derived subgroup $D = [G, G]$ is a solvable group of order 147, generated by {(2,6,4)(3,7,5)(8,22,36)(9,27,39)(10,28,40)(11,23,41)(12,24,42)(13,25,37)(14,26,38)(15,29,43)(16,34,46)(17,35,47)(18,30,48)(19,31,49)(20,32,44)(21,33,45)(1,12,28,20,38,46,30)(2,8,26,21,41,45,32)(3,11,23,15,40,49,34)(4,9,22,19,42,48,31)(5,14,27,17,39,44,29)(6,10,25,16,36,47,35)(7,13,24,18,37,43,33)} and $G/D \cong C_3\times C_4$.

Lower central Series				
Level	Order	Nature	Quotient	Successive quotient
1	147	solvable	cyclic	$C_3 \times C_4$

Derived Series				
Level	Order	Nature	Quotient	Successive quotient
1	147	solvable	cyclic	$C_3 \times C_4$
2	49	abelian	solvable	C_3
3	1	trivial	solvable	C_7^2

Conjugacy classes of maximal subgroups				
Serial	Order	Nature	Conjugacy classes	Generators
1	882	solvable	1	(2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42)(11,27,37)(12,28,38)(13,23,39)(14,24,40)(15,29,43)(16,32,48)(17,33,49)(18,34,44)(19,35,45)(20,30,46)(21,31,47)(1,16,36,23,29,44)(2,15,37,22,30,43)(3,21,38,28,31,49)(4,19,39,26,32,47)(5,18,40,25,33,46)(6,20,41,27,34,48)(7,17,42,24,35,45)(8,9)(10,14)(11,12)
2	588	solvable	1	(2,6,4)(3,7,5)(8,22,36)(9,27,39)(10,28,40)(11,23,41)(12,24,42)(13,25,37)(14,26,38)(15,29,43)(16,34,46)(17,35,47)(18,30,48)(19,31,49)(20,32,44)(21,33,45)(1,26,16,11)(2,12,15,25)(3,33,21,39)(4,5,19,18)(6,47,20,46)(7,40,17,32)(8,22,23,9)(10,29,28,37)(13,43,27,44)(14,36,24,30)(31,35,42,38)(34,49,41,45)
3	588	solvable	3	(1,7,3,2,5,6,4)(8,14,10,9,12,13,11)(15,21,17,16,19,20,18)(22,28,24,23,26,27,25)(29,35,31,30,33,34,32)(36,42,38,37,40,41,39)(43,49,45,44,47,48,46)(2,43,3,8,4,15,5,22,6,29,7,36)(9,46,17,12,25,20,33,28,41,30,49,38)(10,11,18,19,26,27,34,35,42,37,44,45)(13,32,21,40,23,48,31,14,39,16,47,24)
4	36	solvable	49	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)(2,43,3,8,4,15,5,22,6,29,7,36)(9,46,17,12,25,20,33,28,41,30,49,38)(10,11,18,19,26,27,34,35,42,37,44,45)

Proper normal subgroups					
Serial	Order	Nature	Quotient	Subgroups	Generators
1	49	abelian	solvable	--	(1,7,3,2,5,6,4)(8,14,10,9,12,13,11)(15,21,17,16,19,20,18)(22,28,24,23,26,27,25)(29,35,31,30,33,34,32)(36,42,38,37,40,41,39)(43,49,45,44,47,48,46)(1,43,15,8,29,36,22)(2,44,16,9,30,37,23)(3,45,17,10,31,38,24)(4,46,18,11,32,39,25)(5,47,19,12,33,40,26)(6,48,20,13,34,41,27)(7,49,21,14,35,42,28)

2	147	solvable	solvable	1	(2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42)(11,27,37)(12,28,38)(13,23,39)(14,24,40)(15,29,43)(16,32,48)(17,33,49)(18,34,44)(19,35,45)(20,30,46)(21,31,47) (1,43,15,8,29,36,22)(2,44,16,9,30,37,23)(3,45,17,10,31,38,24)(4,46,18,11,32,39,25)(5,47,19,12,33,40,26)(6,48,20,13,34,41,27)(7,49,21,14,35,42,28) (1,7,3,2,5,6,4)(8,14,10,9,12,13,11)(15,21,17,16,19,20,18)(22,28,24,23,26,27,25)(29,35,31,30,33,34,32)(36,42,38,37,40,41,39)(43,49,45,44,47,48,46)
3	98	solvable	solvable	1	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46) (1,43,15,8,29,36,22)(2,44,16,9,30,37,23)(3,45,17,10,31,38,24)(4,46,18,11,32,39,25)(5,47,19,12,33,40,26)(6,48,20,13,34,41,27)(7,49,21,14,35,42,28) (1,7,3,2,5,6,4)(8,14,10,9,12,13,11)(15,21,17,16,19,20,18)(22,28,24,23,26,27,25)(29,35,31,30,33,34,32)(36,42,38,37,40,41,39)(43,49,45,44,47,48,46)
4	294	solvable	dihedral	1,2,3	(1,4)(3,5)(6,7)(8,32)(9,30)(10,33)(11,29)(12,31)(13,35)(14,34)(15,39)(16,37)(17,40)(18,36)(19,38)(20,42)(21,41)(22,46)(23,44)(24,47)(25,43)(26,45)(27,49)(28,48) (1,4,5)(2,6,3)(8,25,40)(9,27,38)(10,23,41)(11,26,36)(12,22,39)(13,24,37)(14,28,42)(15,32,47)(16,34,45)(17,30,48)(18,33,43)(19,29,46)(20,31,44)(21,35,49) (1,42,18,44,10,26)(2,38,19,43,14,25)(3,40,15,49,11,23)(4,37,17,47,8,28)(5,36,21,46,9,24)(6,41,20,48,13,27)(7,39,16,45,12,22)(29,35,32,30,31,33)
5	147	solvable	cyclic	1	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33) (1,39,13,47,23,31,21)(2,38,14,43,25,34,19)(3,42,8,46,27,33,16)(4,41,12,44,24,35,15)(5,37,10,49,22,32,20)(6,40,9,45,28,29,18)(7,36,11,48,26,30,17)
6	441	solvable	cyclic	1,2,5	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33) (1,39,34,5,37,31,7,36,32,6,40,30,3,42,29,4,41,33,2,38,35)(8,11,13,12,9,10,14)(15,25,48,19,23,45,21,22,46,20,26,44,17,28,43,18,27,47,16,24,49)
					(2,7,6,5,4,3)(8,15,22,29,36,43)(9,21,27,33,39,45)(10,16,28,34,40,46)(11,17,23,

7	294	solvable	cyclic	1,3,5	35,41,47)(12,18,24,30,42,48)(13,19,25,31,37,49)(14,20,26,32,38,44)(1,16,32,24,48,14,40)(2,18,31,27,49,12,36)(3,20,35,26,43,9,39)(4,17,34,28,47,8,37)(5,15,30,25,45,13,42)(6,21,33,22,44,11,38)(7,19,29,23,46,10,41)
8	882	solvable	cyclic	1,2,3,4,5,6,7	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)(1,16,43,37,8,30)(2,15,44,36,9,29)(3,21,45,42,10,35)(4,19,46,40,11,33)(5,18,47,39,12,32)(6,20,48,41,13,34)(7,17,49,38,14,31)(22,23)(24,28)(25,26)
9	588	solvable	cyclic	1,3,5,7	(2,6,4)(3,7,5)(8,22,36)(9,27,39)(10,28,40)(11,23,41)(12,24,42)(13,25,37)(14,26,38)(15,29,43)(16,34,46)(17,35,47)(18,30,48)(19,31,49)(20,32,44)(21,33,45)(1,13,42,16)(2,6,41,37)(3,48,39,30)(4,34,38,44)(5,27,40,23)(7,20,36,9)(8,14,21,15)(10,49,18,29)(11,35,17,43)(12,28,19,22)(24,47,25,33)(31,45,46,32)

Sylow subgroups						
p	Order	Conjug. classes	Nature	Normalizer	Normal closure	Generators
2	4	147	cyclic	12	588	(2,29,5,8)(3,36,6,15)(4,43,7,22)(9,30,33,12)(10,37,34,19)(11,44,35,26)(13,16,31,40)(14,23,32,47)(17,38,41,20)(18,45,42,27)(21,24,39,48)(25,46,49,28)
3	9	49	abelian	36	441	(2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42)(11,27,37)(12,28,38)(13,23,39)(14,24,40)(15,29,43)(16,32,48)(17,33,49)(18,34,44)(19,35,45)(20,30,46)(21,31,47)(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)
7	49	1	abelian	*G*	49	(1,36,8,43,22,29,15)(2,37,9,44,23,30,16)(3,38,10,45,24,31,17)(4,39,11,46,25,32,18)(5,40,12,47,26,33,19)(6,41,13,48,27,34,20)(7,42,14,49,28,35,21)(1,48,16,14,32,40,24)(2,49,18,12,31,36,27)(3,43,20,9,35,39,26)(4,47,17,8,34,37,28)(5,45,15,13,30,42,25)(6,44,21,11,33,38,22)(7,46,19,10,29,41,23)

21. Let G be a primitive group of degree 49 with 5 generators. We have $|G| = 1764 = 2^2\times3^2\times7^2$.

	a_1 = (2,8)(3,15)(4,22)(5,29)(6,36)(7,43)(10,16)(11,23)(12,30)(13,37)(14,44)(18,24)(19,31)(20,38)(21,45)(26,32)(27,39)(28,46)(34,40)(35,47)(42,48) (order 2)
	a_2 = (2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46) (order 2)

Generators of G:	a_3 = (2,6,4)(3,7,5)(8,22,36)(9,27,39)(10,28,40)(11,23,41)(12,24,42)(13,25,37)(14,26,38)(15,29,43)(16,34,46)(17,35,47)(18,30,48)(19,31,49)(20,32,44)(21,33,45) (order 3)
	a_4 = (2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42)(11,27,37)(12,28,38)(13,23,39)(14,24,40)(15,29,43)(16,32,48)(17,33,49)(18,34,44)(19,35,45)(20,30,46)(21,31,47) (order 3)
	a_5 = (1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35) (order 7)

This generating set is not minimal. There is a smaller generating set with 2 elements. G is a solvable group. G acts transitively on the set of 49 elements {1, 2, …, 49}. The center of G is trivial.

The derived subgroup $D = [G, G]$ is a solvable group of order 147, generated by {(2,6,4)(3,7,5)(8,22,36)(9,27,39)(10,28,40)(11,23,41)(12,24,42)(13,25,37)(14,26,38)(15,29,43)(16,34,46)(17,35,47)(18,30,48)(19,31,49)(20,32,44)(21,33,45)(1,12,28,20,38,46,30)(2,8,26,21,41,45,32)(3,11,23,15,40,49,34)(4,9,22,19,42,48,31)(5,14,27,17,39,44,29)(6,10,25,16,36,47,35)(7,13,24,18,37,43,33)} and $G/D \cong C_2^2\times C_3$.

Lower central Series				
Level	Order	Nature	Quotient	Successive quotient
1	147	solvable	abelian	$C_2^2\times C_3$

Derived Series				
Level	Order	Nature	Quotient	Successive quotient
1	147	solvable	abelian	$C_2^2\times C_3$
2	49	abelian	solvable	C_3
3	1	trivial	solvable	C_7^2

Conjugacy classes of maximal subgroups				
Serial	Order	Nature	Conjugacy classes	Generators
1	882	solvable	1	(2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42)(11,27,37)(12,28,38)(13,23,39)(14,24,40)(15,29,43)(16,32,48)(17,33,49)(18,34,44)(19,35,45)(20,30,46)(21,31,47)(1,14,29,21,22,42)(2,13,30,20,23,41)(3,11,31,18,24,39)(4,10,32,17,25,38)(5,12,33,19,26,40)(6,9,34,16,27,37)(7,8,35,15,28,36)(43,49)(44,48)(45,46)
2	882	solvable	1	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)(1,43,45,38,37,2)(3,36,44)(4,22,48,31,42,9)(5,15,47,17,40,16)(6,29,49,10,39,23)(7,8,46,24,41,30)(11,25,27,34,35,14)(12,18,26,20,33,21)(13,32,28)
3	882	solvable	1	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)(1,22,24,17,16,2)(3,15,23)(4,43,27,10,21,30)(5,36,26,38,19,37)(6,8,28,31,18,44)(7,29,25,45,20,9)(11,49,34)(12,42,33,39,47,41)(13,14,35,32,46,48)
4	588	solvable	1	(2,3,4,5,6,7)(8,43,36,29,22,15)(9,45,39,33,27,21)(10,46,40,34,28,16)(11,47,41,35,23,17)(12,48,42,30,24,18)(13,49,37,31,25,19)(14,44,38,32,26,20)

				(1,23,17,48,33,8,25,38,49,5,9,18,41,35) (2,16,20,34,29,22,24,45,47,12,11,39,42,7) (3,44,19,13,32,36,28) (4,37,21,6,30,15,27,31,43,26,10,46,40,14)
5	588	solvable	3	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35) (12,30)(13,31)(14,32)(15,36)(16,40)(17,41) (18,42)(19,37)(20,38)(21,39)(22,43)(23,47) (24,48)(25,49)(26,44)(27,45)(28,46) (1,43,45,38,37,2)(3,36,44)(4,22,48,31,42,9) (5,15,47,17,40,16)(6,29,49,10,39,23) (7,8,46,24,41,30)(11,25,27,34,35,14) (12,18,26,20,33,21)(13,32,28)
6	36	solvable	49	(2,22,6,8,4,36)(3,29,7,15,5,43)(9,25,41) (10,32,42,16,26,48)(11,39,37,23,27,13) (12,46,38,30,28,20)(14,18,40,44,24,34) (17,33,49)(19,47,45,31,35,21) (2,7,6,5,4,3)(8,15,22,29,36,43) (9,21,27,33,39,45)(10,16,28,34,40,46) (11,17,23,35,41,47)(12,18,24,30,42)

Proper normal subgroups					
Serial	Order	Nature	Quotient	Subgroups	Generators
1	49	abelian	solvable	--	(1,7,3,2,5,6,4)(8,14,10,9,12,13,11)(15,21, 17,16,19,20,18)(22,28,24,23,26,27,25) (29,35,31,30,33,34,32)(36,42,38,37,40, 41,39)(43,49,45,44,47,48,46)(1,43,15,8, 29,36,22)(2,44,16,9,30,37,23)(3,45,17, 10,31,38,24)(4,46,18,11,32,39,25)(5,47, 19,12,33,40,26)(6,48,20,13,34,41,27) (7,49,21,14,35,42,28)
2	147	solvable	dihedral	1	(2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42) (11,27,37)(12,28,38)(13,23,39)(14,24,40) (15,29,43)(16,32,48)(17,33,49)(18,34,44) (19,35,45)(20,30,46)(21,31,47)(1,43,15,8, 29,36,22)(2,44,16,9,30,37,23)(3,45,17,10, 31,38,24)(4,46,18,11,32,39,25)(5,47,19, 12,33,40,26)(6,48,20,13,34,41,27)(7,49, 21,14,35,42,28)(1,7,3,2,5,6,4)(8,14,10, 9,12,13,11)(15,21,17,16,19,20,18)(22, 28,24,23,26,27,25)(29,35,31,30,33,34,32) (36,42,38,37,40,41,39)(43,49,45,44,47,48,46)
3	98	solvable	solvable	1	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35) (12,30)(13,31)(14,32)(15,36)(16,40) (17,41)(18,42)(19,37)(20,38)(21,39) (22,43)(23,47)(24,48)(25,49)(26,44)(27,45) (28,46)(1,43,15,8,29,36,22)(2,44,16,9,30, 37,23)(3,45,17,10,31,38,24)(4,46,18,11,32, 39,25)(5,47,19,12,33,40,26)(6,48,20,13,34, 41,27)(7,49,21,14,35,42,28)(1,7,3,2,5,6,4) (8,14,10,9,12,13,11)(15,21,17,16,19,20,18) (22,28,24,23,26,27,25)(29,35,31,30,33,34, 32)(36,42,38,37,40,41,39)(43,49,45,44, 47,48,46)
					(1,4,2,7,3,6)(8,18,23,35,38,48)(9,21,24,34, 36,46)(10,20,22,32,37,49)(11,16,28,31,41,

4	294	solvable	dihedral	1,2,3	43)(12,19,26,33,40,47)(13,15,25,30,42,45)(14,17,27,29,39,44)(1,36,43)(2,39,48)(3,40,49)(4,41,44)(5,42,45)(6,37,46)(7,38,47)(8,29,22)(9,32,27)(10,33,28)(11,34,23)(12,35,24)(13,30,25)(14,31,26)(16,18,20)(17,19,21)(1,42,18,44,10,26)(2,38,19,43,14,25)(3,40,15,49,11,23)(4,37,17,47,8,28)(5,36,21,46,9,24)(6,41,20,48,13,27)(7,39,16,45,12,22)(29,35,32,30,31,33)
5	147	solvable	abelian	1	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)(1,39,13,47,23,31,21)(2,38,14,43,25,34,19)(3,42,8,46,27,33,16)(4,41,12,44,24,35,15)(5,37,10,49,22,32,20)(6,40,9,45,28,29,18)(7,36,11,48,26,30,17)
6	441	solvable	abelian	1,2,5	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)(1,39,34,5,37,31,7,36,32,6,40,30,3,42,29,4,41,33,2,38,35)(8,11,13,12,9,10,14)(15,25,48,19,23,45,21,22,46,20,26,44,17,28,43,18,27,47,16,24,49)
7	294	solvable	cyclic	1,5	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)(1,43,46,18,20,13,12,33,30,37,38,24,28,7)(2,36,45,25,21,6,8,47,32,16,41,10,26,35)(3,22,49,4,15,48,11,19,34,9,40,31,23,42)(5,29,44,39,17,27,14)
8	294	solvable	cyclic	1,3,5	(2,7,6,5,4,3)(8,15,22,29,36,43)(9,21,27,33,39,45)(10,16,28,34,40,46)(11,17,23,35,41,47)(12,18,24,30,42,48)(13,19,25,31,37,49)(14,20,26,32,38,44)(1,39,13,47,23,31,21)(2,38,14,43,25,34,19)(3,42,8,46,27,33,16)(4,41,12,44,24,35,15)(5,37,10,49,22,32,20)(6,40,9,45,28,29,18)(7,36,11,48,26,30,17)
9	882	solvable	cyclic	1,2,5,6,7	(1,7,42,38,10,9,44,47,26,27,34,32,18,15)(2,49,40,24,13,30,46,19,22,6,35,39,17,8)(3,14,37,45,12,23,48,33,25,20,29,4,21,36)(5,28,41,31,11,16,43)(1,18,31,41,23,14)(2,11,29,20,24,42)(3,39,30,13,22,21)(4,32,34,27,28,7)(5,46,33,48,26,49)(6,25,35)(8,15,17,38,37,9)(10,36,16)(12,43,19,45,40,44)(1,23,17)(2,24,15)(3,22,16)(4,28,20)(5,26,19)(6,25,21)(7,27,18)(8,30,45)(9,31,43)(10,29,44)(11,35,48)(12,33,47)(13,32,49)(14,34,46)(36,37,38)(39,42,41)
					(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)

10	882	solvable	cyclic	1,2,3,4,5,6,8	(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)(1,39,22,18,29,11)(2,37,23,16,30,9)(3,40,24,19,31,12)(4,36,25,15,32,8)(5,38,26,17,33,10)(6,42,27,21,34,14)(7,41,28,20,35,13)(43,46)(45,47)(48,49)
11	294	solvable	cyclic	1,5	(2,6,4)(3,7,5)(8,22,36)(9,27,39)(10,28,40)(11,23,41)(12,24,42)(13,25,37)(14,26,38)(15,29,43)(16,34,46)(17,35,47)(18,30,48)(19,31,49)(20,32,44)(21,33,45)(1,36,41,13,9,44,49,28,25,32,33,19,17,3)(2,43,42,27,11,30,47,21,24,4,29,40,20,10)(5,15,38,6,8,37,48,14,23,46,35,26,18,31)(7,22,39,34,12,16,45)
12	882	solvable	cyclic	1,2,5,6	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)(1,8,11,46,45,3)(2,15,12,32,49,38)(4,43,10)(5,29,14,39,44,17)(6,22,13,25,48,24)(7,36,9,18,47,31)(16,19,33,35,42,37)(20,26,34,28,41,23)(21,40,30)
13	588	solvable	cyclic	--	(2,3,4,5,6,7)(8,43,36,29,22,15)(9,45,39,33,27,21)(10,46,40,34,28,16)(11,47,41,35,23,17)(12,48,42,30,24,18)(13,49,37,31,25,19)(14,44,38,32,26,20)(1,31,25,14,41,15,33,46,9,6,17,26,49,37)(2,3,24,28,42,36,29,32,11,13,20,19,47,44)(4,10,27,21,40,43,30)(5,45,23,7,38,22,35,39,8,34,18,12,48,16)

Sylow subgroups						
p	Order	Conjug. classes	Nature	Normalizer	Normal closure	Generators
2	4	147	abelian	12	588	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(2,8)(3,15)(4,22)(5,29)(6,36)(7,43)(10,16)(11,23)(12,30)(13,37)(14,44)(18,24)(19,31)(20,38)(21,45)(26,32)(27,39)(28,46)(34,40)(35,47)(42,48)
3	9	49	abelian	36	441	(2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42)(11,27,37)(12,28,38)(13,23,39)(14,24,40)(15,29,43)(16,32,48)(17,33,49)(18,34,44)(19,35,45)(20,30,46)(21,31,47)(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)
7	49	1	abelian	*G*	49	(1,6,2,7,4,5,3)(8,13,9,14,11,12,10)(15,20,16,21,18,19,17)(22,27,23,28,25,26,24)(29,34,30,35,32,33,31)(36,41,37,42,39,40,38)(43,48,44,49,46,47,45)(1,43,15,8,29,36,22)(2,44,16,9,30,37,23)(3,45,17,10,31,38,24)

					(4,46,18,11,32,39,25)(5,47,19,12,33,40,26)(6,48,20,13,34,41,27)(7,49,21,14,35,42,28)

22. Let G be a primitive group of degree 49 with 3 generators. We have $|G| = 2352 = 2^4\times3\times7^2$.

Generators of G:	a_1 = (2,48,5,24)(3,14,6,32)(4,16,7,40)(8,21,29,39)(9,11,33,35)(10,38,34,20)(12,44,30,26)(13,22,31,43)(15,23,36,47)(17,19,41,37)(18,46,42,28)(25,27,49,45) (order 4)
	a_2 = (2,21,30,33,3,23,38,41,4,31,46,49,5,39,12,9,6,47,20,17,7,13,28,25)(8,40,35,42,15,48,37,44,22,14,45,10,29,16,11,18,36,24,19,26,43,32,27,34) (order 24)
	a_3 = (1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35) (order 7)

This generating set is not minimal. There is a smaller generating set with 2 elements. G is a solvable group. G acts transitively on the set of 49 elements {1, 2, …, 49}. The center of G is trivial.

The derived subgroup $D = [G, G]$ is a solvable group of order 196, generated by {(2,38,5,20)(3,46,6,28)(4,12,7,30)(8,37,29,19)(9,13,33,31)(10,24,34,48)(11,43,35,22)(14,18,32,42)(15,45,36,27)(16,26,40,44)(17,21,41,39)(23,49,47,25)(1,28,38,30,12,20,46)(2,26,41,32,8,21,45)(3,23,40,34,11,15,49)(4,22,42,31,9,19,48)(5,27,39,29,14,17,44)(6,25,36,35,10,16,47)(7,24,37,33,13,18,43)} and $G/D \cong C_2^2\times C_3$.

Lower central Series				
Level	Order	Nature	Quotient	Successive quotient
1	196	solvable	abelian	$C_2^2\times C_3$
2	98	solvable	nilpotent	C_2
3	49	abelian	nilpotent	C_2

Derived Series				
Level	Order	Nature	Quotient	Successive quotient
1	196	solvable	abelian	$C_2^2\times C_3$
2	49	abelian	nilpotent	C_4
3	1	trivial	solvable	C_7^2

Conjugacy classes of maximal subgroups				
Serial	Order	Nature	Conjugacy classes	Generators
1	1176	solvable	1	(1,11,27,19,37,45,35)(2,10,28,15,39,48,33)(3,14,22,18,41,47,30)(4,13,26,16,38,49,29)(5,9,24,21,36,46,34)(6,12,23,17,42,43,32)(7,8,25,20,40,44,31)(2,47,46,33,7,39,38,25,6,31,30,17,5,23,28,9,4,21,20,49,3,13,12,41)(8,24,45,42,43,16,37,34,36,14,35,26,29,48,27,18,22,40,19,10,15,32,11,44)
2	1176	solvable	1	(2,48,5,24)(3,14,6,32)(4,16,7,40)(8,21,29,39)(9,11,33,35)(10,38,34,20)(12,44,30,26)(13,22,31,43)(15,23,36,47)(17,19,41,37)(18,46,42,28)(25,27,49,45)(1,14,23,46,13,15,47,37,21,6,39,26)(2,5,27,28,12,11,49,43,18,16,36,41)(3,44,24,20,10,40,45,7,17,25,38,8)(4,29,22,30,9,34,48,33,19,35,42,32)
				(2,43,5,22)(3,8,6,29)(4,15,7,36)(9,48,33,24)(10,13,34,31)(11,20,35,38)(12,27,30,45)(14,

3	1176	solvable	1	41,32,17)(16,49,40,25)(18,21,42,39)(19,28,37,46)(23,44,47,26)(1,41,15,14,43,30,36,3,8,18,29,47)(2,21,17,44,46,38,40,11,13,33,35,6)(4,26,19,27,48,28,42,23,9,24,31,25)(5,45,20,39,49,12,37,34,10,7,32,16)
4	784	solvable	1	(2,31,20,33,5,13,38,9)(3,39,28,41,6,21,46,17)(4,47,30,49,7,23,12,25)(8,14,19,42,29,32,37,18)(10,43,48,11,34,22,24,35)(15,16,27,44,36,40,45,26)(1,11,2,33)(3,22,7,44)(4,38,5,21)(8,36,30,16)(9,27,29,48)(10,49,35,24)(12,31,32,14)(13,18,34,40)(15,17,37,42)(19,47,39,25)(20,28,41,45)(23,46,43,26)
5	48	nilpotent	49	(2,31,20,33,5,13,38,9)(3,39,28,41,6,21,46,17)(4,47,30,49,7,23,12,25)(8,14,19,42,29,32,37,18)(10,43,48,11,34,22,24,35)(15,16,27,44,36,40,45,26)(2,16,3,24,4,32,5,40,6,48,7,14)(8,31,15,39,22,47,29,13,36,21,43,23)(9,27,17,35,25,37,33,45,41,11,49,19)(10,12,18,20,26,28,34,30,42,

Proper normal subgroups					
Serial	Order	Nature	Quotient	Subgroups	Generators
1	49	abelian	nilpotent	--	(1,3,5,4,7,2,6)(8,10,12,11,14,9,13)(15,17,19,18,21,16,20)(22,24,26,25,28,23,27)(29,31,33,32,35,30,34)(36,38,40,39,42,37,41)(43,45,47,46,49,44,48)(1,15,29,22,43,8,36)(2,16,30,23,44,9,37)(3,17,31,24,45,10,38)(4,18,32,25,46,11,39)(5,19,33,26,47,12,40)(6,20,34,27,48,13,41)(7,21,35,28,49,14,42)
2	147	solvable	nilpotent	1	(2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42)(11,27,37)(12,28,38)(13,23,39)(14,24,40)(15,29,43)(16,32,48)(17,33,49)(18,34,44)(19,35,45)(20,30,46)(21,31,47)(1,3,5,4,7,2,6)(8,10,12,11,14,9,13)(15,17,19,18,21,16,20)(22,24,26,25,28,23,27)(29,31,33,32,35,30,34)(36,38,40,39,42,37,41)(43,45,47,46,49,44,48)(1,15,29,22,43,8,36)(2,16,30,23,44,9,37)(3,17,31,24,45,10,38)(4,18,32,25,46,11,39)(5,19,33,26,47,12,40)(6,20,34,27,48,13,41)(7,21,35,28,49,14,42)
3	98	solvable	nilpotent	1	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(1,15,29,22,43,8,36)(2,16,30,23,44,9,37)(3,17,31,24,45,10,38)(4,18,32,25,46,11,39)(5,19,33,26,47,12,40)(6,20,34,27,48,13,41)(7,21,35,28,49,14,42)(1,3,5,4,7,2,6)(8,10,12,11,14,9,13)(15,17,19,18,21,16,20)(22,24,26,25,28,23,27)(29,31,33,32,35,30,34)(36,38,40,39,42,37,41)(43,45,47,46,49,44,48)
					(1,15,29,22,43,8,36)(2,16,30,23,44,9,37)(3,17,31,24,45,10,38)(4,18,32,25,46,11,39)(5,19,33,26,47,12,40)(6,20,34,27,48,13,41)(7,21,35,28,49,14,42)(1,3,5,4,7,2,6)

4	294	solvable	nilpotent	1,2,3	(8,10,12,11,14,9,13)(15,17,19,18,21,16,20)(22,24,26,25,28,23,27)(29,31,33,32,35,30,34)(36,38,40,39,42,37,41)(43,45,47,46,49,44,48)(2,7,6,5,4,3)(8,43,36,29,22,15)(9,49,41,33,25,17)(10,44,42,34,26,18)(11,45,37,35,27,19)(12,46,38,30,28,20)(13,47,39,31,23,21)(14,48,40,32,24,16)
5	196	solvable	abelian	1,3	(2,38,5,20)(3,46,6,28)(4,12,7,30)(8,37,29,19)(9,13,33,31)(10,24,34,48)(11,43,35,22)(14,18,32,42)(15,45,36,27)(16,26,40,44)(17,21,41,39)(23,49,47,25)(1,46,20,12,30,38,28)(2,45,21,8,32,41,26)(3,49,15,11,34,40,23)(4,48,19,9,31,42,22)(5,44,17,14,29,39,27)(6,47,16,10,35,36,25)(7,43,18,13,33,37,24)
6	392	solvable	cyclic	1,3,5	(2,38,5,20)(3,46,6,28)(4,12,7,30)(8,37,29,19)(9,13,33,31)(10,24,34,48)(11,43,35,22)(14,18,32,42)(15,45,36,27)(16,26,40,44)(17,21,41,39)(23,49,47,25)(1,19,17,6)(2,37,18,32)(3,35,15,42)(4,45,16,43)(5,27,20,12)(7,11,21,23)(8,9,24,25)(10,36,22,31)(13,46,26,44)(14,34,28,40)(29,48,38,47)(30,33,39,41)
7	392	solvable	cyclic	1,3,5	(2,31,20,33,5,13,38,9)(3,39,28,41,6,21,46,17)(4,47,30,49,7,23,12,25)(8,14,19,42,29,32,37,18)(10,43,48,11,34,22,24,35)(15,16,27,44,36,40,45,26)(1,39,13,47,23,31,21)(2,38,14,43,25,34,19)(3,42,8,46,27,33,16)(4,41,12,44,24,35,15)(5,37,10,49,22,32,20)(6,40,9,45,28,29,18)(7,36,11,48,26,30,17)
8	588	solvable	abelian	1,2,3,4,5	(1,46,20,12,30,38,28)(2,45,21,8,32,41,26)(3,49,15,11,34,40,23)(4,48,19,9,31,42,22)(5,44,17,14,29,39,27)(6,47,16,10,35,36,25)(7,43,18,13,33,37,24)(2,12,3,20,4,28,5,30,6,38,7,46)(8,11,15,19,22,27,29,35,36,37,43,45)(9,23,17,31,25,39,33,47,41,13,49,21)(10,40,18,48,26,14,34,16,42,24,44,32)
9	1176	solvable	cyclic	1,2,3,4,5,6,8	(2,38,5,20)(3,46,6,28)(4,12,7,30)(8,37,29,19)(9,13,33,31)(10,24,34,48)(11,43,35,22)(14,18,32,42)(15,45,36,27)(16,26,40,44)(17,21,41,39)(23,49,47,25)(1,26,15,32,36,45,22,2,29,21,43,41)(3,4,16,17,42,37,27,28,33,34,46,47)(5,49,18,6,38,19,23,39,35,24,48,30)(7,13,20,12,40,11,25,10,31,9,44,14)
10	1176	solvable	cyclic	1,2,3,4,5,7,8	(2,47,46,33,7,39,38,25,6,31,30,17,5,23,28,9,4,21,20,49,3,13,12,41)(8,24,45,42,43,16,37,34,36,14,35,26,29,48,27,18,22,40,19,10,15,32,11,44)(1,35,45,37,19,27,11)(2,33,48,39,15,28,10)(3,30,47,41,18,22,14)(4,29,49,38,16,26,13)(5,34,46,36,21,24,9)(6,32,43,42,17,23,12)(7,31,44,40,20,25,8)
					(2,34,5,10)(3,42,6,18)(4,44,7,26)(8,17,29,41)(9,22,33,43)(11,13,35,31)(12,40,30,16)(14,46,32,28)(15,25,36,49)(19,21,37,39)

11	392	solvable	cyclic	1,3	(20,48,38,24)(23,45,47,27)(1,31,38,43)(2,36,39,3)(4,27,37,26)(5,12,41,20)(6,49,40,35)(7,18,42,9)(8,23,17,25)(10,19,15,13)(11,46,16,30)(14,29,21,45)(22,47,24,34)(32,33,44,48)
12	784	solvable	cyclic	--	(2,34,5,10)(3,42,6,18)(4,44,7,26)(8,17,29,41)(9,22,33,43)(11,13,35,31)(12,40,30,16)(14,46,32,28)(15,25,36,49)(19,21,37,39)(20,48,38,24)(23,45,47,27)(1,39,29,20,16,33,37,6)(2,21,27,24,15,3,13,14)(4,23,44,11,19,8,43,26)(5,47,17,42,18,46,7,31)(9,38,45,30,22,35,49,36)(10,25,41,40,28,12,34,32)
13	1176	solvable	cyclic	--	(2,20,5,38)(3,28,6,46)(4,30,7,12)(8,19,29,37)(9,31,33,13)(10,48,34,24)(11,22,35,43)(14,42,32,18)(15,27,36,45)(16,44,40,26)(17,39,41,21)(23,25,47,49)(1,5,28,25,12,13,46,45,20,15,38,42)(2,43,23,21,9,40,44,4,16,27,37,10)(3,11,22,48,14,17,47,36,18,7,41,26)(6,31,24,29,8,35,49,33,19,32,39,34)

Sylow subgroups						
p	Order	Conjug. classes	Nature	Normalizer	Normal closure	Generators
2	16	49	nilpotent	48	784	(2,48,5,24)(3,14,6,32)(4,16,7,40)(8,21,29,39)(9,11,33,35)(10,38,34,20)(12,44,30,26)(13,22,31,43)(15,23,36,47)(17,19,41,37)(18,46,42,28)(25,27,49,45)(2,31,20,33,5,13,38,9)(3,39,28,41,6,21,46,17)(4,47,30,49,7,23,12,25)(8,14,19,42,29,32,37,18)(10,43,48,11,34,22,24,35)(15,16,27,44,36,40,45,26)
3	3	49	cyclic	48	147	(2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42)(11,27,37)(12,28,38)(13,23,39)(14,24,40)(15,29,43)(16,32,48)(17,33,49)(18,34,44)(19,35,45)(20,30,46)(21,31,47)
7	49	1	abelian	G	49	(1,35,45,37,19,27,11)(2,33,48,39,15,28,10)(3,30,47,41,18,22,14)(4,29,49,38,16,26,13)(5,34,46,36,21,24,9)(6,32,43,42,17,23,12)(7,31,44,40,20,25,8)(1,36,8,43,22,29,15)(2,37,9,44,23,30,16)(3,38,10,45,24,31,17)(4,39,11,46,25,32,18)(5,40,12,47,26,33,19)(6,41,13,48,27,34,20)(7,42,14,49,28,35,21)

23. Let G be a primitive group of degree 49 with 2 generators. We have $|G| = 2352 = 2^4 \times 3 \times 7^2$.

Generators of G:	a_1 = (2,40,21,35,30,42,33,15,3,48,23,37,38,44,41,22,4,14,31,45,46,10,49,29,5,16,39,11,12,18,9,36,6,24,47,19,20,26,17,43,7,32,13,27,28,34,25,8) (order 48)
	a_2 = (1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35) (order 7)

G is a solvable group. G acts transitively on the set of 49 elements {1, 2, …, 49}. The center of G is trivial. The derived subgroup $D = [G, G]$ is an abelian group of order 49, generated by {(1,30,46,38,20,28,12)(2,32,45,41,21,26,8)(3,34,49,40,15,23,11)(4,31,48,42,19,22,9)(5,29,44,39, 17,27,14)(6,35,47,36,16,25,10)(7,33,43,37,18,24,13)(1,42,10,44,26,34,18)(2,40,13,46,22,35,17)(3,37,12,48,25,29,21)(4,36,14,45,23,33,20)(5,41,11,43,28,31,16)(6,39,8,49,24,30,19)(7,38,9,47,27,32,15)} and $G/D \cong C_3 \times C_{16}$.

Lower central Series				
Level	Order	Nature	Quotient	Successive quotient
1	49	abelian	cyclic	$C_3 \times C_{16}$

Derived Series				
Level	Order	Nature	Quotient	Successive quotient
1	49	abelian	cyclic	$C_3 \times C_{16}$
2	1	trivial	solvable	C_7^2

Conjugacy classes of maximal subgroups				
Serial	Order	Nature	Conjugacy classes	Generators
1	1176	solvable	1	(2,47,46,33,7,39,38,25,6,31,30,17,5,23,28,9,4,21,20,49,3,13,12,41)(8,24,45,42,43,16,37,34,36,14,35,26,29,48,27,18,22,40,19,10,15,32,11,44)(1,48,16,14,32,40,24)(2,49,18,12,31,36,27)(3,43,20,9,35,39,26)(4,47,17,8,34,37,28)(5,45,15,13,30,42,25)(6,44,21,11,33,38,22)(7,46,19,10,29,41,23)
2	784	solvable	1	(2,24,31,35,20,10,33,43,5,48,13,11,38,34,9,22)(3,32,39,37,28,18,41,8,6,14,21,19,46,42,17,29)(4,40,47,45,30,26,49,15,7,16,23,27,12,44,25,36)(1,2,4,3,6,7,5)(8,9,11,10,13,14,12)(15,16,18,17,20,21,19)(22,23,25,24,27,28,26)(29,30,32,31,34,35,33)(36,37,39,38,41,42,40)(43,44,46,45,48,49,47)
3	48	cyclic	49	(2,40,21,35,30,42,33,15,3,48,23,37,38,44,41,22,4,14,31,45,46,10,49,29,5,16,39,11,12,18,9,36,6,24,47,19,20,26,17,43,7,32,13,27,28,34,25,8)

Proper normal subgroups					
Serial	Order	Nature	Quotient	Subgroups	Generators
1	49	abelian	cyclic	--	(1,24,40,32,14,16,48)(2,27,36,31,12,18,49)(3,26,39,35,9,20,43)(4,28,37,34,8,17,47)(5,25,42,30,13,15,45)(6,22,38,33,11,21,44)(7,23,41,29,10,19,46)(1,47,21,13,31,39,23)(2,43,19,14,34,38,25)(3,46,16,8,33,42,27)(4,44,15,12,35,41,24)(5,49,20,10,32,37,22)(6,45,18,9,29,40,28)(7,48,17,11,30,36,26)
2	98	solvable	cyclic	1	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(1,24,40,32,14,16,48)(2,27,36,31,12,18,49)(3,26,39,35,9,20,43)(4,28,37,34,8,17,47)(5,25,42,30,13,15,45)(6,22,38,33,11,21,44)(7,23,41,29,10,19,46)(1,47,21,13,31,39,23)(2,43,19,14,34,38,25)(3,46,16,8,33,42,27)(4,44,15,12,35,41,24)(5,49,20,10,32,37,22)(6,45,18,9,29,40,28)(7,48,17,11,30,36,26)
					(2,20,5,38)(3,28,6,46)(4,30,7,12)(8,19,29,37)9,31,33,13)(10,48,34,24)(11,22,35,43)(14,42,32,18)(15,27,36,45)(16,44,40,26)(17,39,41,21)

3	196	solvable	cyclic	1,2	(23,25,47,49)(1,49,17,9,33,41,25)(2,47,20,11,29,42,24)(3,44,19,13,32,36,28)(4,43,21,10,30,40,27)(5,48,18,8,35,38,23)(6,46,15,14,31,37,26)(7,45,16,12,34,39,22)
4	392	solvable	cyclic	1,2,3	(2,31,20,33,5,13,38,9)(3,39,28,41,6,21,46,17)(4,47,30,49,7,23,12,25)(8,14,19,42,29,32,37,18)(10,43,48,11,34,22,24,35)(15,16,27,44,36,40,45,26)(1,30,46,38,20,28,12)(2,32,45,41,21,26,8)(3,34,49,40,15,23,11)(4,31,48,42,19,22,9)(5,29,44,39,17,27,14)(6,35,47,36,16,25,10)(7,33,43,37,18,24,13)
5	784	solvable	cyclic	1,2,3,4	(1,27,42,5,6,20,11,16,25,7,48,24,28,35,8,30)(2,41,47,31,36,40,26,14,23,49,38,19,46,45,3,13)(4,34,29,43,10,44,17,32,22,21,18,39,12,37,33,15)(1,35,46,18,49,29,3,24)(2,4,12,38,48,45,40,11)(5,44,28,23,47,6,15,20)(7,41,30,8,43,9,34,42)(10,14,19,22,39,36,26,21)(13,25,31,16,37,17,32,27)
6	147	solvable	cyclic	1	(2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42)(11,27,37)(12,28,38)(13,23,39)(14,24,40)(15,29,43)(16,32,48)(17,33,49)(18,34,44)(19,35,45)(20,30,46)(21,31,47)(1,7,3,2,5,6,4)(8,14,10,9,12,13,11)(15,21,17,16,19,20,18)(22,28,24,23,26,27,25)(29,35,31,30,33,34,32)(36,42,38,37,40,41,39)(43,49,45,44,47,48,46)(1,23,39,31,13,21,47)(2,25,38,34,14,19,43)(3,27,42,33,8,16,46)(4,24,41,35,12,15,44)(5,22,37,32,10,20,49)(6,28,40,29,9,18,45)(7,26,36,30,11,17,48)
7	294	solvable	cyclic	1,2,6	(2,7,6,5,4,3)(8,43,36,29,22,15)(9,49,41,33,25,17)(10,44,42,34,26,18)(11,45,37,35,27,19)(12,46,38,30,28,20)(13,47,39,31,23,21)(14,48,40,32,24,16)(1,6,2,7,4,5,3)(8,13,9,14,11,12,10)(15,20,16,21,18,19,17)(22,27,23,28,25,26,24)(29,34,30,35,32,33,31)(36,41,37,42,39,40,38)(43,48,44,49,46,47,45)(1,21,31,23,47,13,39)(2,19,34,25,43,14,38)(3,16,33,27,46,8,42)(4,15,35,24,44,12,41)(5,20,32,22,49,10,37)(6,18,29,28,45,9,40)(7,17,30,26,48,11,36)
8	588	solvable	cyclic	1,2,3,6,7	(2,30,3,38,4,46,5,12,6,20,7,28)(8,35,15,37,22,45,29,11,36,19,43,27)(9,47,17,13,25,21,33,23,41,31,49,39)(10,16,18,24,26,32,34,40,42,48,44,14)(1,30,46,38,20,28,12)(2,32,45,41,21,26,8)(3,34,49,40,15,23,11)(4,31,48,42,19,22,9)(5,29,44,39,17,27,14)(6,35,47,36,16,25,10)(7,33,43,37,18,24,13)
9	1176	solvable	cyclic	1,2,3,4,6,7,8	(2,47,46,33,7,39,38,25,6,31,30,17,5,23,28,9,4,21,20,49,3,13,12,41)(8,24,45,42,43,16,37,34,36,14,35,26,29,48,27,18,22,40,19,10,15,32,11,44)(1,42,10,44,26,34,18)(2,40,13,46,22,35,17)(3,37,12,48,25,29,21)(4,36,14,45,23,33,20)(5,41,11,43,28,31,16)(6,39,8,49,24,30,19)(7,38,9,47,27,32,15)

Sylow subgroups						
p	Order	Conjug. classes	Nature	Normalizer	Normal closure	generators
2	16	49	cyclic	48	784	(2,24,31,35,20,10,33,43,5,48,13,11,38,34,9,22)(3,32,39,37,28,18,41,8,6,14,21,19,46,42,17,29)(4,40,47,45,30,26,49,15,7,16,23,27,12,44,25,36)
3	3	49	cyclic	48	147	(2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42)(11,27,37)(12,28,38)(13,23,39)(14,24,40)(15,29,43)(16,32,48)(17,33,49)(18,34,44)(19,35,45)(20,30,46)(21,31,47)
7	49	1	abelian	G	49	(1,15,29,22,43,8,36)(2,16,30,23,44,9,37)(3,17,31,24,45,10,38)(4,18,32,25,46,11,39)(5,19,33,26,47,12,40)(6,20,34,27,48,13,41)(7,21,35,28,49,14,42)(1,49,17,9,33,41,25)(2,47,20,11,29,42,24)(3,44,19,13,32,36,28)(4,43,21,10,30,40,27)(5,48,18,8,35,38,23)(6,46,15,14,31,37,26)(7,45,16,12,34,39,22)

24. Let G be a primitive group of degree 49 with 3 generators. We have $|G| = 2352 = 2^4\times3\times7^2$.

Generators of G:	a_1 = (2,33)(3,41)(4,49)(5,9)(6,17)(7,25)(10,48)(11,35)(12,23)(13,38)(14,18)(16,26)(19,37)(20,31)(21,46)(24,34)(27,45)(28,39)(30,47)(32,42)(40,44) (order 2)
	a_2 = (2,21,30,33,3,23,38,41,4,31,46,49,5,39,12,9,6,47,20,17,7,13,28,25)(8,40,35,42,15,48,37,44,22,14,45,10,29,16,11,18,36,24,19,26,43,32,27,34) (order 24)
	a_3 = (1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35) (order 7)

This generating set is not minimal. There is a smaller generating set with 2 elements. G is a solvable group. G acts transitively on the set of 49 elements {1, 2, …, 49}. The center of G is trivial.

The derived subgroup $D = [G, G]$ is a solvable group of order 196, generated by {(2,38,5,20)(3,46,6,28)(4,12,7,30)(8,37,29,19)(9,13,33,31)(10,24,34,48)(11,43,35,22)(14,18,32,42)(15,45,36,27)(16,26,40,44)(17,21,41,39)(23,49,47,25)(1,19,35,27,45,11,37)(2,15,33,28,48,10,39)(3,18,30,22,47,14,41)(4,16,29,26,49,13,38)(5,21,34,24,46,9,36)(6,17,32,23,43,12,42)(7,20,31,25,44,8,40)} and $G/D \cong C_2^2\times C_3$.

Lower central Series				
Level	Order	Nature	Quotient	Successive quotient
1	196	solvable	abelian	$C_2^2\times C_3$
2	98	solvable	nilpotent	C_2
3	49	abelian	nilpotent	C_2

Derived Series				
Level	Order	Nature	Quotient	Successive quotient
1	196	solvable	abelian	$C_2^2\times C_3$
2	49	abelian	nilpotent	C_4
3	1	trivial	solvable	C_7^2

Conjugacy classes of maximal subgroups				
Serial	Order	Nature	Conjugacy classes	Generators
				(8,32)(9,31)(10,35)(11,34)(12,30)(13,33)(14,29)(15,40)(16,36)(17,39)(18,37)(19,42)(20,38)(21,41)

1	1176	solvable	1	(22,48)(23,49)(24,43)(25,47)(26,45)(27,44)(28,46)(1,33,42,11,26,6,18,38,34,22,10,21)(2,7,37,40,23,25,16,20,30,31,9,8)(3,24,36,15,28,35,19,12,32,4,13,41)(5,43,39,49,27,47,17,46,29,48,14,45)
2	1176	solvable	1	(2,33)(3,41)(4,49)(5,9)(6,17)(7,25)(10,48)(11,35)(12,23)(13,38)(14,18)(16,26)(19,37)(20,31)(21,46)(24,34)(27,45)(28,39)(30,47)(32,42)(40,44)(1,14,4,17,2,27,7,29,3,39,6,44)(8,22,18,32,23,37,35,49,38,10,48,20)(9,19,21,26,24,33,34,40,36,47,46,12)(11,42,16,45,28,13,31,15,41,25,43,30)
3	1176	solvable	1	(1,6,2,7,4,5,3)(8,13,9,14,11,12,10)(15,20,16,21,18,19,17)(22,27,23,28,25,26,24)(29,34,30,35,32,33,31)(36,41,37,42,39,40,38)(43,48,44,49,46,47,45)(2,49,28,31,7,41,20,23,6,33,12,21,5,25,46,13,4,17,38,47,3,9,30,39)(8,10,27,14,43,44,19,48,36,42,11,40,29,34,45,32,22,26,37,24,15,18,35,16)
4	784	solvable	1	(2,33)(3,41)(4,49)(5,9)(6,17)(7,25)(10,48)(11,35)(12,23)(13,38)(14,18)(16,26)(19,37)(20,31)(21,46)(24,34)(27,45)(28,39)(30,47)(32,42)(40,44)(1,19,6,17,2,15,7,20,4,16,5,21,3,18)(8,23,13,28,9,25,14,26,11,24,12,22,10,27)(29,41,34,37,30,42,35,39,32,40,33,38,31,36)(43,46,48,47,44,45,49)
5	48	nilpotent	49	(8,32)(9,31)(10,35)(11,34)(12,30)(13,33)(14,29)(15,40)(16,36)(17,39)(18,37)(19,42)(20,38)(21,41)(22,48)(23,49)(24,43)(25,47)(26,45)(27,44)(28,46)(2,49,6,33,4,17)(3,9,7,41,5,25)(8,22,36)(10,16,42,48,26,32)(11,45,37,35,27,19)(12,39,38,23,28,13)(14,34,40,18,24,44)(15,29,43)

Proper normal subgroups					
Serial	Order	Nature	Quotient	Subgroups	Generators
1	49	abelian	nilpotent	--	(1,3,5,4,7,2,6)(8,10,12,11,14,9,13)(15,17,19,18,21,16,20)(22,24,26,25,28,23,27)(29,31,33,32,35,30,34)(36,38,40,39,42,37,41)(43,45,47,46,49,44,48)(1,15,29,22,43,8,36)(2,16,30,23,44,9,37)(3,17,31,24,45,10,38)(4,18,32,25,46,11,39)(5,19,33,26,47,12,40)(6,20,34,27,48,13,41)(7,21,35,28,49,14,42)
2	147	solvable	nilpotent	1	(2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42)(11,27,37)(12,28,38)(13,23,39)(14,24,40)(15,29,43)(16,32,48)(17,33,49)(18,34,44)(19,35,45)(20,30,46)(21,31,47)(1,3,5,4,7,2,6)(8,10,12,11,14,9,13)(15,17,19,18,21,16,20)(22,24,26,25,28,23,27)(29,31,33,32,35,30,34)(36,38,40,39,42,37,41)(43,45,47,46,49,44,48)(1,15,29,22,43,8,36)(2,16,30,23,44,9,37)(3,17,31,24,45,10,38)(4,18,32,25,46,11,39)(5,19,33,26,47,12,40)(6,20,34,27,48,13,41)(7,21,35,28,49,14,42)
					(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(1,15,

3	98	solvable	nilpotent	1	29,22,43,8,36)(2,16,30,23,44,9,37)(3,17,31,24,45,10,38)(4,18,32,25,46,11,39)(5,19,33,26,47,12,40)(6,20,34,27,48,13,41)(7,21,35,28,49,14,42)(1,3,5,4,7,2,6)(8,10,12,11,14,9,13)(15,17,19,18,21,16,20)(22,24,26,25,28,23,27)(29,31,33,32,35,30,34)(36,38,40,39,42,37,41)(43,45,47,46,49,44,48)
4	294	solvable	nilpotent	1,2,3	(1,15,29,22,43,8,36)(2,16,30,23,44,9,37)(3,17,31,24,45,10,38)(4,18,32,25,46,11,39)(5,19,33,26,47,12,40)(6,20,34,27,48,13,41)(7,21,35,28,49,14,42)(1,3,5,4,7,2,6)(8,10,12,11,14,9,13)(15,17,19,18,21,16,20)(22,24,26,25,28,23,27)(29,31,33,32,35,30,34)(36,38,40,39,42,37,41)(43,45,47,46,49,44,48)(2,7,6,5,4,3)(8,43,36,29,22,15)(9,49,41,33,25,17)(10,44,42,34,26,18)(11,45,37,35,27,19)(12,46,38,30,28,20)(13,47,39,31,23,21)(14,48,40,32,24,16)
5	196	solvable	abelian	1,3	(2,20,5,38)(3,28,6,46)(4,30,7,12)(8,19,29,37)(9,31,33,13)(10,48,34,24)(11,22,35,43)(14,42,32,18)(15,27,36,45)(16,44,40,26)(17,39,41,21)(23,25,47,49)(1,12,28,20,38,46,30)(2,8,26,21,41,45,32)(3,11,23,15,40,49,34)(4,9,22,19,42,48,31)(5,14,27,17,39,44,29)(6,10,25,16,36,47,35)(7,13,24,18,37,43,33)
6	588	solvable	abelian	1,2,3,4,5	(1,12,28,20,38,46,30)(2,8,26,21,41,45,32)(3,11,23,15,40,49,34)(4,9,22,19,42,48,31)(5,14,27,17,39,44,29)(6,10,25,16,36,47,35)(7,13,24,18,37,43,33)(2,30,3,38,4,46,5,12,6,20,7,28)(8,35,15,37,22,45,29,11,36,19,43,27)(9,47,17,13,25,21,33,23,41,31,49,39)(10,16,18,24,26,32,34,40,42,48,44,14)
7	392	solvable	cyclic	1,3,5	(2,31,20,33,5,13,38,9)(3,39,28,41,6,21,46,17)(4,47,30,49,7,23,12,25)(8,14,19,42,29,32,37,18)(10,43,48,11,34,22,24,35)(15,16,27,44,36,40,45,26)(1,12,28,20,38,46,30)(2,8,26,21,41,45,32)(3,11,23,15,40,49,34)(4,9,22,19,42,48,31)(5,14,27,17,39,44,29)(6,10,25,16,36,47,35)(7,13,24,18,37,43,33)
8	392	solvable	cyclic	1,3,5	(2,20,5,38)(3,28,6,46)(4,30,7,12)(8,19,29,37)(9,31,33,13)(10,48,34,24)(11,22,35,43)(14,42,32,18)(15,27,36,45)(16,44,40,26)(17,39,41,21)(23,25,47,49)(1,12)(2,8)(3,11)(4,9)(5,14)(6,10)(7,13)(15,49)(16,47)(17,44)(18,43)(19,48)(20,46)(21,45)(22,31)(23,34)(24,33)(25,35)(26,32)(27,29)(28,30)
9	1176	solvable	cyclic	1,2,3,4,5,6,7	(1,12,28,20,38,46,30)(2,8,26,21,41,45,32)(3,11,23,15,40,49,34)(4,9,22,19,42,48,31)(5,14,27,17,39,44,29)(6,10,25,16,36,47,35)(7,13,24,18,37,43,33)(2,47,46,33,7,39,38,

					25,6,31,30,17,5,23,28,9,4,21,20,49,3,13,12,41)(8,24,45,42,43,16,37,34,36,14,35,26,29,48,27,18,22,40,19,10,15,32,11,44)
10	1176	solvable	cyclic	1,2,3,4,5,6,8	(2,20,5,38)(3,28,6,46)(4,30,7,12)(8,19,29,37)(9,31,33,13)(10,48,34,24)(11,22,35,43)(14,42,32,18)(15,27,36,45)(16,44,40,26)(17,39,41,21)(23,25,47,49)(1,12,30,20,28,38)(2,9,35,21,22,36)(3,14,33,15,27,37)(4,10,32,19,25,41)(5,13,34,17,24,40)(6,8,31,16,26,42)(7,11,29,18,23,39)(43,49,44)(45,48,47)
11	392	solvable	cyclic	1,3	(2,33)(3,41)(4,49)(5,9)(6,17)(7,25)(10,48)(11,35)(12,23)(13,38)(14,18)(16,26)(19,37)(20,31)(21,46)(24,34)(27,45)(28,39)(30,47)(32,42)(40,44)(1,11,32,6,48,33,40,44,16,38,24,21,14,22)(2,26,31,9,49,3,36,35,18,43,27,39,12,20)(4,17,34,28,47,8,37)(5,7,30,29,45,46,42,41,15,19,25,23,13,10)
12	1176	solvable	cyclic	--	(2,13)(3,21)(4,23)(5,31)(6,39)(7,47)(8,19)(9,38)(10,34)(11,43)(12,25)(15,27)(17,46)(18,42)(20,33)(22,35)(26,44)(28,41)(29,37)(30,49)(36,45)(1,40,2,27,7,10)(3,43,5,30,6,21)(8,33,37,20,28,45)(9,41,42,24,22,12)(11,25,39)(13,49,38,29,26,16)(14,17,36,47,23,34)(15,19,44,48,35,31)(18,32,46)
13	784	solvable	cyclic	--	(2,13)(3,21)(4,23)(5,31)(6,39)(7,47)(8,19)(9,38)(10,34)(11,43)(12,25)(15,27)(17,46)(18,42)(20,33)(22,35)(26,44)(28,41)(29,37)(30,49)(36,45)(1,10,2,13,4,14,3,12,6,8,7,9,5,11)(15,46,16,45,18,48,17,49,20,47,21,43,19,44)(22,29,23,30,25,32,24,31,27,34,28,35,26,33)(36,41,37,42,39,40,38)

Sylow subgroups						
p	Order	Conjug. classes	Nature	Normalizer	Normal closure	Generators
2	16	49	nilpotent	48	784	(2,33)(3,41)(4,49)(5,9)(6,17)(7,25)(10,48)(11,35)(12,23)(13,38)(14,18)(16,26)(19,37)(20,31)(21,46)(24,34)(27,45)(28,39)(30,47)(32,42)(40,44)(8,32)(9,31)(10,35)(11,34)(12,30)(13,33)(14,29)(15,40)(16,36)(17,39)(18,37)(19,42)(20,38)(21,41)(22,48)(23,49)(24,43)(25,47)(26,45)(27,44)(28,46)
3	3	49	cyclic	48	147	(2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42)(11,27,37)(12,28,38)(13,23,39)(14,24,40)(15,29,43)(16,32,48)(17,33,49)(18,34,44)(19,35,45)(20,30,46)(21,31,47)
7	49	1	abelian	*G*	49	(1,15,29,22,43,8,36)(2,16,30,23,44,9,37)(3,17,31,24,45,10,38)(4,18,32,25,46,11,39)(5,19,33,26,47,12,40)(6,20,34,27,48,13,41)(7,21,35,28,49,14,42)(1,38,12,46,28,30,20)(2,41,8,45,26,32,21)(3,40,11,49,23,34,15)(4,42,9,48,22,31,19)(5,39,14,44,27,29,17)(6,36,10,47,25,35,16)(7,37,13,43,24,33,18)

25. Let G be a primitive group of degree 49 with 5 generators. We have $|G| = 2352 = 2^4 \times 3 \times 7^2$.

Generators of G:	a_1 = (2,38,5,20)(3,46,6,28)(4,12,7,30)(8,37,29,19)(9,13,33,31)(10,24,34,48)(11,43,35,22)(14,18,32,42)(15,45,36,27)(16,26,40,44)(17,21,41,39)(23,49,47,25) (order 4)
	a_2 = (2,15,40)(3,22,48)(4,29,14)(5,36,16)(6,43,24)(7,8,32)(9,25,41)(10,11,46)(12,18,19)(13,39,23)(17,33,49)(20,26,27)(21,47,31)(28,34,35)(30,42,37)(38,44,45) (order 3)
	a_3 = (2,8,5,29)(3,15,6,36)(4,22,7,43)(9,12,33,30)(10,19,34,37)(11,26,35,44)(13,40,31,16)(14,47,32,23)(17,20,41,38)(18,27,42,45)(21,48,39,24)(25,28,49,46) (order 4)
	a_4 = (2,21,5,39)(3,23,6,47)(4,31,7,13)(8,24,29,48)(9,44,33,26)(10,41,34,17)(11,12,35,30)(14,15,32,36)(16,22,40,43)(18,49,42,25)(19,20,37,38)(27,28,45,46) (order 4)
	a_5 = (1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35) (order 7)

This generating set is not minimal. There is a smaller generating set with 2 elements. G is a solvable group. G acts transitively on the set of 49 elements {1, 2, …, 49}. The center of G is trivial.

The derived subgroup $D = [G, G]$ is a solvable group of order 1176, generated by {(2,8,5,29)(3,15,6,36)(4,22,7,43)(9,12,33,30)(10,19,34,37)(11,26,35,44)(13,40,31,16)(14,47,32,23)(17,20,41,38)(18,27,42,45)(21,48,39,24)(25,28,49,46)(1,14,13)(2,25,5)(3,38,45)(4,19,30)(6,43,42)(7,34,15)(8,21,48)(9,39,40)(10,31,24)(11,47,16)(12,23,32)(18,26,37)(20,36,28)(22,49,27)(29,35,41)(33,44,46)} and $G/D \cong C_2$.

Lower central Series				
Level	Order	Nature	Quotient	Successive quotient
1	1176	solvable	cyclic	C_2

Derived Series				
Level	Order	Nature	Quotient	Successive quotient
1	1176	solvable	cyclic	C_2
2	392	solvable	dihedral	C_3
3	98	solvable	solvable	C_2^2
4	49	abelian	solvable	C_2
5	1	trivial	solvable	C_7^2

Conjugacy classes of maximal subgroups				
Serial	Order	Nature	Conjugacy classes	Generators
1	1176	solvable	1	(2,21,5,39)(3,23,6,47)(4,31,7,13)(8,24,29,48)(9,44,33,26)(10,41,34,17)(11,12,35,30)(14,15,32,36)(16,22,40,43)(18,49,42,25)(19,20,37,38)(27,28,45,46)(1,22,21)(2,25,18)(3,26,17)(4,27,19)(5,28,16)(6,23,15)(7,24,20)(8,33,47)(9,30,45)(10,35,48)(11,31,43)(12,34,46)(13,29,44)(14,32,49)(36,38,39)(37,42,40)
2	784	solvable	3	(2,34,5,10)(3,42,6,18)(4,44,7,26)(8,17,29,41)(9,22,33,43)(11,13,35,31)(12,40,30,16)(14,46,32,28)(15,25,36,49)(19,21,37,39)(20,48,38,24)(23,45,47,27)(1,46,15,49)(2,39,19,35)(3,25,20,14)(4,18,21,7)(5,32,16,42)(6,11,17,28)(8,45,22,48)(9,38,26,34)(10,24,27,13)(12,31,23,41)(29,44,36,47)(30,37,40,33)

3	588	solvable	4	(2,6,4)(3,7,5)(8,23,40)(9,28,37)(10,26,42)(11,25,38)(12,27,41)(13,24,36)(14,22,39)(15,31,48)(16,29,47)(17,30,45)(18,34,44)(19,33,46)(20,35,49)(21,32,43)(1,43,30,9)(2,41,29,27)(3,28,35,38)(4,16,33,15)(5,31,32,7)(6,11,34,47)(8,10,44,49)(12,26,46,39)(13,42,48,24)(14,18,45,19)(17,36,21,23)(22,40,37,25)
4	48	solvable	49	(2,47,22)(3,13,29)(4,21,36)(5,23,43)(6,31,8)(7,39,15)(9,45,10)(11,18,17)(12,28,38)(14,40,24)(16,48,32)(19,26,25)(20,30,46)(27,34,33)(35,42,41)(37,44,49)(2,41,48,37,5,17,24,19)(3,49,14,45,6,25,32,27)(4,9,16,11,7,33,40,35)(8,34,21,20,29,10,39,38)(12,22,44,31,30,43,26,13

Maximal normal subgroups				
Serial	Order	Nature	Quotient	Generators
1	1176	solvable	cyclic	(2,21,5,39)(3,23,6,47)(4,31,7,13)(8,24,29,48)(9,44,33,26)(10,41,34,17)(11,12,35,30)(14,15,32,36)(16,22,40,43)(18,49,42,25)(19,20,37,38)(27,28,45,46)(1,22,21)(2,25,18)(3,26,17)(4,27,19)(5,28,16)(6,23,15)(7,24,20)(8,33,47)(9,30,45)(10,35,48)(11,31,43)

Sylow subgroups						
p	Order	Conjug. classes	Nature	Normalizer	Normal closure	Generators
2	16	147	nilpotent	16	*G*	(2,29,5,8)(3,36,6,15)(4,43,7,22)(9,30,33,12)(10,37,34,19)(11,44,35,26)(13,16,31,40)(14,23,32,47)(17,38,41,20)(18,45,42,27)(21,24,39,48)(25,46,49,28)(2,34,5,10)(3,42,6,18)(4,44,7,26)(8,17,29,41)(9,22,33,43)(11,13,35,31)(12,40,30,16)(14,46,32,28)(15,25,36,49)(19,21,37,39)(20,48,38,24)(23,45,47,27)
3	3	196	cyclic	12	1176	(2,4,6)(3,5,7)(8,40,23)(9,37,28)(10,42,26)(11,38,25)(12,41,27)(13,36,24)(14,39,22)(15,48,31)(16,47,29)(17,45,30)(18,44,34)(19,46,33)(20,49,35)(21,43,32)
7	49	1	abelian	*G*	49	(1,6,2,7,4,5,3)(8,13,9,14,11,12,10)(15,20,16,21,18,19,17)(22,27,23,28,25,26,24)(29,34,30,35,32,33,31)(36,41,37,42,39,40,38)(43,48,44,49,46,47,45)(1,15,29,22,43,8,36)(2,16,30,23,44,9,37)(3,17,31,24,45,10,38)(4,18,32,25,46,11,39)(5,19,33,26,47,12,40)(6,20,34,27,48,13,41)(7,21,35,28,49,14,42)

26. Let G be a primitive group of degree 49 with 6 generators. We have $|G| = 3528 = 2^3\times3^2\times7^2$.

	a_1 = (2,8)(3,15)(4,22)(5,29)(6,36)(7,43)(10,16)(11,23)(12,30)(13,37)(14,44)(18,24)(19,31)(20,38)(21,45)(26,32)(27,39)(28,46)(34,40)(35,47)(42,48) (order 2)
	a_2 = (8,29)(9,30)(10,31)(11,32)(12,33)(13,34)(14,35)(15,36)(16,37)(17,38)(18,39)(19,40)(20,41)(21,42)(22,43)(23,44)(24,45)(25,46)(26,47)(27,48)(28,49) (order 2)
	a_3 = (2,5)(3,6)(4,7)(9,12)(10,13)(11,14)(16,19)(17,20)(18,21)(23,26)(24,27)(25,28)(30,33)(31,34)(32,35)(37,40)(38,41)(39,42)(44,47)(45,48)(46,49) (order 2)

Generators of G:	a_4 = (2,6,4)(3,7,5)(8,22,36)(9,27,39)(10,28,40)(11,23,41)(12,24,42)(13,25,37)(14,26,38)(15,29,43)(16,34,46)(17,35,47)(18,30,48)(19,31,49)(20,32,44)(21,33,45) (order 3)
	a_5 = (2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42)(11,27,37)(12,28,38)(13,23,39)(14,24,40)(15,29,43)(16,32,48)(17,33,49)(18,34,44)(19,35,45)(20,30,46)(21,31,47) (order 3)
	a_6 = (1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35) (order 7)

This generating set is not minimal. There is a smaller generating set with 2 elements. G is a solvable group. G acts transitively on the set of 49 elements {1, 2, …, 49}. The center of G is trivial.

The derived subgroup D = [G, G] is a solvable group of order 294, generated by {(1,42,10,44,26,34,18)(2,40,13,46,22,35,17)(3,37,12,48,25,29,21)(4,36,14,45,23,33,20)(5,41,11,43,28,31,16)(6,39,8,49,24,30,19)(7,38,9,47,27,32,15)(2,7,6,5,4,3)(8,15,22,29,36,43)(9,21,27,33,39,45)(10,16,28,34,40,46)(11,17,23,35,41,47)(12,18,24,30,42,48)(13,19,25,31,37,49)(14,20,26,32,38,44)} and $G/D \cong C_2^2 \times C_3$.

Lower central Series				
Level	Order	Nature	Quotient	Successive quotient
1	294	solvable	abelian	$C_2^2 \times C_3$
2	147	solvable	nilpotent	C_2

Derived Series				
Level	Order	Nature	Quotient	Successive quotient
1	294	solvable	abelian	$C_2^2 \times C_3$
2	49	abelian	solvable	$C_2 \times C_3$
3	1	trivial	solvable	C_7^2

Conjugacy classes of maximal subgroups				
Serial	Order	Nature	Conjugacy classes	Generators
1	1764	solvable	1	(2,4,6)(3,5,7)(8,43,36,29,22,15)(9,46,41,30,25,20)(10,47,42,31,26,21)(11,48,37,32,27,16)(12,49,38,33,28,17)(13,44,39,34,23,18)(14,45,40,35,24,19)(1,30,48,40,21,25,8,2,34,47,42,18,22,9,6,33,49,39,15,23,13,5,35,46,36,16,27,12,7,32,43,37,20,26,14,4,29,44,41,19,28,11)(3,31,45,38,17,24,10)
2	1764	solvable	1	(2,8)(3,15)(4,22)(5,29)(6,36)(7,43)(10,16)(11,23)(12,30)(13,37)(14,44)(18,24)(19,31)(20,38)(21,45)(26,32)(27,39)(28,46)(34,40)(35,47)(42,48)(1,21,5,18,6,17)(2,16)(3,15,7,19,4,20)(8,28,12,25,13,24)(9,23)(10,22,14,26,11,27)(29,42,33,39,34,38)(30,37)(31,36,35,40,32,41)(43,49,47,46,48,45)
3	1764	solvable	1	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)(1,19,8,20,36,21,29,16,43,17,22,18)(2,47,10,27,39,7,33,9,48,38,28,32)(3,26,11,6,40,14,34,37,49,31,23,46)(4,5,12,13,41,42,35,30,44,45,24,25)
				(2,6,4)(3,7,5)(8,43,36,29,22,15)(9,48,39,30,27,18)(10,49,40,31,28,19)(11,44,41,32,23,20)(12,45,42,

4	1176	solvable	1	33,24,21)(13,46,37,34,25,16)(14,47,38,35,26,17)(1,13,49,32,17,36,9,28,33,3,41,44,25,19)(2,27,47,4,20,43,11,21,29,10,42,30,24,40)(5,6,48,46,18,15,8,14,35,31,38,37,23,26)(7,34,45,39,16,22,12)
5	1176	solvable	3	(8,29)(9,30)(10,31)(11,32)(12,33)(13,34)(14,35)(15,36)(16,37)(17,38)(18,39)(19,40)(20,41)(21,42)(22,43)(23,44)(24,45)(25,46)(26,47)(27,48)(28,49)(1,29,35,28,27,6)(2,8,32,21,26,41)(3,43,31,49,24,48)(4,15,33,42,23,13)(5,36,30,14,25,20)(7,22,34)(9,11,18,19,40,37)(10,46,17,47,38,44)(12,39,16)
6	72	solvable	49	(2,6,4)(3,7,5)(8,43,36,29,22,15)(9,48,39,30,27,18)(10,49,40,31,28,19)(11,44,41,32,23,20)(12,45,42,33,24,21)(13,46,37,34,25,16)(14,47,38,35,26,17)(2,22,6,8,4,36)(3,29,7,15,5,43)(9,25,41)(10,32,42,16,26,48)(11,39,37,23,27,13)(12,46,38,30,28,20)(14,18,40,44,24)

Proper normal subgroups					
Serial	Order	Nature	Quotient	Subgroups	Generators
1	49	abelian	solvable	--	(1,5,7,6,3,4,2)(8,12,14,13,10,11,9)(15,19,21,20,17,18,16)(22,26,28,27,24,25,23)(29,33,35,34,31,32,30)(36,40,42,41,38,39,37)(43,47,49,48,45,46,44)(1,43,15,8,29,36,22)(2,44,16,9,30,37,23)(3,45,17,10,31,38,24)(4,46,18,11,32,39,25)(5,47,19,12,33,40,26)(6,48,20,13,34,41,27)(7,49,21,14,35,42,28)
2	147	solvable	solvable	1	(2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42)(11,27,37)(12,28,38)(13,23,39)(14,24,40)(15,29,43)(16,32,48)(17,33,49)(18,34,44)(19,35,45)(20,30,46)(21,31,47)(1,5,7,6,3,4,2)(8,12,14,13,10,11,9)(15,19,21,20,17,18,16)(22,26,28,27,24,25,23)(29,33,35,34,31,32,30)(36,40,42,41,38,39,37)(43,47,49,48,45,46,44)(1,43,15,8,29,36,22)(2,44,16,9,30,37,23)(3,45,17,10,31,38,24)(4,46,18,11,32,39,25)(5,47,19,12,33,40,26)(6,48,20,13,34,41,27)(7,49,21,14,35,42,28)
3	98	solvable	solvable	1	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(1,5,7,6,3,4,2)(8,12,14,13,10,11,9)(15,19,21,20,17,18,16)(22,26,28,27,24,25,23)(29,33,35,34,31,32,30)(36,40,42,41,38,39,37)(43,47,49,48,45,46,44)(1,43,15,8,29,36,22)(2,44,16,9,30,37,23)(3,45,17,10,31,38,24)(4,46,18,11,32,39,25)(5,47,19,12,33,40,26)(6,48,20,13,34,41,27)(7,49,21,14,35,42,28)

4	294	solvable	dihedral	1,2,3	(1,11,27,19,37,45,35)(2,10,28,15,39,48,33)(3,14,22,18,41,47,30)(4,13,26,16,38,49,29)(5,9,24,21,36,46,34)(6,12,23,17,42,43,32)(7,8,25,20,40,44,31)(1,36,15,43,8,22)(2,42,20,47,11,24)(3,37,21,48,12,25)(4,38,16,49,13,26)(5,39,17,44,14,27)(6,40,18,45,9,28)(7,41,19,46,10,23)(30,35,34,33,32,31)(1,40,32)(2,38,34)(3,41,30)(4,36,33)(5,39,29)(6,37,31)(7,42,35)(8,12,11)(9,10,13)(15,26,46)(16,24,48)(17,27,44)(18,22,47)(19,25,43)(20,23,45)(21,28,49)
5	196	solvable	solvable	1,3	(1,15)(2,19)(3,20)(4,21)(5,16)(6,17)(7,18)(8,22)(9,26)(10,27)(11,28)(12,23)(13,24)(14,25)(29,36)(30,40)(31,41)(32,42)(33,37)(34,38)(35,39)(44,47)(45,48)(46,49)(1,26,36,33,8,19,43,5,22,40,29,12,15,47)(2,28,37,35,9,21,44,7,23,42,30,14,16,49)(3,24,38,31,10,17,45)(4,27,39,34,11,20,46,6,25,41,32,13,18,48)
6	588	solvable	dihedral	1,2,3,4,5	(1,3)(2,4)(5,6)(8,10)(9,11)(12,13)(15,17)(16,18)(19,20)(22,24)(23,25)(26,27)(29,31)(30,32)(33,34)(36,38)(37,39)(40,41)(43,45)(44,46)(47,48)(1,11,44,7,10,48)(2,14,45,6,8,46)(3,13,43,4,9,49)(5,12,47)(15,32,37,21,31,41)(16,35,38,20,29,39)(17,34,36,18,30,42)(19,33,40)(22,25,23,28,24,27),(1,31,21,13,23,47)(2,33,15,10,28,48)(3,35,20,9,26,43)(4,32,18,11,25,46)(5,29,17,14,27,44)(6,30,19,8,24,49)(7,34,16,12,22,45)(36,38,42,41,37,40)
7	147	solvable	nilpotent	1	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)(1,47,21,13,31,39,23)(2,43,19,14,34,38,25)(3,46,16,8,33,42,27)(4,44,15,12,35,41,24)(5,49,20,10,32,37,22)(6,45,18,9,29,40,28)(7,48,17,11,30,36,26)
8	441	solvable	nilpotent	1,2,7	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)(1,47,42,6,45,39,2,43,40,7,48,38,4,44,36,5,49,41,3,46,37)(8,26,35,13,24,32,9,22,33,14,27,31,11,23,29,12,28,34,10,25,30)(15,19,21,20,17,18,16)
					(2,7,6,5,4,3)(8,15,22,29,36,43)(9,21,27,33,39,45)(10,16,28,34,40,46)(11,17,23,35,41,47)(12,18,24,30,42,48)(13,19,25,

9	294	solvable	abelian	1,3,7	31,37,49)(14,20,26,32,38,44)(1,47,21, 13,31,39,23)(2,43,19,14,34,38,25)(3,46, 16,8,33,42,27)(4,44,15,12,35,41,24)(5, 49,20,10,32,37,22)(6,45,18,9,29,40,28) (7,48,17,11,30,36,26)
10	882	solvable	abelian	1,2,3,4,7,8,9	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40, 28)(11,41,23)(12,42,24)(13,37,25)(14, 38,26)(15,43,29)(16,46,34)(17,47,35) (18,48,30)(19,49,31)(20,44,32)(21,45, 33)(1,11,36,32,43,25)(2,9,37,30,44,23) (3,12,38,33,45,26)(4,8,39,29,46,22)(5, 10,40,31,47,24)(6,14,41,35,48,28)(7, 13,42,34,49,27)(15,18)(17,19)(20,21)
11	588	solvable	cyclic	1,3,7,9	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40, 28)(11,41,23)(12,42,24)(13,37,25)(14, 38,26)(15,43,29)(16,46,34)(17,47,35) (18,48,30)(19,49,31)(20,44,32)(21,45, 33)(1,36,37,2)(3,29,42,44)(4,15,40,9) (5,8,39,16)(6,22,41,23)(7,43,38,30) (10,32,21,47)(11,18,19,12)(13,25, 20,26)(14,46,17,33)(24,34,28,48) (31,35,49,45)
12	588	solvable	cyclic	1,3,5,7,9	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11, 35)(12,30)(13,31)(14,32)(15,36)(16, 40)(17,41)(18,42)(19,37)(20,38)(21, 39)(22,43)(23,47)(24,48)(25,49)(26, 44)(27,45)(28,46)(1,47,34,22,40,20) (2,44,30,23,37,16)(3,49,32,24,42,18) (4,45,35,25,38,21)(5,48,29,26,41,15) (6,43,33,27,36,19)(7,46,31,28,39,17) (8,12,13)(10,14,11)
13	1764	solvable	cyclic	1,2,3,4,7,8,9,10,11	(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40, 28)(11,41,23)(12,42,24)(13,37,25)(14, 38,26)(15,43,29)(16,46,34)(17,47,35) (18,48,30)(19,49,31)(20,44,32)(21,45, 33)(1,36,39,25,23,16,21,35,31,10,13, 6)(2,15,42,32,24,9,20,7,29,38,11,27) (3,8,41,4,22,37,18,28,30,17,14,34) (5,43,40,46,26,44,19,49,33,45,12,48)
14	1764	solvable	cyclic	1,2,3,4,5,6,7,8,9,10,12	(2,7,6,5,4,3)(8,15,22,29,36,43)(9,21,27, 33,39,45)(10,16,28,34,40,46)(11,17,23, 35,41,47)(12,18,24,30,42,48)(13,19,25, 31,37,49)(14,20,26,32,38,44)(1,39,27, 19,30,10,7,36,25,20,33,9,3,42,22,18,34, 12,2,38,28,15,32,13,5,37,24,21,29,11,6, 40,23,17,35,8,4,41,26,16,31,14)(43,46, 48,47,44,45,49)
15	588	solvable	cyclic	1,3,7,9	(2,29)(3,36)(4,43)(5,8)(6,15)(7,22)(9, 33)(10,40)(11,47)(13,19)(14,26)(16, 34)(17,41)(18,48)(21,27)(23,35)(24, 42)(25,49)(31,37)(32,44)(39,45)(1,15, 21,35,31,24,23,44,47,12,13,41,39,4) (2,43,19,14,34,38,25)(3,22,16,49,33, 10,27,37,46,5,8,20,42,32)(6,36,18,7,

					29,17,28,30,45,26,9,48,40,11)
16	1764	solvable	cyclic	1,2,3,4,7,8,9,10	(2,29)(3,36)(4,43)(5,8)(6,15)(7,22)(9,33)(10,40)(11,47)(13,19)(14,26)(16,34)(17,41)(18,48)(21,27)(23,35)(24,42)(25,49)(31,37)(32,44)(39,45)(1,29,31,24,23,2)(3,22,30)(4,8,34,17,28,37)(5,43,33,45,26,44)(6,15,35,38,25,9)(7,36,32,10,27,16)(11,13,20,21,42,39)(12,48,19,49,40,46)(14,41,18)
17	1176	solvable	cyclic	--	(2,6,4)(3,7,5)(8,43,36,29,22,15)(9,48,39,30,27,18)(10,49,40,31,28,19)(11,44,41,32,23,20)(12,45,42,33,24,21)(13,46,37,34,25,16)(14,47,38,35,26,17)(1,34,28,11,38,15,30,49,12,3,20,23,46,40)(2,48,26,4,41,22,32,42,8,31,21,9,45,19)(5,6,27,25,39,36,29,35,14,10,17,16,44,47)(7,13,24,18,37,43,33)

Sylow subgroups						
p	Order	Conjug. classes	Nature	Normalizer	Normal closure	Generators
2	8	147	nilpotent	24	1176	(2,8)(3,15)(4,22)(5,29)(6,36)(7,43)(10,16)(11,23)(12,30)(13,37)(14,44)(18,24)(19,31)(20,38)(21,45)(26,32)(27,39)(28,46)(34,40)(35,47)(42,48)(8,29)(9,30)(10,31)(11,32)(12,33)(13,34)(14,35)(15,36)(16,37)(17,38)(18,39)(19,40)(20,41)(21,42)(22,43)(23,44)(24,45)(25,46)(26,47)(27,48)(28,49)
3	9	49	abelian	72	441	(2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42)(11,27,37)(12,28,38)(13,23,39)(14,24,40)(15,29,43)(16,32,48)(17,33,49)(18,34,44)(19,35,45)(20,30,46)(21,31,47)(2,4,6)(3,5,7)(8,36,22)(9,39,27)(10,40,28)(11,41,23)(12,42,24)(13,37,25)(14,38,26)(15,43,29)(16,46,34)(17,47,35)(18,48,30)(19,49,31)(20,44,32)(21,45,33)
7	49	1	abelian	*G*	49	(1,22,36,29,8,15,43)(2,23,37,30,9,16,44)(3,24,38,31,10,17,45)(4,25,39,32,11,18,46)(5,26,40,33,12,19,47)(6,27,41,34,13,20,48)(7,28,42,35,14,21,49)(1,32,48,40,16,24,14)(2,31,49,36,18,27,12)(3,35,43,39,20,26,9)(4,34,47,37,17,28,8)(5,30,45,42,15,25,13)(6,33,44,38,21,22,11)(7,29,46,41,19,23,10)

27. Let G be a primitive group of degree 49 with 5 generators. We have $|G| = 3528 = 2^3\times3^2\times7^2$.

	a_1 = (2,15,40)(3,22,48)(4,29,14)(5,36,16)(6,43,24)(7,8,32)(9,25,41)(10,11,46)(12,18,19)(13,39,23)(17,33,49)(20,26,27)(21,47,31)(28,34,35)(30,42,37)(38,44,45) (order 3)
	a_2 = (2,8,5,29)(3,15,6,36)(4,22,7,43)(9,12,33,30)(10,19,34,37)(11,26,35,44)(13,40,31,16)(14,47,32,23)(17,20,41,38)(18,27,42,45)(21,48,39,24)(25,28,49,46) (order 4)
Generators of *G*:	a_3 = (2,21,5,39)(3,23,6,47)(4,31,7,13)(8,24,29,48)(9,44,33,26)(10,41,34,17)(11,12,35,30)(14,15,32,36)(16,22,40,43)(18,49,42,25)(19,20,37,38)(27,28,45,46) (order 4)

	a_4 = (2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42)(11,27,37)(12,28,38)(13,23,39)(14,24,40)(15,29,43)(16,32,48)(17,33,49)(18,34,44)(19,35,45)(20,30,46)(21,31,47) (order 3)
	a_5 = (1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35) (order 7)

This generating set is not minimal. There is a smaller generating set with 2 elements. G is a solvable group. G acts transitively on the set of 49 elements {1, 2, …, 49}. The center of G is trivial.

The derived subgroup D = [G, G] is a solvable group of order 392, generated by {(2,24,5,48)(3,32,6,14)(4,40,7,16)(8,39,29,21)(9,35,33,11)(10,20,34,38)(12,26,30,44)(13,43,31,22)(15,47,36,23)(17,37,41,19)(18,28,42,46)(25,45,49,27)(1,39,37,5)(2,13,36,20)(3,23,42,22)(4,47,40,32)(6,31,41,49)(7,21,38,10)(8,26,16,25)(9,30,15,43)(11,17,19,14)(12,48,18,34)(24,46,28,33)(29,35,44,45)} and $G/D \cong C_3^2$.

Lower central Series				
Level	Order	Nature	Quotient	Successive quotient
1	392	solvable	abelian	C_3^2

Derived Series				
Level	Order	Nature	Quotient	Successive quotient
1	392	solvable	abelian	C_3^2
2	98	solvable	solvable	C_2^2
3	49	abelian	solvable	C_2
4	1	trivial	solvable	C_7^2

Conjugacy classes of maximal subgroups				
Serial	Order	Nature	Conjugacy classes	Generators
1	1176	solvable	1	(2,48,5,24)(3,14,6,32)(4,16,7,40)(8,21,29,39)(9,11,33,35)(10,38,34,20)(12,44,30,26)(13,22,31,43)(15,23,36,47)(17,19,41,37)(18,46,42,28)(25,27,49,45)(1,22,20,3,24,15,5,26,17,4,25,19,7,28,18,2,23,21,6,27,16)(8,32,43,10,35,45,12,30,47,11,34,46,14,29,49,9,31,44,13,33,48)(36,37,39,38,41,42,40)
2	1176	solvable	1	(2,39,5,21)(3,47,6,23)(4,13,7,31)(8,48,29,24)(9,26,33,44)(10,17,34,41)(11,30,35,12)(14,36,32,15)(16,43,40,22)(18,25,42,49)(19,38,37,20)(27,46,45,28)(1,8,4)(2,36,37)(3,15,47)(5,43,17)(6,29,28)(7,22,34)(9,18,40)(10,25,48)(11,32,14)(12,39,16)(13,46,24)(19,26,20)(21,33,35)(23,27,41)(30,49,38)(31,42,44)
3	1176	solvable	1	(2,29,5,8)(3,36,6,15)(4,43,7,22)(9,30,33,12)(10,37,34,19)(11,44,35,26)(13,16,31,40)(14,23,32,47)(17,38,41,20)(18,45,42,27)(21,24,39,48)(25,46,49,28)(1,33,39,12,23,5,21,40,31,26,13,19)(2,22,42,18,24,30,20,14,29,3,11,41)(4,44,37,49,28,45,17,48,34,43,8,46)(6,38,36,27,25,15,16,32,35,9,10,7)
4	1176	solvable	1	(2,8,5,29)(3,15,6,36)(4,22,7,43)(9,12,33,30)(10,19,34,37)(11,26,35,44)(13,40,31,16)(14,47,32,23)(17,20,41,38)(18,27,42,45)(21,48,39,24)(25,28,49,46)(1,22,26,9,16,15,25,39,37,17,45,46,41,34,31,49,7,6,33,12,14)(2,21,5,11,40,8,24,43,23,20,30,18,42,4,38,47,10,48,29,27,35)(3,44,19,13,32,36,28)

5	882	solvable	4	(1,43,20,16,44,6)(2,22,3,15,9,21)(4,29,39,19,37,33)(5,8,35,18,23,38)(7,36,26,17,30,11)(10,42,12,28,31,25)(13,14,49,27,24,45)(32,46,41,40,47,34)(1,43,26,23,2,36,39,25,30,31,38,11,13,34,17,21,14,48,47,19,7)(3,22,9,27,37,18,42,32,45,33,10,6,8,20,28,16,49,40,46,5,29)(4,15,35,24,44,12,41)
6	72	solvable	49	(8,39,24)(9,38,27)(10,42,26)(11,41,28)(12,37,25)(13,40,22)(14,36,23)(15,47,32)(16,43,31)(17,46,35)(18,44,34)(19,49,30)(20,45,33)(21,48,29)(2,32,13)(3,40,21)(4,48,23)(5,14,31)(6,16,39)(7,24,47)(8,36,22)(9,18,20)(10,12,49)(11,27,37)(15,43,29)(17,26,28)(19,35,45)(25,34,30)

Proper normal subgroups					
Serial	Order	Nature	Quotient	Subgroups	Generators
1	49	abelian	solvable	--	(1,3,5,4,7,2,6)(8,10,12,11,14,9,13)(15,17,19,18,21,16,20)(22,24,26,25,28,23,27)(29,31,33,32,35,30,34)(36,38,40,39,42,37,41)(43,45,47,46,49,44,48)(1,15,29,22,43,8,36)(2,16,30,23,44,9,37)(3,17,31,24,45,10,38)(4,18,32,25,46,11,39)(5,19,33,26,47,12,40)(6,20,34,27,48,13,41)(7,21,35,28,49,14,42)
2	147	solvable	solvable	1	(2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42)(11,27,37)(12,28,38)(13,23,39)(14,24,40)(15,29,43)(16,32,48)(17,33,49)(18,34,44)(19,35,45)(20,30,46)(21,31,47)(1,3,5,4,7,2,6)(8,10,12,11,14,9,13)(15,17,19,18,21,16,20)(22,24,26,25,28,23,27)(29,31,33,32,35,30,34)(36,38,40,39,42,37,41)(43,45,47,46,49,44,48)(1,15,29,22,43,8,36)(2,16,30,23,44,9,37)(3,17,31,24,45,10,38)(4,18,32,25,46,11,39)(5,19,33,26,47,12,40)(6,20,34,27,48,13,41)(7,21,35,28,49,14,42)
3	98	solvable	solvable	1	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(1,15,29,22,43,8,36)(2,16,30,23,44,9,37)(3,17,31,24,45,10,38)(4,18,32,25,46,11,39)(5,19,33,26,47,12,40)(6,20,34,27,48,13,41)(7,21,35,28,49,14,42)(1,3,5,4,7,2,6)(8,10,12,11,14,9,13)(15,17,19,18,21,16,20)(22,24,26,25,28,23,27)(29,31,33,32,35,30,34)(36,38,40,39,42,37,41)(43,45,47,46,49,44,48)
4	294	solvable	solvable	1,2,3	(1,15,29,22,43,8,36)(2,16,30,23,44,9,37)(3,17,31,24,45,10,38)(4,18,32,25,46,11,39)(5,19,33,26,47,12,40)(6,20,34,27,48,13,41)(7,21,35,28,49,14,42)(1,3,5,4,7,2,6)(8,10,12,11,14,9,13)(15,17,19,18,21,16,20)(22,24,26,25,28,23,27)(29,31,33,32,35,30,34)(36,38,40,39,42,37,41)(43,45,47,46,49,44,48)(2,7,6,5,4,3)(8,43,36,29,22,15)(9,49,41,33,25,17)(10,44,42,34,26,18)(11,45,37,35,27,19)(12,46,38,30,28,20)(13,47,39,31,23,21)(14,48,40,32,24,16)

5	392	solvable	abelian	1,3	(2,29,5,8)(3,36,6,15)(4,43,7,22)(9,30,33,12)(10,37,34,19)(11,44,35,26)(13,16,31,40)(14,23,32,47)(17,38,41,20)(18,45,42,27)(21,24,39,48)(25,46,49,28)(1,22,10,31)(2,6,11,12)(3,21,8,49)(4,44,9,18)(5,38,13,36)(7,33,14,27)(15,37,45,39)(16,28,46,35)(17,47,43,20)(19,34,48,26)(23,29,32,24)(25,41,30,40)
6	1176	solvable	cyclic	1,2,3,4,5	(2,29,5,8)(3,36,6,15)(4,43,7,22)(9,30,33,12)(10,37,34,19)(11,44,35,26)(13,16,31,40)(14,23,32,47)(17,38,41,20)(18,45,42,27)(21,24,39,48)(25,46,49,28)(1,22,5,33,3,38,2,44,4,11,7,21)(8,10,19,16,24,25,30,35,39,36,49,47)(9,41,18,48,28,13,29,20,40,27,45,34)(12,46,17,14,23,15,32,26,42,31,43,37)
7	1176	solvable	cyclic	1,3,5	(2,24,5,48)(3,32,6,14)(4,40,7,16)(8,39,29,21)(9,35,33,11)(10,20,34,38)(12,26,30,44)(13,43,31,22)(15,47,36,23)(17,37,41,19)(18,28,42,46)(25,45,49,27)(1,8,11,30,2,3,46,32,34,38,45,49,20,41,40,28,21,15,12,26,23)(4,33,25,31,43,10,48,37,6,42,18,35,19,24,47,22,13,36,9,7,16)(5,27,39,29,14,17,44)
8	1176	solvable	cyclic	1,3,5	(2,21,5,39)(3,23,6,47)(4,31,7,13)(8,24,29,48)(9,44,33,26)(10,41,34,17)(11,12,35,30)(14,15,32,36)(16,22,40,43)(18,49,42,25)(19,20,37,38)(27,28,45,46)(1,22,20,3,24,15,5,26,17,4,25,19,7,28,18,2,23,21,6,27,16)(8,32,43,10,35,45,12,30,47,11,34,46,14,29,49,9,31,44,13,33,48)(36,37,39,38,41,42,40)
9	1176	solvable	cyclic	1,3,5	(2,39,5,21)(3,47,6,23)(4,13,7,31)(8,48,29,24)(9,26,33,44)(10,17,34,41)(11,30,35,12)(14,36,32,15)(16,43,40,22)(18,25,42,49)(19,38,37,20)(27,46,45,28)(1,8,4)(2,36,37)(3,15,47)(5,43,17)(6,29,28)(7,22,34)(9,18,40)(10,25,48)(11,32,14)(12,39,16)(13,46,24)(19,26,20)(21,33,35)(23,27,41)(30,49,38)(31,42,44)

Sylow subgroups						
p	Ordre	Conjug. classes	Nature	Normalizer	Normal closure	Generators
2	8	49	nilpotent	72	392	(2,48,5,24)(3,14,6,32)(4,16,7,40)(8,21,29,39)(9,11,33,35)(10,38,34,20)(12,44,30,26)(13,22,31,43)(15,23,36,47)(17,19,41,37)(18,46,42,28)(25,27,49,45)(2,29,5,8)(3,36,6,15)(4,43,7,22)(9,30,33,12)(10,37,34,19)(11,44,35,26)(13,16,31,40)(14,23,32,47)(17,38,41,20)(18,45,42,27)(21,24,39,48)(25,46,49,28)
3	9	196	abelian	18	*G*	(2,15,40)(3,22,48)(4,29,14)(5,36,16)(6,43,24)(7,8,32)(9,25,41)(10,11,46)(12,18,19)(13,39,23)(17,33,49)(20,26,27)(21,47,31)(28,34,35)(30,42,37)(38,44,45)(2,29,24)(3,36,32)(4,43,40)(5,8,48)(6,15,14)(7,22,16)(9,41,25)(10,27,30)(11,20,42)(12,34,45)(17,49,33)(18,35,38)

						(19,28,44)(26,37,46)
7	49	1	abelian	G	49	(1,6,2,7,4,5,3)(8,13,9,14,11,12,10)(15,20,16, 21,18,19,17)(22,27,23,28,25,26,24)(29,34,30, 35,32,33,31)(36,41,37,42,39,40,38)(43,48,44, 49,46,47,45)(1,15,29,22,43,8,36)(2,16,30,23, 44,9,37)(3,17,31,24,45,10,38)(4,18,32,25,46, 11,39)(5,19,33,26,47,12,40)(6,20,34,27,48, 13,41)(7,21,35,28,49,14,42)

28. Let G be a primitive group of degree 49 with 3 generators. We have $|G| = 4704 = 2^5\times3\times7^2$.

Generators of G:	a_1 = (2,33)(3,41)(4,49)(5,9)(6,17)(7,25)(10,48)(11,35)(12,23) (13,38)(14,18)(16,26)(19,37)(20,31)(21,46)(24,34)(27,45) (28,39)(30,47)(32,42)(40,44) (order 2)
	a_2 = (2,40,21,35,30,42,33,15,3,48,23,37,38,44,41,22,4,14,31, 45,46,10,49,29,5,16,39,11,12,18,9,36,6,24,47,19,20,26,17, 43,7,32,13,27,28,34,25,8) (order 48)
	a_3 = (1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17, 38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13, 27,20,41,48,34)(7,14,28,21,42,49,35) (order 7)

This generating set is not minimal. There is a smaller generating set with 2 elements. G is a solvable group. G acts transitively on the set of 49 elements {1, 2, …, 49}. The center of G is trivial.

The derived subgroup D = [G, G] is a solvable group of order 392, generated by {(2,33,38,31,5,9,20,13)(3,41,46,39,6,17,28,21)(4,49,12,47,7,25,30,23)(8,42,37,14,29,18,19,32)(10,11,24,43,34,35,48,22)(15,44,45,16,36,26,27,40)(1,30,46,38,20,28,12)(2,32,45,41,21,26,8)(3,34,49,40,15,23,11)(4,31,48,42,19,22,9)(5,29,44,39,17,27,14)(6,35,47,36,16,25,10)(7,33,43,37,18,24,13)} and $G/D \cong C_2^2\times C_3$.

Lower central Series				
Level	Order	Nature	Quotient	Successive quotient
1	392	solvable	abelian	$C_2^2\times C_3$
2	196	solvable	nilpotent	C_2
3	98	solvable	nilpotent	C_2
4	49	abelian	nilpotent	C_2

Derived Series				
Level	Order	Nature	Quotient	Successive quotient
1	392	solvable	abelian	$C_2^2\times C_3$
2	49	abelian	nilpotent	C_8
3	1	trivial	solvable	C_7^2

Conjugacy classes of maximal subgroups				
Serial	Order	Nature	Conjugacy classes	Generators
1	2352	solvable	1	(1,6,2,7,4,5,3)(8,13,9,14,11,12,10)(15,20,16,21, 18,19,17)(22,27,23,28,25,26,24)(29,34,30,35,32, 33,31)(36,41,37,42,39,40,38)(43,48,44,49,46,47, 45)(2,40,21,35,30,42,33,15,3,48,23,37,38,44,41, 22,4,14,31,45,46,10,49,29,5,16,39,11,12,18,9,36, 6,24,47,19,20,26,17,43,7,32,13,27,28,34,25,8)
2	2352	solvable	1	(2,33)(3,41)(4,49)(5,9)(6,17)(7,25)(10,48)(11,35) (12,23)(13,38)(14,18)(16,26)(19,37)(20,31)(21,46) (24,34)(27,45)(28,39)(30,47)(32,42)(40,44)(1,46, 28,12,20,30)(2,48,25,8,19,35)(3,44,24,11,17,33)

				(4,47,26,9,16,32)(5,43,23,14,18,34)(6,45,22,10,21,31)(7,49,27,13,15,29)(36,41,42)(37,40,39)
3	2352	solvable	1	(2,34,5,10)(3,42,6,18)(4,44,7,26)(8,17,29,41)(9,22,33,43)(11,13,35,31)(12,40,30,16)(14,46,32,28)(15,25,36,49)(19,21,37,39)(20,48,38,24)(23,45,47,27)(1,23,40,43,46,8,41,25,28,18,37,14,12,35,36,19,20,5,39,34,30,48,42,2)(3,21,49,27,45,33,26,9,24,6,13,15,10,44,16,32,17,22,29,7,31,11,4,47)
4	1568	solvable	1	(2,24,31,35,20,10,33,43,5,48,13,11,38,34,9,22)(3,32,39,37,28,18,41,8,6,14,21,19,46,42,17,29)(4,40,47,45,30,26,49,15,7,16,23,27,12,44,25,36)(1,6,2,7,4,5,3)(8,33,9,29,11,30,10,32,13,31,14,34,12,35)(15,38,16,41,18,42,17,40,20,36,21,37,19,39)(22,44,23,46,25,45,24,48,27,49,28,47,26,43)
5	96	nilpotent	49	(2,24,31,35,20,10,33,43,5,48,13,11,38,34,9,22)(3,32,39,37,28,18,41,8,6,14,21,19,46,42,17,29)(4,40,47,45,30,26,49,15,7,16,23,27,12,44,25,36)(2,4,6)(3,5,7)(8,48,36,32,22,16)(9,47,41,31,25,21)(10,45,42,35,26,19)(11,44,37,34,27,18)(12,46,38,30,28,20)(13,49,39,33,23,

Proper normal subgroups					
Serial	Order	Nature	Quotient	Subgroups	Generators
1	49	abelian	nilpotent	--	(1,7,3,2,5,6,4)(8,14,10,9,12,13,11)(15,21,17,16,19,20,18)(22,28,24,23,26,27,25)(29,35,31,30,33,34,32)(36,42,38,37,40,41,39)(43,49,45,44,47,48,46)(1,12,28,20,38,46,30)(2,8,26,21,41,45,32)(3,11,23,15,40,49,34)(4,9,22,19,42,48,31)(5,14,27,17,39,44,29)(6,10,25,16,36,47,35)(7,13,24,18,37,43,33)
2	147	solvable	nilpotent	1	(2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42)(11,27,37)(12,28,38)(13,23,39)(14,24,40)(15,29,43)(16,32,48)(17,33,49)(18,34,44)(19,35,45)(20,30,46)(21,31,47)(1,12,28,20,38,46,30)(2,8,26,21,41,45,32)(3,11,23,15,40,49,34)(4,9,22,19,42,48,31)(5,14,27,17,39,44,29)(6,10,25,16,36,47,35)(7,13,24,18,37,43,33)(1,7,3,2,5,6,4)(8,14,10,9,12,13,11)(15,21,17,16,19,20,18)(22,28,24,23,26,27,25)(29,35,31,30,33,34,32)(36,42,38,37,40,41,39)(43,49,45,44,47,48,46)
3	98	solvable	nilpotent	1	(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11,35)(12,30)(13,31)(14,32)(15,36)(16,40)(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(1,21,31,23,47,13,39)(2,19,34,25,43,14,38)(3,16,33,27,46,8,42)(4,15,35,24,44,12,41)(5,20,32,22,49,10,37)(6,18,29,28,45,9,40)(7,17,30,26,48,11,36)(1,2,4,3,6,7,5)(8,9,11,10,13,14,12)(15,16,18,17,20,21,19)(22,23,25,24,27,28,26)(29,30,32,31,34,35,33)(36,37,39,38,41,42,40)(43,44,46,45,48,49,47)

4	294	solvable	nilpotent	1,2,3	(1,21,31,23,47,13,39)(2,19,34,25,43,14,38)(3,16,33,27,46,8,42)(4,15,35,24,44,12,41)(5,20,32,22,49,10,37)(6,18,29,28,45,9,40)(7,17,30,26,48,11,36)(1,2,4,3,6,7,5)(8,9,11,10,13,14,12)(15,16,18,17,20,21,19)(22,23,25,24,27,28,26)(29,30,32,31,34,35,33)(36,37,39,38,41,42,40)(43,44,46,45,48,49,47)(2,7,6,5,4,3)(8,43,36,29,22,15)(9,49,41,33,25,17)(10,44,42,34,26,18)(11,45,37,35,27,19)(12,46,38,30,28,20)(13,47,39,31,23,21)(14,48,40,32,24,16)
5	196	solvable	nilpotent	1,3	(2,38,5,20)(3,46,6,28)(4,12,7,30)(8,37,29,19)(9,13,33,31)(10,24,34,48)(11,43,35,22)(14,18,32,42)(15,45,36,27)(16,26,40,44)(17,21,41,39)(23,49,47,25)(1,2,4,3,6,7,5)(8,9,11,10,13,14,12)(15,16,18,17,20,21,19)(22,23,25,24,27,28,26)(29,30,32,31,34,35,33)(36,37,39,38,41,42,40)(43,44,46,45,48,49,47)
6	588	solvable	nilpotent	1,2,3,4,5	(1,46,20,12,30,38,28)(2,45,21,8,32,41,26)(3,49,15,11,34,40,23)(4,48,19,9,31,42,22)(5,44,17,14,29,39,27)(6,47,16,10,35,36,25)(7,43,18,13,33,37,24)(2,12,3,20,4,28,5,30,6,38,7,46)(8,11,15,19,22,27,29,35,36,37,43,45)(9,23,17,31,25,39,33,47,41,13,49,21)(10,40,18,48,26,14,34,16,42,24,44,32)
7	392	solvable	abelian	1,3,5	(2,31,20,33,5,13,38,9)(3,39,28,41,6,21,46,17)(4,47,30,49,7,23,12,25)(8,14,19,42,29,32,37,18)(10,43,48,11,34,22,24,35)(15,16,27,44,36,40,45,26)(1,39,13,47,23,31,21)(2,38,14,43,25,34,19)(3,42,8,46,27,33,16)(4,41,12,44,24,35,15)(5,37,10,49,22,32,20)(6,40,9,45,28,29,18)(7,36,11,48,26,30,17)
8	784	solvable	cyclic	1,3,5,7	(2,31,20,33,5,13,38,9)(3,39,28,41,6,21,46,17)(4,47,30,49,7,23,12,25)(8,14,19,42,29,32,37,18)(10,43,48,11,34,22,24,35)(15,16,27,44,36,40,45,26)(1,37,42,6)(2,19,41,12)(3,11,39,17)(4,27,38,23)(5,45,40,32)(7,35,36,43)(8,33,21,47)(9,48,20,30)(10,15,18,14)(13,28,16,22)(24,49,25,29)(31,34,46,44)
9	784	solvable	cyclic	1,3,5,7	(2,24,31,35,20,10,33,43,5,48,13,11,38,34,9,22)(3,32,39,37,28,18,41,8,6,14,21,19,46,42,17,29)(4,40,47,45,30,26,49,15,7,16,23,27,12,44,25,36)(1,32,48,40,16,24,14)(2,31,49,36,18,27,12)(3,35,43,39,20,26,9)(4,34,47,37,17,28,8)(5,30,45,42,15,25,13)(6,33,44,38,21,22,11)(7,29,46,41,19,23,10)
10	1176	solvable	abelian	1,2,3,4,5,6,7	(1,39,13,47,23,31,21)(2,38,14,43,25,34,19)(3,42,8,46,27,33,16)(4,41,12,44,24,35,15)(5,37,10,49,22,32,20)(6,40,9,45,28,29,18)(7,36,11,48,26,30,17)(2,47,46,33,7,39,38,25,6,31,30,17,5,23,28,9,4,21,20,49,3,13,12,41)(8,24,45,42,43,16,37,34,36,14,35,26,29,48,27,18,22,40,19,10,15,32,11,44)

11	2352	solvable	cyclic	1,2,3,4,5,6,7,8,10	(2,31,20,33,5,13,38,9)(3,39,28,41,6,21,46,17)(4,47,30,49,7,23,12,25)(8,14,19,42,29,32,37,18)(10,43,48,11,34,22,24,35)(15,16,27,44,36,40,45,26)(1,33,36,13,22,7,15,37,29,24,8,18)(2,39,38,26,25,20,19,35,34,9,14,3)(4,28,40,16,27,31,21,11,30,5,10,41)(6,48,42,49,23,44,17,45,32,46,12,47)
12	2352	solvable	cyclic	1,2,3,4,5,6,7,9,10	(2,40,21,35,30,42,33,15,3,48,23,37,38,44,41,22,4,14,31,45,46,10,49,29,5,16,39,11,12,18,9,36,6,24,47,19,20,26,17,43,7,32,13,27,28,34,25,8)(1,39,13,47,23,31,21)(2,38,14,43,25,34,19)(3,42,8,46,27,33,16)(4,41,12,44,24,35,15)(5,37,10,49,22,32,20)(6,40,9,45,28,29,18)(7,36,11,48,26,30,17)
13	784	solvable	cyclic	1,3,5,7	(8,32)(9,31)(10,35)(11,34)(12,30)(13,33)(14,29)(15,40)(16,36)(17,39)(18,37)(19,42)(20,38)(21,41)(22,48)(23,49)(24,43)(25,47)(26,45)(27,44)(28,46)(1,25,22,39,36,32,29,11,8,18,15,46,43,4)(2,9,23,16,37,44,30)(3,33,24,12,38,19,31,47,10,5,17,26,45,40)(6,49,27,7,41,28,34,42,13,35,20,14,48,21)
14	1568	solvable	cyclic	1,3,5,7,8,9,13	(8,32)(9,31)(10,35)(11,34)(12,30)(13,33)(14,29)(15,40)(16,36)(17,39)(18,37)(19,42)(20,38)(21,41)(22,48)(23,49)(24,43)(25,47)(26,45)(27,44)(28,46)(1,34,7,28,26,39,21,24,43,31,46,18,16,14,25,20)(2,8,30,10,45,49,27,3,47,36,33,41,6,4,17,48)(5,37,11,15,32,19,40,38,44,12,42,22,35,23,9,13)
15	2352	solvable	cyclic	1,2,3,4,5,6,7,10	(2,31,20,33,5,13,38,9)(3,39,28,41,6,21,46,17)(4,47,30,49,7,23,12,25)(8,14,19,42,29,32,37,18)(10,43,48,11,34,22,24,35)(15,16,27,44,36,40,45,26)(1,48,27,12,19,29)(2,47,23,8,16,34)(3,45,25,11,21,35)(4,44,28,9,17,30)(5,46,22,14,20,31)(6,49,26,10,15,32)(7,43,24,13,18,33)(36,40,41)(38,42,39)

Sylow subgroups						
p	Ordre	Conjug. classes	Nature	Normalizer	Normal closure	Generators
2	32	49	nilpotent	96	1568	(8,32)(9,31)(10,35)(11,34)(12,30)(13,33)(14,29)(15,40)(16,36)(17,39)(18,37)(19,42)(20,38)(21,41)(22,48)(23,49)(24,43)(25,47)(26,45)(27,44)(28,46)(2,24,31,35,20,10,33,43,5,48,13,11,38,34,9,22)(3,32,39,37,28,18,41,8,6,14,21,19,46,42,17,29)(4,40,47,45,30,26,49,15,7,16,23,27,12,44,25,36)
3	3	49	cyclic	96	147	(2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42)(11,27,37)(12,28,38)(13,23,39)(14,24,40)(15,29,43)(16,32,48)(17,33,49)(18,34,44)(19,35,45)(20,30,46)(21,31,47)
						(1,33,49,41,17,25,9)(2,29,47,42,20,24,11)(3,32,44,36,19,28,13)(4,30,43,40,21,27,10)(5,35,48,38,18,23,8)(6,31,46,37,15,26,14)

7	49	1	abelian	*G*	49	(7,34,45,39,16,22,12)(1,43,15,8,29,36,22)(2,44,16,9,30,37,23)(3,45,17,10,31,38,24)(4,46,18,11,32,39,25)(5,47,19,12,33,40,26)(6,48,20,13,34,41,27)(7,49,21,14,35,42,28)

29. Let G be a primitive group of degree 49 with 6 generators. We have $|G| = 7056 = 2^4 \times 3^2 \times 7^2$.

Generators of G:	a_1 = (2,38,5,20)(3,46,6,28)(4,12,7,30)(8,37,29,19)(9,13,33,31)(10,24,34,48)(11,43,35,22)(14,18,32,42)(15,45,36,27)(16,26,40,44)(17,21,41,39)(23,49,47,25) (order 4)
	a_2 = (2,15,40)(3,22,48)(4,29,14)(5,36,16)(6,43,24)(7,8,32)(9,25,41)(10,11,46)(12,18,19)(13,39,23)(17,33,49)(20,26,27)(21,47,31)(28,34,35)(30,42,37)(38,44,45) (order 3)
	a_3 = (2,8,5,29)(3,15,6,36)(4,22,7,43)(9,12,33,30)(10,19,34,37)(11,26,35,44)(13,40,31,16)(14,47,32,23)(17,20,41,38)(18,27,42,45)(21,48,39,24)(25,28,49,46) (order 4)
	a_4 = (2,21,5,39)(3,23,6,47)(4,31,7,13)(8,24,29,48)(9,44,33,26)(10,41,34,17)(11,12,35,30)(14,15,32,36)(16,22,40,43)(18,49,42,25)(19,20,37,38)(27,28,45,46) (order 4)
	a_5 = (2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42)(11,27,37)(12,28,38)(13,23,39)(14,24,40)(15,29,43)(16,32,48)(17,33,49)(18,34,44)(19,35,45)(20,30,46)(21,31,47) (order 3)
	a_6 = (1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35) (order 7)

This generating set is not minimal. There is a smaller generating set with 2 elements. G is a solvable group. G acts transitively on the set of 49 elements {1, 2, …, 49}. The center of G is trivial.

The derived subgroup $D = [G, G]$ is a solvable group of order 1176, generated by {(2,8,5,29)(3,15,6,36)(4,22,7,43)(9,12,33,30)(10,19,34,37)(11,26,35,44)(13,40,31,16)(14,47,32,23)(17,20,41,38)(18,27,42,45)(21,48,39,24)(25,28,49,46)(1,18,38)(2,40,21)(3,29,11)(4,45,22)(5,28,44)(7,9,33)(8,46,10)(12,42,23)(13,27,41)(14,16,19)(15,25,24)(17,43,32)(20,34,48)(26,35,30)(31,36,39)(37,47,49)} and $G/D \cong C_2 \times C_3$.

Lower central Series				
Level	Order	Nature	Quotient	Successive quotient
1	1176	solvable	cyclic	$C_2 \times C_3$

Derived Series				
Level	Order	Nature	Quotient	Successive quotient
1	1176	solvable	cyclic	$C_2 \times C_3$
2	392	solvable	solvable	C_3
3	98	solvable	solvable	C_2^2
4	49	abelian	solvable	C_2
5	1	trivial	solvable	C_7^2

Conjugacy classes of maximal subgroups				
Serial	Order	Nature	Conjugacy classes	Generators
1	3528	solvable	1	(2,16,3,24,4,32,5,40,6,48,7,14)(8,31,15,39,22,47,29,13,36,21,43,23)(9,27,17,35,25,37,33,45,41,11,49,19)(10,12,18,20,26,28,34,30,42,38,44,46)(1,33,39,5,35,37,7,34,36,6,31,40,3,32,42,4,30,41,2,29,38)(8,11,13,12,9,10,14)(15,44,23,19,43,22,21,47,

				26,20,49,28,17,48,27,18,45,24,16,46,25)
2	2352	solvable	1	(2,47,22)(3,13,29)(4,21,36)(5,23,43)(6,31,8)(7,39, 15)(9,45,10)(11,18,17)(12,28,38)(14,40,24)(16,48, 32)(19,26,25)(20,30,46)(27,34,33)(35,42,41)(37, 44,49)(1,2,42,25,31,32,28,37)(3,47,45,7,29,13,8, 35)(4,11,5,20,30,44,34,19)(6,24,15,40,33,36,17, 27)(9,18,43,22,46,16,10,38)(12,23,26,14,48,39, 41,49)
3	2352	solvable	3	(2,38,5,20)(3,46,6,28)(4,12,7,30)(8,37,29,19) (9,13,33,31)(10,24,34,48)(11,43,35,22)(14,18, 32,42)(15,45,36,27)(16,26,40,44)(17,21,41,39) (23,49,47,25)(1,38,18,13,44,29,42,4,10,16,34, 49)(2,31,21,6,45,15,41,46,8,37,32,14)(3,17, 20,48,43,36,39,11,9,30,35,7)(5,24,19,27,47, 22,40,25,12,23,33,28)
4	1764	solvable	4	(2,15,40)(3,22,48)(4,29,14)(5,36,16)(6,43,24) (7,8,32)(9,25,41)(10,11,46)(12,18,19)(13,39, 23)(17,33,49)(20,26,27)(21,47,31)(28,34,35) (30,42,37)(38,44,45)(1,19,47,3,31,44,23,32, 39,28,21,36)(2,42,46,15,35,5,22,45,40,30,17, 25)(4,48,49,34,29,27,26,41,38,20,16,6)(7,9, 43,11,33,14,24,8,37,12,18,10)
5	144	solvable	49	(2,40,15)(3,48,22)(4,14,29)(5,16,36)(6,24,43) (7,32,8)(9,41,25)(10,46,11)(12,19,18)(13,23, 39)(17,49,33)(20,27,26)(21,31,47)(28,35,34) (30,37,42)(38,45,44)(2,33,32,19,7,25,24,11, 6,17,16,45,5,9,14,37,4,49,48,35,3,41,40,27) (8,26,47,38,43,18,39,30,36,10,31,28,29,44, 23,20,22,

Proper normal subgroups					
Serial	Order	Nature	Quotient	Subgroups	Generators
1	49	abelian	solvable	--	(1,3,5,4,7,2,6)(8,10,12,11,14,9,13)(15,17, 19,18,21,16,20)(22,24,26,25,28,23,27) (29,31,33,32,35,30,34)(36,38,40,39,42, 37,41)(43,45,47,46,49,44,48)(1,15,29, 22,43,8,36)(2,16,30,23,44,9,37)(3,17, 31,24,45,10,38)(4,18,32,25,46,11,39) (5,19,33,26,47,12,40)(6,20,34,27,48, 13,41)(7,21,35,28,49,14,42)
2	147	solvable	solvable	1	(2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42) (11,27,37)(12,28,38)(13,23,39)(14,24,40) (15,29,43)(16,32,48)(17,33,49)(18,34,44) (19,35,45)(20,30,46)(21,31,47)(1,3,5,4,7, 2,6)(8,10,12,11,14,9,13)(15,17,19,18,21, 16,20)(22,24,26,25,28,23,27)(29,31,33, 32,35,30,34)(36,38,40,39,42,37,41) (43,45,47,46,49,44,48)(1,15,29,22, 43,8,36)(2,16,30,23,44,9,37)(3,17,31, 24,45,10,38)(4,18,32,25,46,11,39) (5,19,33,26,47,12,40)(6,20,34,27,48, 13,41)(7,21,35,28,49,14,42)
					(2,5)(3,6)(4,7)(8,29)(9,33)(10,34)(11, 35)(12,30)(13,31)(14,32)(15,36)(16,40)

3	98	solvable	solvable	1	(17,41)(18,42)(19,37)(20,38)(21,39)(22,43)(23,47)(24,48)(25,49)(26,44)(27,45)(28,46)(1,15,29,22,43,8,36)(2,16,30,23,44,9,37)(3,17,31,24,45,10,38)(4,18,32,25,46,11,39)(5,19,33,26,47,12,40)(6,20,34,27,48,13,41)(7,21,35,28,49,14,42)(1,3,5,4,7,2,6)(8,10,12,11,14,9,13)(15,17,19,18,21,16,20)(22,24,26,25,28,23,27)(29,31,33,32,35,30,34)(36,38,40,39,42,37,41)(43,45,47,46,49,44,48)
4	294	solvable	solvable	1,2,3	(1,15,29,22,43,8,36)(2,16,30,23,44,9,37)(3,17,31,24,45,10,38)(4,18,32,25,46,11,39)(5,19,33,26,47,12,40)(6,20,34,27,48,13,41)(7,21,35,28,49,14,42)(1,3,5,4,7,2,6)(8,10,12,11,14,9,13)(15,17,19,18,21,16,20)(22,24,26,25,28,23,27)(29,31,33,32,35,30,34)(36,38,40,39,42,37,41)(43,45,47,46,49,44,48)(2,7,6,5,4,3)(8,43,36,29,22,15)(9,49,41,33,25,17)(10,44,42,34,26,18)(11,45,37,35,27,19)(12,46,38,30,28,20)(13,47,39,31,23,21)(14,48,40,32,24,16)
5	392	solvable	solvable	1,3	(2,29,5,8)(3,36,6,15)(4,43,7,22)(9,30,33,12)(10,37,34,19)(11,44,35,26)(13,16,31,40)(14,23,32,47)(17,38,41,20)(18,45,42,27)(21,24,39,48)(25,46,49,28)(1,22,10,31)(2,6,11,12)(3,21,8,49)(4,44,9,18)(5,38,13,36)(7,33,14,27)(15,37,45,39)(16,28,46,35)(17,47,43,20)(19,34,48,26)(23,29,32,24)(25,41,30,40)
6	1176	solvable	dihedral	1,2,3,4,5	(2,29,5,8)(3,36,6,15)(4,43,7,22)(9,30,33,12)(10,37,34,19)(11,44,35,26)(13,16,31,40)(14,23,32,47)(17,38,41,20)(18,45,42,27)(21,24,39,48)(25,46,49,28)(1,22,5,33,3,38,2,44,4,11,7,21)(8,10,19,16,24,25,30,35,39,36,49,47)(9,41,18,48,28,13,29,20,40,27,45,34)(12,46,17,14,23,15,32,26,42,31,43,37)
7	1176	solvable	cyclic	1,3,5	(2,21,5,39)(3,23,6,47)(4,31,7,13)(8,24,29,48)(9,44,33,26)(10,41,34,17)(11,12,35,30)(14,15,32,36)(16,22,40,43)(18,49,42,25)(19,20,37,38)(27,28,45,46)(1,22,21)(2,25,18)(3,26,17)(4,27,19)(5,28,16)(6,23,15)(7,24,20)(8,33,47)(9,30,45)(10,35,48)(11,31,43)(12,34,46)(13,29,44)(14,32,49)(36,38,39)(37,42,40)
8	3528	solvable	cyclic	1,2,3,4,5,6,7	(2,4,6)(3,5,7)(8,40,23)(9,37,28)(10,42,26)(11,38,25)(12,41,27)(13,36,24)(14,39,22)(15,48,31)(16,47,29)(17,45,30)(18,44,34)(19,46,33)(20,49,35)(21,43,32)(1,22,25,14,21,15,24,38,42,16,44,45,40,33,30,48,6,5,32,11,13)(2,49,18,12,31,36,27)(3,37,46,9,47,29,26,34,7,20,4,10,39,8,23,43,28,19,35,17,41)

9	2352	solvable	cyclic	1,3,5,7	(2,32,13)(3,40,21)(4,48,23)(5,14,31)(6, 16,39)(7,24,47)(8,36,22)(9,18,20)(10, 12,49)(11,27,37)(15,43,29)(17,26,28) (19,35,45)(25,34,30)(33,42,38)(41,44,46) (1,22,2,34,49,21,48,30)(3,7,12,36,46,43, 40,14)(4,16,18,5,45,27,24,47)(6,39,31,9, 44,10,32,41)(8,13,19,23,42,37,26,20)(11, 28,29,17,38,15,35,25)

Sylow subgroups						
p	Ordre	Conjug. classes	Nature	Normalizer	Normal closure	Generators
2	16	147	nilpotent	48	2352	(2,8,5,29)(3,15,6,36)(4,22,7,43)(9,12,33,30) (10,19,34,37)(11,26,35,44)(13,40,31,16) (14,47,32,23)(17,20,41,38)(18,27,42,45) (21,48,39,24)(25,28,49,46)(2,38,5,20) (3,46,6,28)(4,12,7,30)(8,37,29,19) (9,13,33,31)(10,24,34,48)(11,43,35,22) (14,18,32,42)(15,45,36,27)(16,26,40,44) (17,21,41,39)(23,49,47,25)
3	9	196	abelian	36	3528	(2,4,6)(3,5,7)(8,22,36)(9,25,41)(10,26,42) (11,27,37)(12,28,38)(13,23,39)(14,24,40) (15,29,43)(16,32,48)(17,33,49)(18,34,44) (19,35,45)(20,30,46)(21,31,47)(2,40,15) (3,48,22)(4,14,29)(5,16,36)(6,24,43) (7,32,8)(9,41,25)(10,46,11)(12,19,18) (13,23,39)(17,49,33)(20,27,26)(21,31,47) (28,35,34)(30,37,42)(38,45,44)
7	49	1	abelian	*G*	49	(1,8,22,15,36,43,29)(2,9,23,16,37,44,30) (3,10,24,17,38,45,31)(4,11,25,18,39,46,32) (5,12,26,19,40,47,33)(6,13,27,20,41,48,34) (7,14,28,21,42,49,35)(1,34,44,42,18,26,10) (2,35,46,40,17,22,13)(3,29,48,37,21,25,12) (4,33,45,36,20,23,14)(5,31,43,41,16,28,11) (6,30,49,39,19,24,8)(7,32,47,38,15,27,9)

30. Let G be the primitive group 7^2: $GL(2,7)$ ◊ 1. We have $|G| = 49 = 7^2$. G is a cyclic group generated by (1,8,22,15,36,43,29), (2,9,23,16,37,44,30), (3,10,24,17,38,45,31), (4,11,25,18,39,46,32), (5,12,26,19,40,47,33), (6,13,27,20,41,48,34), (7,14,28,21,42,49,35).

Non-trivial orbits of G: {1,8,22,15,36,43,29}, {2,9,23,16,37,44,30}, {3,10,24,17,38,45,31}, {4,11,25,18,39,46,32}, {5,12,26,19,40,47,33}, {6,13,27,20,41,48,34}, {7,14,28,21,42,49,35}.

31. Let G be the primitive group 7^2: $GL(2,7)$ ◊ 2. We have $|G| = 98 = 2\times7^2$.

Generators of G:	a_1 = (8,29)(9,30)(10,31)(11,32)(12,33)(13,34)(14,35)(15,36) (16,37)(17,38)(18,39)(19,40)(20,41)(21,42)(22,43)(23,44) (24,45)(25,46)(26,47)(27,48)(28,49) (order 2)
	a_2 = (1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17, 38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13, 27,20,41,48,34)(7,14,28,21,42,49,35) (order 7)

G is a solvable group. Non-trivial orbits of G: {1,8,29,22,43,15,36}, {2,9,30,23,44,16,37}, {3,10,31,24,45,17,38}, {4,11,32,25,46,18,39}, {5,12,33,26,47,19,40}, {6,13,34,27,48,20,41}, {7,14,35,28,49,21,42}. The center of G is trivial.

The derived subgroup $D = [G, G]$ is a cyclic group of order 7, generated by {(1,43,15,8,29,36,22)(2,44,16,9,30,37,23)(3,45,17,10,31,38,24)(4,46,18,11,32,39,25)(5,47,19,12, 33,40,26)(6,48,20,13,34,41,27)(7,49,21,14,35,42,28)} and $G/D \cong C_2$.

Conjugacy classes of subgroups					
Serial	Order	Nature	Conjugacy classes	Maximal subgroup classes	Generators
1	1	trivial	1	--	-
2	2	cyclic	7	1	(8,29)(9,30)(10,31)(11,32)(12,33)(13,34)(14,35)(15,36)(16,37)(17,38)(18,39)(19,40)(20,41)(21,42)(22,43)(23,44)(24,45)(25,46)(26,47)(27,48)(28,49)
3	7	cyclic	1	1	(1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35)
4	14	dihedral	1	2,3	(8,29)(9,30)(10,31)(11,32)(12,33)(13,34)(14,35)(15,36)(16,37)(17,38)(18,39)(19,40)(20,41)(21,42)(22,43)(23,44)(24,45)(25,46)(26,47)(27,48)(28,49)(1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35)

Maximal normal subgroups				
Serial	Order	Nature	Quotient	Generators
1	7	cyclic	cyclic	(1,15,29,22,43,8,36)(2,16,30,23,44,9,37)(3,17,31,24,45,10,38)(4,18,32,25,46,11,39)(5,19,33,26,47,12,40)(6,20,34,27,48,13,41)

Sylow subgroups						
p	Ordre	Conjug. classes	Nature	Normalizer	Normal closure	Generators
2	2	7	cyclic	2	14	(8,29)(9,30)(10,31)(11,32)(12,33)(13,34)(14,35)(15,36)(16,37)(17,38)(18,39)(19,40)(20,41)(21,42)(22,43)(23,44)(24,45)(25,46)(26,47)(27,48)(28,49)
7	7	1	cyclic	14	7	(1,15,29,22,43,8,36)(2,16,30,23,44,9,37)(3,17,31,24,45,10,38)(4,18,32,25,46,11,39)(5,19,33,26,47,12,40)(6,20,34,27,48,13,41)(7,21,35,28,49,14,42)

32. Let G be the primitive group 7^2: $GL(2,7)$ ◊ 3. We have $|G| = 147 = 3\times 7^2$.

Generators of G:	a_1 = (8,22,36)(9,23,37)(10,24,38)(11,25,39)(12,26,40)(13,27,41)(14,28,42)(15,29,43)(16,30,44)(17,31,45)(18,32,46)(19,33,47)(20,34,48)(21,35,49) (order 3)
	a_2 = (1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35) (order 7)

G is a solvable group. Non-trivial orbits of G: {1,8,22,36,15,43,29}, {2,9,23,37,16,44,30}, {3,10,24,38,17,45,31}, {4,11,25,39,18,46,32}, {5,12,26,40,19,47,33}, {6,13,27,41,20,48,34}, {7,14,28,42,21,49,35}. The center of G is trivial.

The derived subgroup $D = [G, G]$ is a cyclic group of order 7, generated by {(1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35)} and $G/D \cong C_3$.

Conjugacy classes of subgroups					
Serial	Order	Nature	Conjugacy classes	Maximal subgroup classes	Generators
1	1	trivial	1	--	-
2	3	cyclic	7	1	(8,22,36)(9,23,37)(10,24,38)(11,25,39)(12,26,40)(13,27,41)(14,28,42)(15,29,43)(16,30,44)(17,31,45)(18,32,46)(19,33,47)(20,34,48)(21,35,49)
3	7	cyclic	1	1	(1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35)
4	21	solvable	1	2,3	(8,22,36)(9,23,37)(10,24,38)(11,25,39)(12,26,40)(13,27,41)(14,28,42)(15,29,43)(16,30,44)(17,31,45)(18,32,46)(19,33,47)(20,34,48)(21,35,49)(1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35)

Maximal normal subgroups				
Serial	Order	Nature	Quotient	Generators
1	7	cyclic	cyclic	(1,29,43,36,15,22,8)(2,30,44,37,16,23,9)(3,31,45,38,17,24,10)(4,32,46,39,18,25,11)(5,33,47,40,19,26,12)(6,34,48,41,20,27,13)

Sylow subgroups						
p	Ordre	Conjug. classes	Nature	Normalizer	Normal closure	Generators
3	3	7	cyclic	3	21	(8,22,36)(9,23,37)(10,24,38)(11,25,39)(12,26,40)(13,27,41)(14,28,42)(15,29,43)(16,30,44)(17,31,45)(18,32,46)(19,33,47)(20,34,48)(21,35,49)
7	7	1	cyclic	21	7	(1,29,43,36,15,22,8)(2,30,44,37,16,23,9)(3,31,45,38,17,24,10)(4,32,46,39,18,25,11)(5,33,47,40,19,26,12)(6,34,48,41,20,27,13)(7,35,49,42,21,28,14)

33. Let G be the primitive group 7^2: $GL(2,7)$ ◊ 4. We have $|G|$ = 294 = $2\times3\times7^2$.

Generators of G:	a_1 = (8,22,36)(9,23,37)(10,24,38)(11,25,39)(12,26,40)(13,27,41)(14,28,42)(15,29,43)(16,30,44)(17,31,45)(18,32,46)(19,33,47)(20,34,48)(21,35,49) (order 3)
	a_2 = (8,15,22,29,36,43)(9,16,23,30,37,44)(10,17,24,31,38,45)(11,18,25,32,39,46)(12,19,26,33,40,47)(13,20,27,34,41,48)(14,21,28,35,42,49) (order 6)
	a_3 = (1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35) (order 7)

This generating set is not minimal. There is a smaller generating set with 2 elements. G is a solvable group. Non-trivial orbits of G: {1,8,22,15,36,29,43}, {2,9,23,16,37,30,44}, {3,10,24,17,38,31,45}, {4,11,25,18,39,32,46}, {5,12,26,19,40,33,47}, {6,13,27,20,41,34,48}, {7,14,28,21,42,35,49}. The center of G is trivial.

The derived subgroup D = [G, G] is a cyclic group of order 7, generated by {(1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35)} and $G/D \cong C_2 \times C_3$.

Derived Series				
Level	Order	Nature	Quotient	Successive quotient
1	7	cyclic	cyclic	$C_2 \times C_3$
2	1	trivial	solvable	C_7

Conjugacy classes of subgroups					
Serial	Order	Nature	Conjugacy classes	Maximal subgroup classes	Generators
1	1	trivial	1	--	-
2	2	cyclic	7	1	(8,29)(9,30)(10,31)(11,32)(12,33)(13,34)(14,35)(15,36)(16,37)(17,38)(18,39)(19,40)(20,41)(21,42)(22,43)(23,44)(24,45)(25,46)(26,47)(27,48)(28,49)
3	3	cyclic	7	1	(8,22,36)(9,23,37)(10,24,38)(11,25,39)(12,26,40)(13,27,41)(14,28,42)(15,29,43)(16,30,44)(17,31,45)(18,32,46)(19,33,47)(20,34,48)(21,35,49)
4	6	cyclic	7	2,3	(8,22,36)(9,23,37)(10,24,38)(11,25,39)(12,26,40)(13,27,41)(14,28,42)(15,29,43)(16,30,44)(17,31,45)(18,32,46)(19,33,47)(20,34,48)(21,35,49) (8,29)(9,30)(10,31)(11,32)(12,33)(13,34)(14,35)(15,36)(16,37)(17,38)(18,39)(19,40)(20,41)(21,42)(22,43)(23,44)(24,45)(25,46)(26,47)(27,48)(28,49)
5	7	cyclic	1	1	(1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35)
6	14	dihedral	1	2,5	(8,29)(9,30)(10,31)(11,32)(12,33)(13,34)(14,35)(15,36)(16,37)(17,38)(18,39)(19,40)(20,41)(21,42)(22,43)(23,44)(24,45)(25,46)(26,47)(27,48)(28,49)(1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35)
7	21	solvable	1	--	(8,22,36)(9,23,37)(10,24,38)(11,25,39)(12,26,40)(13,27,41)(14,28,42)(15,29,43)(16,30,44)(17,31,45)(18,32,46)(19,33,47)(20,34,48)(21,35,49)(1,8,22,15,36,43,29)(2,9,23,16,37,44,30)(3,10,24,17,38,45,31)(4,11,25,18,39,46,32)(5,12,26,19,40,47,33)(6,13,27,20,41,48,34)(7,14,28,21,42,49,35)
8	42	solvable	1	--	(8,43,36,29,22,15)(9,44,37,30,23,16)(10,45,38,31,24,17)(11,46,39,32,25,18)(12,47,40,33,26,19)(13,48,41,34,27,20)(14,49,42,35,28,21)(1,22,36,29,8,15,43)(2,23,37,30,9,16,44)(3,24,38,31,10,17,45)(4,25,39,32,11,18,46)(5,26,40,33,12,19,47)(6,27,41,34,13,20,48)(7,28,42,35,14,21,49)

Maximal normal subgroups				
Serial	Order	Nature	Quotient	Generators
1	21	solvable	cyclic	(8,22,36)(9,23,37)(10,24,38)(11,25,39)(12,26,40)(13,27,41)(14,28,42)(15,29,43)(16,30,44)(17,31,45)(18,32,46)(19,33,47)(20,34,48)(21,35,49)(1,29,43,36,15,22,8)(2,30,44,37,16,23,9)(3,31,45,38,17,24,10)(4,32,46,39,18,25,11)(5,33,47,40,19,26,12)(6,34,48,41,20,27,13)(7,35,49,42,21,28,14)
2	14	dihedral	cyclic	(8,29)(9,30)(10,31)(11,32)(12,33)(13,34)(14,35)(15,36)(16,37)(17,38)(18,39)(19,40)(20,41)(21,42)(22,43)(23,44)(24,45)(25,46)(26,47)(27,48)(28,49)(1,29,43,36,15,22,8)(2,30,44,37,16,23,9)(3,31,45,38,17,24,10)(4,32,46,39,18,25,11)(5,33,47,40,19,26,12)(6,34,48,41,20,27,13)

Sylow subgroups						
p	Ordre	Conjug. classes	Nature	Normalizer	Normal closure	Generators
2	2	7	cyclic	6	14	(8,29)(9,30)(10,31)(11,32)(12,33)(13,34)(14,35)(15,36)(16,37)(17,38)(18,39)(19,40)(20,41)(21,42)(22,43)(23,44)(24,45)(25,46)(26,47)(27,48)(28,49)
3	3	7	cyclic	6	21	(8,22,36)(9,23,37)(10,24,38)(11,25,39)(12,26,40)(13,27,41)(14,28,42)(15,29,43)(16,30,44)(17,31,45)(18,32,46)(19,33,47)(20,34,48)(21,35,49)
7	7	1	cyclic	42	7	(1,22,36,29,8,15,43)(2,23,37,30,9,16,44)(3,24,38,31,10,17,45)(4,25,39,32,11,18,46)(5,26,40,33,12,19,47)(6,27,41,34,13,20,48)(7,28,42,35,14,21,49)

34. Let G be the primitive group $L2$, 7^2 # 49.1. We have $|G| = 56448 = 2^7 \times 3^2 \times 7^2$.

Generators of G:	a_1 = (1,2,4)(3,7,5)(8,9,11)(10,14,12)(15,16,18)(17,21,19)(22,23,25)(24,28,26)(29,30,32)(31,35,33)(36,37,39)(38,42,40)(43,44,46)(45,49,47) (order 3)
	a_2 = (1,8,22)(2,9,23)(3,10,24)(4,11,25)(5,12,26)(6,13,27)(7,14,28)(15,43,29)(16,44,30)(17,45,31)(18,46,32)(19,47,33)(20,48,34)(21,49,35) (order 3)
	a_3 = (1,4,7)(2,6,5)(8,11,14)(9,13,12)(15,18,21)(16,20,19)(22,25,28)(23,27,26)(29,32,35)(30,34,33)(36,39,42)(37,41,40)(43,46,49)(44,48,47) (order 3)
	a_4 = (1,22,43)(2,23,44)(3,24,45)(4,25,46)(5,26,47)(6,27,48)(7,28,49)(8,36,29)(9,37,30)(10,38,31)(11,39,32)(12,40,33)(13,41,34)(14,42,35) (order 3)
	a_5 = (2,8)(3,15)(4,22)(5,29)(6,36)(7,43)(10,16)(11,23)(12,30)(13,37)(14,44)(18,24)(19,31)(20,38)(21,45)(26,32)(27,39)(28,46)(34,40)(35,47)(42,48) (order 2)

This generating set is not minimal. There is a smaller generating set with 2 elements. G acts transitively on the set of 49 elements {1, 2, …, 49}. The center of G is trivial.

The derived subgroup $D = [G, G]$ is a perfect group of order 28224, generated by {(1,24,20,44,4,26,21,43,3,27,16,46,5,28,15,45,6,23,18,47,7,22,17,48,2,25,19,49)(8,10,13,9,11,12,14)(29,38,34,37,32,40,35,36,31,41,30,39,33,42)(1,35,18,24,43,42,11,3,29,21,25,45,36,14,4,31,15,

28,46,38,8,7,32,17,22,49,39,10)(2,34,16,27,44,41,9,6,30,20,23,48,37,13)(5,33,19,26,47,40,12)}
and $G/D \cong C_2$.

Maximal normal subgroups				
Serial	Order	Nature	Quotient	Generators
1	28224	perfect	cyclic	(1,5,2,7)(4,6)(8,47,23,14,43,26,9,49,22,12,44,28)(10,45,24)(11,48,25,13,46,27)(15,40,30,21,36,33,16,42,29,19,37,35)(17,38,31)(18,41,32,20,39,34), (1,20,2,15,6,16)(3,21,5,17,7,19)(4,18)(8,13,9)(10,14,12)(22,41,44,29,27,37,43,34,23,36,48,30)(24,42,47,31,28,

Sylow subgroups						
p	Order	Conjug. classes	Nature	Normalizer	Normal closure	Generators
2	128	441	nilpotent	128	*G*	(8,15)(9,16)(10,17)(11,18)(12,19)(13,20)(14,21)(36,43)(37,44)(38,45)(39,46)(40,47)(41,48)(42,49)(2,36)(3,43)(4,15)(5,8)(6,29)(7,22)(9,40)(10,47)(11,19)(13,33)(14,26)(16,39)(17,46)(20,32)(21,25)(23,42)(24,49)(27,35)(30,41)(31,48)(38,44)(2,6)(3,7)(9,13)(10,14)(16,20)(17,21)(23,27)(24,28)(30,34)(31,35)(37,41)(38,42)(44,48)(45,49)
3	9	784	abelian	72	28224	(8,36,22)(9,37,23)(10,38,24)(11,39,25)(12,40,26)(13,41,27)(14,42,28)(15,43,29)(16,44,30)(17,45,31)(18,46,32)(19,47,33)(20,48,34)(21,49,35)(2,4,7)(3,5,6)(9,11,14)(10,12,13)(16,18,21)(17,19,20)(23,25,28)(24,26,27)(30,32,35)(31,33,34)(37,39,42)(38,40,41)(44,46,49)(45,47,48)
7	49	64	abelian	882	28224	(1,6,3,2,5,7,4)(8,13,10,9,12,14,11)(15,20,17,16,19,21,18)(22,27,24,23,26,28,25)(29,34,31,30,33,35,32)(36,41,38,37,40,42,39)(43,48,45,44,47,49,46)(1,42,16,48,25,33,10)(2,41,18,47,24,29,14)(3,36,21,44,27,32,12)(4,40,17,43,28,30,13)(5,38,15,49,23,34,11)(6,39,19,45,22,35,9)(7,37,20,46,26,31,8)

35. Let G be the primitive group $A7$ on 1-sets[2.1]. We have $|G| = 12700800 = 2^7\times3^4\times5^2\times7^2$.

Generators of *G*:	a_1 = (1,2,3,4,5,6,7)(8,9,10,11,12,13,14)(15,16,17,18,19,20,21)(22,23,24,25,26,27,28)(29,30,31,32,33,34,35)(36,37,38,39,40,41,42)(43,44,45,46,47,48,49) (order 7)
	a_2 = (1,8,15,22,29,36,43)(2,9,16,23,30,37,44)(3,10,17,24,31,38,45)(4,11,18,25,32,39,46)(5,12,19,26,33,40,47)(6,13,20,27,34,41,48)(7,14,21,28,35,42,49) (order 7)
	a_3 = (5,6,7)(12,13,14)(19,20,21)(26,27,28)(33,34,35)(40,41,42)(47,48,49) (order 3)
	a_4 = (29,36,43)(30,37,44)(31,38,45)(32,39,46)(33,40,47)(34,41,48)(35,42,49) (order 3)
	a_5 = (2,8)(3,15)(4,22)(5,29)(6,36)(7,43)(10,16)(11,23)(12,30)(13,37)(14,44)(18,24)(19,31)(20,38)(21,45)(26,32)(27,39)(28,46)(34,40)(35,47)(42,48) (order 2)

This generating set is not minimal. There is a smaller generating set with 2 elements. G acts transitively on the set of 49 elements {1, 2, …, 49}. The center of G is trivial.

The derived subgroup $D = [G, G]$ is a perfect group of order 6350400, generated by {(1,11,47,21,2,8,46,19,7,9,43,18,5,14,44,15,4,12,49,16)(3,10,45,17)(6,13,48,20)(22,25,26,28,23)(29,39,33,42,30,36,32,40,35,37)(31,38)(34,41)(1,28,43,35,36,14,15,7,22,49,29,42,8,21)(2,23,44,30,37,9,16)(3,26,46,34,38,12,18,6,24,47,32,41,10,19,4,27,45,33,39,13,17,5,25,48,31,40,11,20)} and $G/D \cong C_2$.

Derived Series				
Level	Order	Nature	Quotient	Successive quotient
1	6350400	perfect	cyclic	C_2

Maximal normal subgroups				
Serial	Order	Nature	Quotient	Generators
1	6350400	perfect	cyclic	(1,46,34,23,12,42,17)(2,47,35,24,8,39,20)(3,43,32,27,9,40,21)(4,48,30,26,14,38,15)(5,49,31,22,11,41,16)(6,44,33,28,10,36,18)(7,45,29,25,13,37,19)(1,49,12,22,7,47,8,28,5,43,14,26)(2,45,11,23,3,46,9,24,4,44,10,25)(6,48,13,27)(15,42,19,36,21,40)

Sylow subgroups						
p	Ordre	Conjug. classes	Nature	Normalizer	Normal closure	Generators
2	128	99225	nilpotent	128	G	(15,22)(16,23)(17,24)(18,25)(19,26)(20,27)(21,28)(36,43)(37,44)(38,45)(39,46)(40,47)(41,48)(42,49), (8,29)(9,30)(10,31)(11,32)(12,33)(13,34)(14,35)(22,36)(23,37)(24,38)(25,39)(26,40)(27,41)(28,42)(2,15)(3,8)(4,43)(5,22)(6,29)(7,36)(9,17)(11,45)(12,24)(13,31)(14,38)(18,44)(19,23)(20,30)(21,37)(25,47)(27,33)(28,40)(32,48)(35,41)(39,49)
3	81	4900	abelian	2592	6350400	(3,7,6)(10,14,13)(17,21,20)(24,28,27)(31,35,34)(38,42,41)(45,49,48)(2,4,5)(9,11,12)(16,18,19)(23,25,26)(30,32,33)(37,39,40)(44,46,47), (22,43,36)(23,44,37)(24,45,38)(25,46,39)(26,47,40)(27,48,41)(28,49,42)(8,29,15)(9,30,16)(10,31,17)(11,32,18)(12,33,19)(13,34,20)(14,35,21)
5	25	15876	abelian	800	6350400	(15,22,29,36,43)(16,23,30,37,44)(17,24,31,38,45)(18,25,32,39,46)(19,26,33,40,47)(20,27,34,41,48)(21,28,35,42,49)(3,5,6,7,4)(10,12,13,14,11)(17,19,20,21,18)(24,26,27,28,25)(31,33,34,35,32)(38,40,41,42,39)(45,47,48,49,46)
7	49	14400	abelian	882	6350400	(1,5,7,4,6,2,3)(8,12,14,11,13,9,10)(15,19,21,18,20,16,17)(22,26,28,25,27,23,24)(29,33,35,32,34,30,31)(36,40,42,39,41,37,38)(43,47,49,46,48,44,45)(1,23,39,33,45,20,14)(2,25,40,31,48,21,8)(3,27,42,29,44,18,

						12)(4,26,38,34,49,15,9)(5,24,41,35, 43,16,11)(6,28,36,30,46,19,10) (7,22,37,32,47,17,13)

36. Let G be the primitive group $A7$ on 1-sets[2.2]. We have $|G| = 25401600 = 2^8\times3^4\times5^2\times7^2$.

Generators of G:	a_1 = (1,2,3,4,5,6,7)(8,9,10,11,12,13,14)(15,16,17,18,19,20,21)(22,23,24,25,26,27,28)(29,30,31,32,33,34,35)(36,37,38,39,40, 41,42)(43,44,45,46,47,48,49) (order 7)
	a_2 = (1,8,15,22,29,36,43)(2,9,16,23,30,37,44)(3,10,17,24,31, 38,45)(4,11,18,25,32,39,46)(5,12,19,26,33,40,47)(6,13,20, 27,34,41,48)(7,14,21,28,35,42,49) (order 7)
	a_3 = (5,6,7)(12,13,14)(19,20,21)(26,27,28)(33,34,35)(40,41,42)(47,48,49) (order 3)
	a_4 = (29,36,43)(30,37,44)(31,38,45)(32,39,46)(33,40,47)(34,41,48)(35,42,49) (order 3)
	a_5 = (2,8)(3,15)(4,22)(5,29)(6,36)(7,43)(10,16)(11,23)(12,30)(13,37)(14,44)(18,24)(19,31)(20,38)(21,45)(26,32)(27,39)(28,46)(34,40)(35,47)(42,48) (order 2)
	a_6 = (6,7)(13,14)(20,21)(27,28)(34,35)(36,43)(37,44)(38,45)(39,46)(40,47)(41,49)(42,48) (order 2)

This generating set is not minimal. There is a smaller generating set with 2 elements. G acts transitively on the set of 49 elements {1, 2, …, 49}. The center of G is trivial.

The derived subgroup $D = [G, G]$ is a perfect group of order 6350400, generated by {(1,27,46,19,29,41,11,5,22,48,18,33,36,13,4,26,43,20,32,40,8,6,25,47,15,34,39,12)(2,24,44,17,30, 38,9,3,23,45,16,31,37,10)(7,28,49,21,35,42,14)(1,33,41,7,31,36,5,34,42,3,29,40,6,35,38)(2,30,37)(4,32,39)(8,19,27,14,17,22,12,20,28,10,15,26,13,21,24)(9,16,23)(11,18,25)(43,47,48,49,45)} and $G/D \cong C_2^2$.

Maximal normal subgroups				
Serial	Order	Nature	Quotient	Generators
1	12700800	---	cyclic	(1,8,13,48,46,4)(2,36,9,41,44,39)(3,15,14,34,47,25)(5,22,10,20,49,32)(6,43,11)(7,29,12,27,45,18)(16,42,30,40,23,38)(17,21,35,33,26,24)(19,28,31)(1,10,40,23,15,14,33,27)(2,17,42,30,20,7,31,41)(3,38,37,16,21,35,34,6)(4,45,39,44,18,49,32,48)(5,24,36,9,19,28,29,13)(8,12,26,22)(11,47,25,43)
2	12700800	---	cyclic	(1,25,12,21,30,6,22,11,19,35,2,27,8,18,33,7,23,13, 15,32,5,28,9,20,29,4,26,14,16,34)(3,24,10,17,31)(36,46,40,49,37,48)(38,45)(39,47,42,44,41,43)(1,46,31,9,40,22,18,3,44,33,8,39,24,16,5,43,32, 10,37,26,15,4,45,30,12,36,25,17,2,47,29,11,38, 23,19)(6,48,34,13,41,27,20)(7,49,35,14,42,28,21)
3	12700800	---	cyclic	(1,14,43,21)(2,9,44,16)(3,10,45,17)(4,12,46,19)(5,11,47,18)(6,13,48,20)(7,8,49,15)(22,35)(23,30)(24,31)(25,33)(26,32)(27,34)(28,29)(36,42)(39,40)(1,38,32,26)(2,17,30,19)(3,31,33,5)(4,24,29,40)(6,10,34,12)(7,45,35,47)(8,41,11,27)(9,20)(14,48)

Sylow subgroups						
p	Order	Conjug. classes	Nature	Normalizer	Normal closure	Generators
						(6,7)(13,14)(20,21)(22,43)(23,44)(24,45)(25,46)(26,47)(27,49)(28,48)(34,35)(41,42)

2	256	99225	nilpotent	256	G	(8,43)(9,44)(10,45)(11,46)(12,47)(13,48)(14,49)(22,36)(23,37)(24,38)(25,39)(26,40)(27,41)(28,42), (15,29)(16,30)(17,31)(18,32)(19,33)(20,34)(21,35)(22,43)(23,44)(24,45)(25,46)(26,47)(27,48)(28,49)(2,8)(3,36)(4,15)(5,29)(6,22)(7,43)(10,37)(11,16)(12,30)(13,23)(14,44)(17,39)(19,32)(20,25)(21,46)(24,41)(26,34)(28,48)(31,40)(35,47)(42,45)
3	81	4900	abelian	5184	6350400	(3,4,6)(10,11,13)(17,18,20)(24,25,27)(31,32,34)(38,39,41)(45,46,48)(8,29,43)(9,30,44)(10,31,45)(11,32,46)(12,33,47)(13,34,48)(14,35,49), (2,5,7)(8,43,29)(9,47,35)(10,45,31)(11,46,32)(12,49,30)(13,48,34)(14,44,33)(16,19,21)(23,26,28)(37,40,42), (15,36,22)(16,37,23)(17,38,24)(18,39,25)(19,40,26)(20,41,27)(21,42,28)
5	25	15876	abelian	1600	6350400	(15,43,29,36,22)(16,44,30,37,23)(17,45,31,38,24)(18,46,32,39,25)(19,47,33,40,26)(20,48,34,41,27)(21,49,35,42,28)(3,4,6,7,5)(10,11,13,14,12)(17,18,20,21,19)(24,25,27,28,26)(31,32,34,35,33)(38,39,41,42,40)(45,46,48,49,47)
7	49	14400	abelian	1764	6350400	(1,6,2,5,3,4,7)(8,13,9,12,10,11,14)(15,20,16,19,17,18,21)(22,27,23,26,24,25,28)(29,34,30,33,31,32,35)(36,41,37,40,38,39,42)(43,48,44,47,45,46,49)(1,49,32,17,26,37,13)(2,48,29,21,25,38,12)(3,47,30,20,22,42,11)(4,45,33,16,27,36,14)(5,44,34,15,28,39,10)(6,43,35,18,24,40,9)(7,46,31,19,23,41,8)

37. Let G be the primitive group $A7$ on 1-sets[2.3]. We have $|G| = 25401600 = 2^8 \times 3^4 \times 5^2 \times 7^2$.

Generators of G:	a_1 = (1,2,3,4,5,6,7)(8,9,10,11,12,13,14)(15,16,17,18,19,20,21)(22,23,24,25,26,27,28)(29,30,31,32,33,34,35)(36,37,38,39,40,41,42)(43,44,45,46,47,48,49) (order 7)
	a_2 = (1,8,15,22,29,36,43)(2,9,16,23,30,37,44)(3,10,17,24,31,38,45)(4,11,18,25,32,39,46)(5,12,19,26,33,40,47)(6,13,20,27,34,41,48)(7,14,21,28,35,42,49) (order 7)
	a_3 = (5,6,7)(12,13,14)(19,20,21)(26,27,28)(33,34,35)(40,41,42)(47,48,49) (order 3)
	a_4 = (29,36,43)(30,37,44)(31,38,45)(32,39,46)(33,40,47)(34,41,48)(35,42,49) (order 3)
	a_5 = (2,8)(3,15)(4,22)(5,29)(6,36,7,43)(10,16)(11,23)(12,30)(13,37,14,44)(18,24)(19,31)(20,38,21,45)(26,32)(27,39,28,46)(34,40,35,47)(41,42,49,48) (order 4)
	a_6 = (6,7)(13,14)(20,21)(27,28)(34,35)(36,43)(37,44)(38,45)(39,46)(40,47)(41,49)(42,48) (order 2)

This generating set is not minimal. There is a smaller generating set with 2 elements. G acts transitively on the set of 49 elements {1, 2, …, 49}. The center of G is trivial.

The derived subgroup $D = [G, G]$ is a perfect group of order 6350400, generated by {(1,28,18,33,36,49,11,5,22,21,32,40,43,14,4,26,15,35,39,47,8,7,25,19,29,42,46,12)(2,24,16,31,37,45,9,3,23,17,30,38,44,10)(6,27,20,34,41,48,13)(1,33,28,48,3,29,26,49,6,31,22,47,7,34,24,43,5,35, 27,45)(2,30,23,44)(4,32,25,46)(8,19,14,20,10,15,12,21,13,17)(9,16)(11,18)(36,40,42,41,38)} and $G/D \cong C_4$.

Derived Series				
Level	Order	Nature	Quotient	Successive quotient
1	6350400	perfect	cyclic	C_4

Maximal normal subgroups				
Serial	Order	Nature	Quotient	Generators
1	12700800	---	cyclic	(1,12,32,17)(2,14,30,21)(3,8,33,18)(4,10,29,19)(5,11,31,15)(6,13,34,20)(7,9,35,16)(22,47,25,45)(23,49)(24,43,26,46)(27,48)(28,44)(36,40,39,38)(37,42)(1,16,11,5,17,13)(2,18,12,3,20,8)(4,19,10,6,15,9)(7,21,14)(22,44,39,33,24,48,36,30,25,47,38,34)

Sylow subgroups						
p	Order	Conjug. classes	Nature	Normalizer	Normal closure	Generators
2	256	99225	nilpotent	256	G	(4,5)(11,12)(15,22,36,43)(16,23,37,44)(17,24,38,45)(18,26,39,47)(19,25,40,46)(20,27,41,48)(21,28,42,49)(32,33)(2,8,6,29)(3,15)(4,43)(5,22)(7,36)(9,13,34,30)(10,20,31,16)(11,48,32,44)(12,27,33,23)(14,41,35,37)(18,45)(19,24)(21,38)(25,47)(28,40)(39,49)(8,29)(9,30)(10,31)(11,32)(12,33)(13,34)(14,35)(15,36)(16,37)(17,38)(18,39)(19,40)(20,41)(21,42)
3	81	4900	abelian	5184	6350400	(4,7,5)(11,14,12)(18,21,19)(25,28,26)(32,35,33)(39,42,40)(46,49,47)(2,3,6)(9,10,13)(16,17,20)(23,24,27)(30,31,34)(37,38,41)(44,45,48)(15,43,36)(16,44,37)(17,45,38)(18,46,39)(19,47,40)(20,48,41)(21,49,42)(8,22,29)(9,23,30)(10,24,31)(11,25,32)(12,26,33)(13,27,34)(14,28,35)
5	25	15876	abelian	1600	6350400	(15,29,43,22,36)(16,30,44,23,37)(17,31,45,24,38)(18,32,46,25,39)(19,33,47,26,40)(20,34,48,27,41)(21,35,49,28,42)(3,6,7,4,5)(10,13,14,11,12)(17,20,21,18,19)(24,27,28,25,26)(31,34,35,32,33)(38,41,42,39,40)(45,48,49,46,47)
7	49	14400	abelian	1764	6350400	(1,7,3,4,2,6,5)(8,14,10,11,9,13,12)(15,21,17,18,16,20,19)(22,28,24,25,23,27,26)(29,35,31,32,30,34,33)(36,42,38,39,37,41,40)(43,49,45,46,44,48,47)(1,36,43,29,15,22,8)(2,37,44,30,16,23,9)(3,38,45,31,17,24,10)(4,39,46,32,18,25,11)(5,40,47,33,19,26,12)(6,41,48,34,20,27,13)(7,42,49,35,21,28,14)

38. Let G be the primitive group $A7$ on 1-sets[2.4]. We have $|G| = 50803200 = 2^9\times3^4\times5^2\times7^2$.

	a_1 = (1,2,3,4,5,6,7)(8,9,10,11,12,13,14)(15,16,17,18,19,20,21)(22,23,24,25,26,27,28)(29,30,31,32,33,34,35)(36,37,38,39,40,41,42)(43,44,45,46,47,48,49) (order 7)

Generators of G:	a_2 = (1,8,15,22,29,36,43)(2,9,16,23,30,37,44)(3,10,17,24,31,38,45)(4,11,18,25,32,39,46)(5,12,19,26,33,40,47)(6,13,20,27,34,41,48)(7,14,21,28,35,42,49) (order 7)
	a_3 = (5,6,7)(12,13,14)(19,20,21)(26,27,28)(33,34,35)(40,41,42)(47,48,49) (order 3)
	a_4 = (29,36,43)(30,37,44)(31,38,45)(32,39,46)(33,40,47)(34,41,48)(35,42,49) (order 3)
	a_5 = (36,43)(37,44)(38,45)(39,46)(40,47)(41,48)(42,49) (order 2)
	a_6 = (6,7)(13,14)(20,21)(27,28)(34,35)(41,42)(48,49) (order 2)
	a_7 = (2,8)(3,15)(4,22)(5,29)(6,36)(7,43)(10,16)(11,23)(12,30)(13,37)(14,44)(18,24)(19,31)(20,38)(21,45)(26,32)(27,39)(28,46)(34,40)(35,47)(42,48) (order 2)

This generating set is not minimal. There is a smaller generating set with 2 elements. G acts transitively on the set of 49 elements {1, 2, …, 49}. The center of G is trivial.

The derived subgroup D = [G, G] is of order 12700800, generated by {(1,18,29,4,15,32)(2,20,30,6,16,34)(3,17,31)(5,21,33,7,19,35)(8,39,43,25)(9,41,44,27)(10,38,45,24)(11,36,46,22)(12,42,47,28)(13,37,48,23)(14,40,49,26)(1,38,4,42)(2,40)(3,39,7,36)(5,37)(6,41)(8,10,11,14)(9,12)(15,24,46,35)(16,26,44,33)(17,25,49,29)(18,28,43,31)(19,23,47,30)(20,27,48,34)(21,22,45,32)} and $G/D \cong C_2^2$.

Maximal normal subgroups				
Serial	Order	Nature	Quotient	Generators
1	25401600	---	cyclic	(1,5,33,34,27,25,39,42,14,10,45,43)(2,19,30,20,23,18,37,21,9,17,44,15)(3,47,29,6,26,32,41,28,11,38,49,8)(4,40,35,13,24,46,36,7,12,31,48,22)(1,40,45,18,29,37,3,19,43,39,31,16)(2,5,47,46,32,30)(4,33,44)(6,12,48,11,34,9)(7,26,49,25,35,23)(8,41,10,20)(14,27)(15,36,38,17)(21,22,42,24)
2	25401600	---	cyclic	(1,5,47,44,30,29)(2,33,43)(3,12,45,9,31,8)(4,40,48,16,35,22)(6,19,49,23,32,36)(7,26,46,37,34,15)(11,38,13,17,14,24)(18,42,27)(20,21,28,25,39,41)(1,14,3,35,2,49,4,28,6,21)(5,42)(8,10,31,30,44,46,25,27,20,15)(9,45,32,23,48,18,22,13,17,29)(11,24,34,16,43)(12,38,33,37,47,39,26,41,19,36)
3	25401600	---	cyclic	(1,16,45,36,2,17,43,37,3,15,44,38)(4,20,46,41)(5,21,47,42)(6,18,48,39)(7,19,49,40)(8,9,10)(11,13)(12,14)(22,23,24)(25,27)(26,28)(29,30,31)(32,34)(33,35)(1,26,14,30,6,24,8,33,7,23,13,31)(2,27,10,29,5,28,9,34,3,22,12,35)(4,25,11,32)(15,40,21,37,20,38)(16,41,17,36,19,42)

Sylow subgroups						
p	Order	Conjug. classes	Nature	Normalizer	Normal closure	Generators
						(36,43)(37,44)(38,45)(39,46)(40,47)(41,48)(42,49)(2,8)(3,22)(4,15)(5,29)(6,36)(7,43)(10,23)(11,16)(12,30)(13,37)(14,44)(17,25)(19,32)(20,39)(21,46)(26,31)

2	512	99225	nilpotent	512	*G*	(27,38)(28,45)(34,40)(35,47)(42,48)(2,3)(4,5)(9,10)(11,12)(16,17)(18,19)(23,24)(25,26)(30,31)(32,33)(37,38)(39,40)(44,45)(46,47)(2,4)(9,11)(16,18)(23,25)(30,32)(37,39)(44,46)
3	81	4900	abelian	10368	6350400	(5,7,6)(12,14,13)(19,21,20)(26,28,27)(33,35,34)(40,42,41)(47,49,48)(2,4,3)(9,11,10)(16,18,17)(23,25,24)(30,32,31)(37,39,38)(44,46,45), (22,36,43)(23,37,44)(24,38,45)(25,39,46)(26,40,47)(27,41,48)(28,42,49)(8,15,29)(9,16,30)(10,17,31)(11,18,32)(12,19,33)(13,20,34)(14,21,35)
5	25	15876	abelian	3200	6350400	(15,22,36,43,29)(16,23,37,44,30)(17,24,38,45,31)(18,25,39,46,32)(19,26,40,47,33)(20,27,41,48,34)(21,28,42,49,35)(3,6,4,7,5)(10,13,11,14,12)(17,20,18,21,19)(24,27,25,28,26)(31,34,32,35,33)(38,41,39,42,40)(45,48,46,49,47)
7	49	14400	abelian	3528	6350400	(1,2,3,6,5,7,4)(8,9,10,13,12,14,11)(15,16,17,20,19,21,18)(22,23,24,27,26,28,25)(29,30,31,34,33,35,32)(36,37,38,41,40,42,39)(43,44,45,48,47,49,46)(1,30,45,41,26,21,11)(2,31,48,40,28,18,8)(3,34,47,42,25,15,9)(4,29,44,38,27,19,14)(5,35,46,36,23,17,13)(6,33,49,39,22,16,10)(7,32,43,37,24,20,12)

8. DISCUSSION

In this article the problem of cigarette combustion globally is considered. The research showed that this problem is neither easy nor simple, just opposite it is a very hard scientific problem, which includes other complex subproblems, as those of problem of balancing chemical reaction of tobacco combustion, field temperature problem in the combustion zone, smoke filtration problem, and the problem of groups' formation of reaction coefficients. No one of the mentioned subproblems is fully solved. Their solutions are quite tied with usage of update hardware and software. For instance, with current softwear it is imposible to be balanced the chemical reaction (4. 28).

Now, this question naturally arises: *Is it solvable any chemical equation by a computer*?

The reply of this question is very simple: No! Reason why this reply is negative lies in inappropriate chemical methods, which most of them are paradoxal, but from other hand sophisticated mathematical methods are not computer adapted for daily usage. Right now, the hardest problem in chemistry as well as mathematics is balancing of $\aleph$ reactions, *i.e.*, the continuum reactions, which need special treatment with new mathematical methods, as this developed in this article.

Since the chemical reaction of tobacco combustion (4. 28) is very complex reaction, which belongs to the class of $\aleph$ reactions, its necessary and sufficient conditions are not determined. In fact, it is a shortcoming of the mathematical model, which does not provide precise necessary and sufficient conditions when the reaction (4. 28) is possible.

Of course, there are other obstacles, which must be overcome. For instace, for the reaction (4. 28) is not developed topology of its solutions.

These remarks creates a new scientific requirement that for balancing $\aleph$ reactions should be built a computer package as soon as possible, because this kind of chemical reactions is not studied enough in chemistry and mathematics too.

Generaly speaking it is a first work where the chemical reaction (4. 28) is considered, and as every pioneering job it does not provide a complete treatmant, but it opens doors of next research.

9. CONCLUSION

The global cigarette combustion problem in this work is treated only from the mathematical point of view, for whom is given a completely new approach toward its solution. It is considered as a complex mathematical problem, which includes few subproblems: the problem of balancing chemical reaction of tobacco combustion, field temperature problem in the combustion zone, smoke filtration problem, and the problem of groups' formation of reaction coefficients. In fact, these problems are particularly solved for the certain simulation conditions. For instance, the smoke infiltration problem is founded and solved by virtue of partial differential equation of first order. The field temperature problem in the combustion zone is modeled by the two-dimensional heat transfer equation which is solved by quadratures. The chemical reaction (4. 28), which describes tobacco combustion is a completely new reaction and it includes all important alkaloids and toxins. This reaction has four generators. In fact, it is a very hard chemical reaction, which cannot be balanced by a computer, because right now in the theory of computer sciences there is not powerful software, which can be used for its balance. Unique way to balance this reaction is by the usage of mathematical method.

Since, the reaction of tobacco combustion is very complicated we found only its general solution and one particular solution. This reaction spans real vector spaces. For the reaction coefficients are calculated a symmetric group S_{49}, an alternating group A_{49} and 38 primitive groups.

The strengths of the mathematical model are:

1. This model provides an alternative approach for balancing $\aleph$ chemical reactions.
2. Since this model is well formalized, it belongs to the class of consistent models for balancing chemical reaction.
3. This model showed that any chemical reactions can be treated as *n*-dimensional geometrical entity.
4. In fact, here-offered model simplifies mathematical operations provided by the previous well-known matrix methods and is very easily acceptable for daily practice. The model has this advantage, because it fits for all $\aleph$ chemical reactions, which previously were balanced only by the methods of generalized matrix inverses.
5. The mathematical model provides the dimension of the solution space.
6. Also, by this method a basis of the solution space is determined.
7. This method gives an opportunity to be extended with other numerical calculations necessary for $\aleph$ reactions.
8. It can predict quantitative relations among reaction coefficients.
9. The mathematical model can predict reaction stability.
10. Offered mathematical model represents a well basis for building a software package.

The weak sides of the mathematical model are:

1. By this model the minimal reaction coefficients cannot be determined.
2. Also, this model cannot recognize when chemical reaction reduces to one generator reaction.
3. This model cannot arrange molecules disposition.
4. It does not provide precise necessary and sufficient conditions when the $\aleph$ reaction (4. 28) is possible.
5. This model cannot generate topology of reaction.

This model wild opens the doors in chemistry and mathematics too, for a new research of $\aleph$ chemical reactions, which unfortunately today cannot be balanced by usage of computer, because there is not such method. Here developed mathematical model is a big challenge for researchers to extend and adapt it for a computer application. Sure that it is not easy and simple job, but it deserves to be realized as soon as possible.

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SYMBOLS

α – fraction, $(0 \le \alpha \le 1)$,
β – tobacco absorption coefficient, 1/mm

C_i – concentration of components X_i, ($1 \le i \le n$), mg/mm
L – total cigarette length, mm
L_0 – initial length of the cigarette, mm
ΔL – burning cigarette length, mm
x – distance from the burning end, mm
y – coordinate, mm
t – time, s
T– temperature, ºC
k – tobacco thermal diffusivity, m^2/s.

REFERENCES

[1] T. S. Laszlo, F. M. Watson, *A scanning infrared technique for cigarette coal peak temperature measurements*, Beit. Tabakforsch. Int. **7** (1974), 269-275.

[2] A. T. Lendvay, T. S. Laszlo, *Cigarette peak coal temperature measurements*, Beit. Tabakforsch. Int. **7** (1974), 276-281.

[3] R. R. Baker, *Temperature variation within a cigarette combustion coal during the smoke cycle*, High Temperature Sci. **7** (1975), 236-247.

[4] D. F. Boyd, C. D. Briggs, P. W. Darby, Dependence *of the gas phase composition of smoke on the combustion temperature of tobacco products*, Tobacco Sci. **13** (1972), 160-165.

[5] K. Gugan, *Natural smolder in cigarettes*, Combustion & Flame **10** (1966), 161-164.

[6] F. Miller, W. J. Freeman, R. L. Stedman, *The effect of additives on the combustion temperature of cigarettes,* Beit. Tabakforsch. Int. **4** (1968), 269-274.

[7] F. E. Resnik, W. G. Houck, W. A. Geiszler, J. E. Wickman, *Factors affecting static burn rate*, Tobacco Sci. **21** (1977), 103-107.

[8] M. Z. Debardeleben, W. E. Clafin, W. F. Gannon, *Role of cigarette physical characteristics on smoke composition*, Recent Adv. Tobacco Sci. **4** (1978), 85-111.

[9] K. Aarany-Fuzessery, I. Hamza-Nagy, I. Nagy-Buday, *The effect of physical characteristics of cigarettes on their burning properties and on the major components of the main stream smoke*, Acta Alimentaria **11** (1982), 143-155.

[10] R. L. Rice, R. S. Perrier, V. C. Runeckles, *A weight loss technique for determining rate of static burn*, Tobacco Sci. **14** (1970), 173-175.

[11] D. A. Eichman, *Cigarette burning rates*, Fire & Arson Investigator **30** (3) (1980), 10-19.

[12] P. Halmos, *Finite Dimensional Vector Spaces*, Springer Vlg, Berlin 1998.

[13] S. Kurepa, *Konačno dimezionalni vektorski prostori i primjene*, 5-to izd., Sveučilisna naklada Liber, Zagreb 1990.

[14] H. U. Besche, B. Eick, *Construction of finite groups*, J. Symb. Comput. **27** (1999), 387-404.

[15] I. B. Risteski, *A new topology for the trajectories of the meniscus during continuous steel casting,* Mat. & Technol. **41** (2007), 213-226.

[16] I. B. Risteski, *A new class of quasicyclic complex vector functional equations*, Math. J. Okayama Univ. **50** (2008), 1-61.

[17] I. Risteski, V. Covachev, *Complex Vector Functional Equations*, World Scientific, New Jersey - London - Singapore - Hong Kong 2001.

[18] I. B. Risteski, *Matrix method for solving linear complex vector functional equations*, Internat. J. Math. & Math. Sci. **29** (2002), 217-238.

[19] I. B. Risteski, *New wide classes of complex vector functional equations,* Kumamoto J. Math. **20** (2007), 51-87.

[20] I. B. Risteski, *A new pseudoinverse matrix method for balancing chemical equations and their stability*, J. Korean Chem. Soc. **52** (2008), 223-238.

[21] I. B. Risteski, *The new algebraic criterions to even out the chemical reactions,* 22nd October Meeting of Miners & Metallurgists, Collection of Papers, Oct. 1-2, Bor 1990, pp. 313-318.

[22] M. Polonijo, D. Crnković, T. B. Kirigin, M. Bombardelli, Z. Franušić, R. Sušanj, *Euklidski prostori*, Sveučilište u Zagrebu, Zagreb 2009, pp. 15-16.

[23] J. L. Leffingwell, *Leaf chemistry. Basic chemical constituents of tobacco leaf and differences among tobacco types*. In: D. L. Davis, M. T. Nielsen, editors. Tobacco production, chemistry and technology, Blackwell Science, Oxford 1999, pp. 265-284.

[24] R. R. Baker, *Smoke chemistry*, In: In: D. L. Davis, M. T. Nielsen, editors. Tobacco production, chemistry and technology, Blackwell Science, Oxford 1999, pp. 398-439.

[25] C. R. Green, A. Rodgman, *The Tobacco Chemists' Research Conference: a half century forum for advances in analytical methodology of tobacco and its products*, Recent Adv. Tobacco Sci. **22** (1996), 131-304.

[26] M. Muramatsu, *An approach to modelling a burning cigarette*, Beitr. Tabakforsch. Int. **21** (2005), 286-293.

[27] R. R. Baker, *Smoke generation inside a burning cigarette: Modifying combustion to develop cigarettes that may be less hazardous to health*, Prog. Energy Combust. Sci. **32** (2006), 373-385.

[28] M. Borgerding, H. Klus, *Analysis of complex mixtures-cigarette smoke*, Exp. Toxicol. Path. **57** (2005), 43-73.

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INVESTIGATION OF EFFICIENCY OF CIGARETTE PAPER FILTERS

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ABSTRACT

Filter design and materials is a major tool to modify the filtration efficiency for cigarette smoke.Despite the variety of filters in the market, cellulose acetatecontinued to be most common used filter in cigarettes. Filters within a similar smoke removal efficiency range are also manufactured from paper. It is estimated that in recent yearsthe productionof cellulose acetate tow willbelessthanthe demand forcellulosefilters.

The aim of our research is to determine the efficiency of paper filter rod on the smoke composition and retention of particulate matter and nicotine compared to the cellulose acetate filter rod, as control. The content of tar and nicotine were measured according to ISO 4387 and ISO 10315,respectivelly. For certifying the pressure drop of the filter rod and cigarette was used ISO 6565:2011. The results have shown that removal efficiency of paper filter was comparable to cellulose acetate filter.

Key words: paper, cellulose acetate, filter rod, efficiency, particular matter, nicotine, smoke

ИСТРАЖУВАЊЕ НА ЕФИКАСНОСТА НА ХАРИЕНИТЕ ФИЛТРИ ЗА ЦИГАРИ

Дизајнот и материјалите кои се користат за изработка на филтер-стапчетосе главна алатка за модифицирање на ефикасноста на филтрирањето на чадот од цигарите. И покрај широкиот спектар на цигарни филтер-стапчиња на пазарот, ацетатното продолжи да биде најчесто користено филтер-стапче. Со слична ефикасност за отстранување чадот е филтер-стапчетоизработено од хартија. Се проценува дека во последниве години производство на целулозно ацетатни филаментиуспоруваи ќе биде помало од побарувачката на ацетатно целулозните филтри. Целта на нашето истражување е да се утврди ефикасноста на хартиеното филтер–стапче врз составот на чадот и на задржувањето на цврстите честички и никотинот во споредба со филтер-стапчето од целулоза ацетат како контрола. Содржината на катранот и никотинот се одредува според ISO 4387 и ISO 10315, последователно. За сертифицирање на падот на притисок на филтер-стачето и цигари беше употребен ISO 6565. Резултатите покажаа дека ефикасноста на отстранување на хартиеното филтер–стапче е споредлива софилтер-стапчето од целулоза ацетат.

Клучни зборови: хартиено, целулозно ацетатно, филтер-стапче, ефикасност, цврсти честички, никотин, чад

INTRODUCTION

Cigarettes marketed today may be perceived as havingessentially two sections—a column of tobacco and a filter. Large-scale manufacture of filtered cigarettes began in the early 1950s in response to demand for lower smoke yields. The purpose of the first filters on cigarettes was primarily to aesthetic and hygienic function. Gradually, this function shifts and filters nowadays are regarded as a major modifier of tobacco smoke.Filtered cigarettes gained popularity rapidly, and today the majority of cigarettes sold are filtered.Moreover, even tobacco for rolling cigarettes available for consumption with filter cartridges with slices. Because filter materials influence yields of some smoke constituents to varying degrees, cigarette taste changes with the filter material used (Kirkova, 2007, 2007a).

Probably the first functioning filters were made from creped paper that was corrugated and formed into a cylinder. These filters were and are efficient smoke removers, but they were rapidly replaced by cellulose acetate filters when they became available (Browne,1990, Norman, 1999).

For most of the world, the primary cigarette filter medium is cellulose acetate, with a smaller specialty market primarily using dual or triple filters containing cellulose acetate with corrugated paper and/or activated charcoal (Norman, 1999, Georgiev, 2002, Nikolić, 2004).

The filter—whether made of cellulose acetate tow, paper, or a combination of these—serves several purposes. Functionally, it reduces the amount of smoke, captures particulate matter from the smoke and absorbs vapours. Particulate removal occurs largely through mechanical filtration of the smoke aerosol particles. Smoke particles can be captured via internal impaction, diffusion or direct interception. About 65% of the total particulate removal is attributed to diffusional deposition, and 35% to direct interception (Eaker, 1990, Georgiev, 2002).

Cellulose acetate and paper filter materials have only small influences on vapour phase retention (Browne,1990, Hoffmannet al., 1995, Norman, 1999).

Factors that influence preferential retention include several properties of both the compounds of interests (particulate matter, semi-volatile, volatile, carbon monoxide) and of the fiber (Georgiev, 2002, Nikolić, 2004, Kirkova, 2004, 2004a).

By estimates, however, consumption of acetate filament as the main raw material for production of filter rods in the coming years will exceed supply. In this regard, many companies have directed their efforts to develop advanced technologies for production of filter material from non-traditional and readily available materials.

The problem with these developments is to maintain and/or increase the attenuation of the filters with minimum modifying effect, and preservation of the smoking characteristics typical for the brand. For this reason of particular interest are paper filters. The aim of our research is to determine what is the efficiency of the paper filter from the known and preferred efficiency of cellulose acetate filter.

MATERIAL AND METHODS

The object of our research are filter rods made of acetate filament 3.0 Y35 000-control and made of a special corrugated paper-test sample.

Experiments were performed on paper sheet with two width and varying depths of longitudinally corrugating. Filter rods used have different paper width - 300 mm and 330 mm and depth of corrugating from 0.00 mm to 0.50 mm. Final length of filter rod was 126 mm.

To determine the retention efficiency of the filters cigarettes were made on same cigarette making machine (Molins 9N, UK). The filter rod was 20 mm long and the length of the cork-paper was 24mm. The total cigarette length was 84 mm.

Using standardized methods for analysis we determine the physical properties of the filter rods (corrugation depth, diameter, mass, pressure drop and hardness) and thickness of tipping paper. Used tipping papers used in both filters were no perforated, and with very close valuesof their masses. Cigarette paper and wrapping paper are regular manufactured.

Determination of cigarette mass, resistance to draw, cigarettes diameter, hardness and statistical processing of the results were made on SODIMAT.

Cigarettes are smoked under standard conditions, the cigarette smoke condensate collected on a Cambridge filter pad as described in ISO 4387:2000, and vapor phase for determination of carbon monoxide in ISO 8454:2007. This condensate is extracted as per nicotine determination described in ISO 10315:2000. The number of puffs in control and test samples was equal. It follows that for both cigarettes (control and test samples) are provided with equal conditions of combustion, which provides uniformity of the smoking parameters.

RESULTS AND DISCUSSION

1.Characterization of paper physical properties

Values of the most important physical parameters of paper filter rods investigated in this experiment (corrugation depth, diameter, mass, pressure drop and hardness) and the coefficient of variation (CV) were presented in Tables 1.

Table 1. Physical parameters of paper filter rods

Corrugation depth (%)	Diameter (mm)	Weight (g)	Pressure drop (mm WG)			Hardness (%)	
			L=126 mm	CV	L=20 mm	%	CV
Paper width 300 mm							
0.00	7.84	1.10	362.4	3.0	57.5	92.6	0.9
0.10	7.87	1.09	434.2	1.9	68.9	92.5	0.8
0.20	7.84	1.09	485.0	2.1	77.0	93.2	0.7
0.30	7.85	1.10	573.1	2.2	91.0	95.1	1.1
Paper width 330 mm							
0.10	7.86	1.20	627.3	2.2	99.6	95.0	1.0
0.50	7.87	1.19	736.8	2.4	117.0	95.7	1.3

The results show that with increasing width of the paper increases pressure drop and hardness of filter rods at a relatively constant weight and diameter. The coefficient of variation also is increased. It is obvious that with increasing depth of corrugating increasing values of pressure drop and hardness. This is explained by the increase of specific surface of filter. Compared with acetate filters, variation in pressure drop for the same amount is significantly higher, which means higher possibility to adjust the efficiency for smoke filtering. Increase the depth of corrugating gives proportionally increasing values of pressure drop and strength.

The coefficient of variation was biggest at depth of flute of 2.40 mm. Acceptable values for pressure drop, hardness and coefficient of variation are obtained at a depth of flute of 0.2 mm. Therefore, further research continued with the paper width 300 mm and depth of flute of 0.2 mm.

2. Comparison of the physical properties of the control and test sample filter rod

In Table 2 were present the mean values of average performance on the test filter rods of acetate filament - control and paper – sample, both both with a length of 120 mm.

Table 2. Comparison of parameter between paper and cellulose acetate filters

Properties	Cellulose acetate			Paper		
	м	у	CV	м	у	CV
Weight (g)	0.774	7.1	0.92	1.143	8.6	0.76
Pressure drop (mm)	367	86	249	345	15.9	4.95
Diameter (mm)	7.86	0.03	0.35	7.86	0.02	0.22
Hardness (%)	84.36	-	-	89,49	-	-
Weight (mm)	0.041	1.957	4.77	0.042	1.342	3.23

Compared with acetate filter, paper filter have a higher weight. The fact that the paper filter had higher weight due to the higher specific surface area, allows higher removal efficiency. Subsequently and hardness values are higher in the paper filter. It is noteworthy that the pressure drop was slightly reduced. The remaining results are similar to the values obtained for acetate filter.

3. Retention of particulate matter and nicotine

The primary effect of fibrous cigarette filters (paper or cellulose acetate) is removal of particulate from the smoke. The major mechanism of smoke retention in fibrous cigarette filters is mechanical capture of aerosol particles. The nicotine removal is dependent on particulate filtration. In general, cellulose acetate remove tar and nicotine particles in this size range with an efficiency of 40-50% (Norman, 1999). According to Georgiev, (2002) paper filters without additives are characterized by medium efficiency for retention of nicotine and condensate (30% and 40%, respectively).

In Tables 3 are summarized values of the most important physical parameters on cigarettes with cellulose acetate and paper filters.

Table 3. Physical parameters of investigated cigarette and filter

Parameter	Cellulose acetate	Paper
Average cigarette weight (g)	103.39	111.12
Cigarette weight (g)	1.020-1.040	1.090-1.110
Filter weight (g)	0.147	0.213
Other materials weight (g)	0.197	0.263
Draw resistance of cigarette (mmWG)	99-104	103-104
Pressure drop of filter (mmWG)	61.30	68.85
Diameter of cigarette (mm)	7.96-7.98	7.96-7.98
Hardness of cigarette (%)	78.54	77.89
Length of cigarette (mm)	84.42	84.26
Length of filter (mm)	19.94	20.13

It was found that cigarettes with paper filter shown the higher retention for nicotine and tar in comparison to acetate. Retention efficiency ranged from 12.1% higher for nicotine to 8.19% higher for particulate matter (Table 4).

Table 4. Comparison of retention between paper and cellulose acetate filters

Retention (%)		Particulate matter	Nicotine
Cellulose acetate	A	42.42	39.68
	B	43.30	37.91
	Mean value	42.86	38.79
Paper	A	52.50	51.18
	B	49.60	50.59
	Mean value	51.05	50.89

Obtained results are graphically presented on Figure 1.

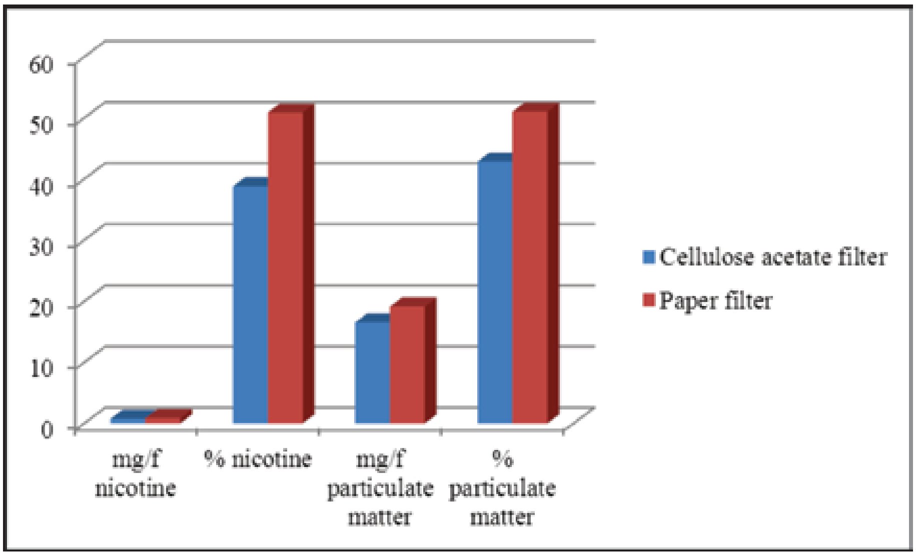

Figure 1. Retention efficiency of paper and cellulose acetate filter

4. **Removal efficiency of paper and cellulose acetate filters for particulate matter, nicotine and carbon monoxide from mainstream smoke**

In Tables 5 were presented values of the physical parameters on cigarettes with cellulose acetate and with paper filter.

Table 5. Physical parameters of cigarettes and filter rods

Parameter	Cellulose acetate	Paper
Cigarette weight (g)	1.030	1.110
Tobacco weight (g)	0.840	0.845
Other materials weight (g)	0.197	0.263
Cigarette length (mm)	84.0	84.0
Filter length (mm)	20.0	20.0
Diameter (mm)	7.95	7.95
Draw resistance of cigarette (mmWG)	102	105
Pressure drop of filter (mmWG)	62	68
Hardness of cigarette (%)	78.5	77.9
Permeability (CU)	35.0	35.2
Free combustion (mm)	10.3	10.4

The results show the increase in weight of cigarettes with relatively equal weight of tobacco. It results because the weight of other materials in cigarette was bigger. Cigarettes were analyzed to determine the amount of nicotine, carbon monoxide and total particulate matter in mainstream smoke.

Obtained results are graphically presented on Figure 2.

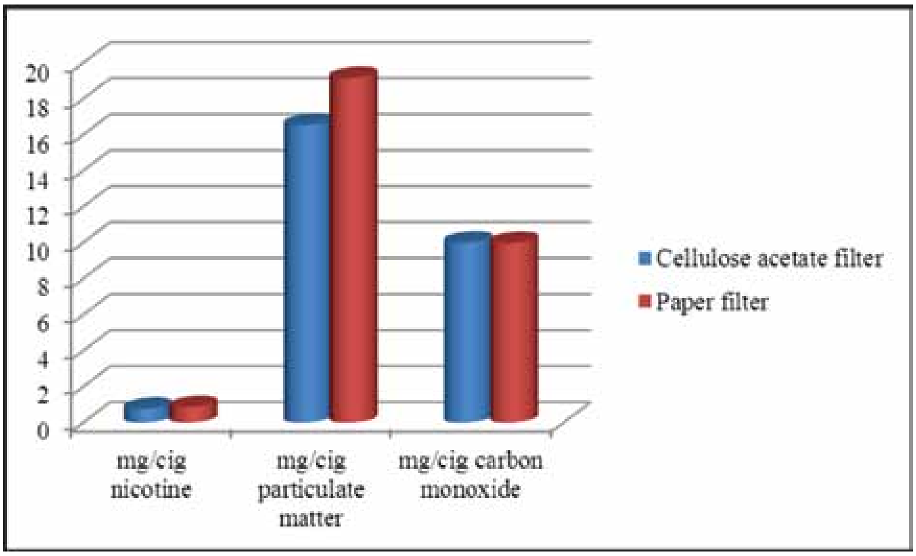

Figure 2. Chemical composition of cigarettes with paper and cellulose acetate filter

From the results it can be concluded that although it uses the same tobacco blend and uniform design cigarettes, nicotine in cigarette with paper filter was lower by 0.3 mg/cig, or 26.50% than cigarette with acetate filter. Also, particulate matter in cigarette with paper filter was lower by 3.71 mg/cig, or 16.80% than cigarette with acetate filter, as control. As regards the retention of carbon monoxide both filters have the same capacity.

CONCLUSION

The results of this study demonstrate the possibilities of paper filters and their effectiveness against the harmful constituents of smoke - nicotine and particulate matter, while preserving the smoking characteristics of cigarettes.

Investigated paper filter is characterized by high efficiency for retention of nicotine and particulate matter from mainstream smoke. Both type of filters, paper and cellulose acetate have only small influences on carbon monoxide retention.

REFERENCES

1. Browne C.L., 1990. The design of cigarettes, 3rd ed: Hoechst Celanese, 59-89.
2. Eaker D.W., 1990. Dynamic behaviour and filtration of mainstram smoke in the tobacco column and filter. Rec.Adv.Tob. Sci16.103-187.
3. Hoffmann D., Djordjevic M.V., Brunnemann K.D., 1995.Changes in cigarette design and composition over time and how they influence the yields of smoke constituents. J SmokingRelated Disorders; 6:9–23.
4. ISO 10315:2000 Cigarettes -- Determination of nicotine in smoke condensates -- Gas-chromatographic method.
5. ISO 4387:2000 Determination of Total and Nicotine-Free Dry Particulate Matter using a Routine Analytical Cigarette-Smoking Machine Determination of Total Particulate Matter and Preparation for Water and Nicotine Measurements.
6. ISO 6565:2011 Tobacco and tobacco products -- Draw resistance of cigarettes and pressure drop of filter rods -- Standard conditions and measurement.
7. ISO 8454:2007 Cigarettes -- Determination of carbon monoxide in the vapour phase of cigarette smoke -- NDIR method.
8. Kirkova, S., 2004. Studies on the influence of some factors in lowering the levels of nicotine, tar and carbon monoxide in cigarette smoke, 15th National Conference with International Participation “Quality-for better life’2004”, 113-118.
9. Kirkova, S., 2004a.Study the interdependence between the content of nicotine, tar and carbon monoxide in cigarette smoke, Scientific conference with international participation “in ‘60 ITTI” of HT ITTI, 345-350.
10. Kirkova, S., 2007. Studies on the interdependence of nicotine, tar and carbon monoxide in cigarette smoke and its manageability, Scientific conference with international participation “Food Science and Technology 2007”, UFT, HT-LIV, Iss.1, 243-246.
11. Kirkova, S., 2007a. Studies on the design element of tobacco to reduce nicotine, tar and carbon monoxide in smoke Scientific conference with international participation “Food Science and Technology 2007”, UFT, HT-LIV, Iss. 1, 253-257.
12. Nikolić M., 2004. Tehnologija prerade duvana, Univerzitet u Beogradu, Poljoprivredni fakultet, 73-83.
13. Norman A., 1999. Cigarette design and materials, in Davis D. L, Tobacco Production, Chemistry and Technology, 346-352.
14. Георгиев С. 2002. Технология на тютюневите изделия, второ издание, Антоан Георгиев, Пловдив, 270-326.

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AN OVERVIEW OF USE AND REGULATION OF ADDITIVES IN CIGARETTES

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ABSTRACT

In many countries around the world, tobacco control measures are becoming stricter and its directly reflect on to manufacturing and consumption of cigarettes.

In 2012 new regulatory framework is required in which the cigarette manufacturer is obliged disclose all additives used in cigarette by brand. This should include the impact of additives on smoking behaviour, passive smoking, direct or indirect pharmacological influence and fire risks.

The regulatory should permit additives necessary for the manufacture and storage of cigarettes providing these are safe, but should challenge all additives that may influence smoking behaviour.

Although it is impossible to make a safe cigarette, it is reasonable to prevent use of flavourings and additives that are harmful and increases the addictiveness of cigarette.

The aim of this work was to classify types of additives used in cigarettes, examine their legal status and reasons for the use or ban.

Key words: cigarettes, additives, regulation, use, smoke, tobacco

ПРЕГЛЕД НА УПОТРЕБАТА И ЗАКОНСКАТА РЕГУЛИТИВА НА АДИТИВИ ЗА ЦИГАРИТЕ

Во многу земји низ целиот свет, мерки за контрола на тутунот стануваат се построги и директно се одразуваат на производството и потрошувачката на цигари. Во 2012 година потребно е да се донесе нова регулаторна рамка со која производителот на цигари е должен објави сите адитиви што се користат во одреден бренд на цигара. Објавата треба да го вклучува влијанието на адитивите врз зависност од пушење, пасивното пушење, директно или индиректно фармаколошко влијание и ризикот од запалување. Регулаторите треба да дозволат да се применуваат адитиви кои се неопходни за производство и складирање на цигари и да се осигураат дека тие се безбедни, но треба да ги отстранат сите адитиви кои може да предизвикаат на зависност од пушењето. Иако е невозможно да се направи безбедна цигара, разумно е да се спречи употребата на аромите и адитиви кои се штетни и ја зголемува зависноста од цигарите. Целта на овој труд е да се класифицираат видови на адитиви што се користат во цигарите, да ги испита нивниот правен статус и причините за употреба или забрана.

Клучни зборови: цигари, адитиви, регулатива, употреба, чад, тутун

INTRODUCTION

Cigarettes are industrial form of tobacco use with a rather complex design providing the consumer with a product of high and consistent quality. Design features encompass a wide range of variables such as tobacco type and blend, chemical processing and ingredients, and in addition, physical features such as paper, filter and ventilation (Browne, 1990; Norman, 1999; Wigand, 2006).

The definition for additives from the European Union (Directive 2001/37/EC,2001), is "*Any substance or any constituent except for tobacco leaf and other natural or unprocessed tobacco plant parts used in the manufacture or preparation of a tobacco product and still present in the finished product, even if in altered form, including paper, filter, inks, and adhesives*".

The term "additive" means "*any substance the intended use of which results or may reasonably be expected to result, directly or indirectly, in its becoming a component or otherwise affecting the characteristic of any tobacco product (including any substances intended for use as a flavoring or coloring or in producing, manufacturing, packing, processing, preparing, treating, packaging, transporting, or holding), except that such term does not include tobacco or a pesticide chemical residue in or on raw tobacco*" (FDA, 2012)

In the present work the term additives is use for added substances or ingredients. Additives are generally understood to be substances added to the basic components, such as tobacco, paper, filter materials, inks and adhesives, to impart specific desirable properties on them and control the performance of the cigarettes while being smoked. They are intentionally added to cigarettes by the tobacco industry to modify flavour, regulate combustion, moisturise the smoke, preserve the cigarettes, and in some instances to act as solvents for other additives (Rabinoff, 2007)

Cigarette styles are characterized by their tobacco blend. There are two major styles of cigarettes worldwide: flue-cured cigarettes, which use very few additives, and traditional blended cigarettes, which use a number of additives. Flue-cured cigarettes contain only flue-cured tobacco and do not use flavor ingredients. These cigarettes are popular in most of the British Commonwealth (Australia, Canada, India, Malaysia, Pakistan, Nigeria, the U.K. and South Africa).

Traditional blended cigarettes utilize three different types of tobacco − flue-cured, burley, and oriental − that are blended together during the manufacturing process. Blended cigarettes are the most popular cigarettes in the United States, most of Europe, Latin America, Eastern Europe and many Asian countries. Various additives are combined into the shredded tobacco product mixtures, with humectants such as propylene glycol or glycerol, as well as flavouring products and enhancers such as cocoa solids, licorice, tobacco extracts, and various sugars, which are known collectively as "casings". The leaf tobacco will then be shredded, along with a specified amount of small laminate, expanded tobacco, blended and reconstituted leaf sheet, expanded and improved stems. A perfume-like flavour/fragrance, called the "topping" or "toppings", which is most often formulated by flavor companies, will then be blended into the tobacco mixture to improve the consistency in flavour and taste of the cigarettes associated with a certain brand name. Additionally, they replace lost flavours due to the repeated wetting and drying used in processing the tobacco. Finally the tobacco mixture will be filled into cigarettes tubes and packaged (Fisher, 1999, Георгиев С, 2002; Nikolić, 2004).

Beyond the core ingredients in cigarettes (tobacco leaf and paper), there is a list of at least 599 ingredients that are known to have been added to cigarette tobacco. This list of ingredients was submitted by the five major American tobacco companies during 1994 congressional hearings investigating the tobacco industry. The list contains a number of common flavoring agents, such as vanilla. It also includes hundreds of other chemicals approved for use in food products by the Food and Drug Administration (FDA).

The number of additives used may vary with anywhere from 30 to 150 different flavours being used for one brand (Manus, 1989). The tobacco industry claimed it had 1400 additives that could be put into cigarettes (Manus, 1989).

Additives can account for over ten percent of the total weight of a cigarette (Bates et al., 1999). The characteristics of each brand depend on the tobacco type and blend, how it is cured, the additives used and other technical characteristics of the cigarette. Ingredient mixtures differ between brands and even within a given brand because of country-specific preferences.

Many countries regulate tobacco product ingredients. Over 50 countries require tobacco manufacturers to report the ingredients used in their products to regulators. These countries include all of the European Union countries, Brazil, Mexico, Ukraine, Turkey, Israel and Thailand. Several countries, including Germany, the United Kingdom and France, also regulate the ingredients that are permitted for use in tobacco products.

The countries of south-eastern Europe, Albania, Bosnia and Herzegovina, Bulgaria, Croatia, Macedonia, Montenegro, Romania and Serbia have recognized that they have significant public health, economic and social problems relating the lack of tobacco control.

More than 170 countries adopted in November 2010 at the conference of the World Health Organization (WHO) in Uruguay measures to control the use and sale of tobacco products, including those related to adding flavors in cigarettes. It has recommended that additives used to make cigarettes more appealing to new smokers should be restricted or banned. Recommendations from the Uruguay conference were to determine whether countries should forbid addition of all new additives and explicitly address the possibility of reducing the use of additives that make tobacco products more attractive and/or taste better. … (WHO, 2010).

A new regulatory framework is required these countries in which the manufacturer is obliged to submit information about additives. For each additive that the tobacco company used for cigarettes they manufacture and, for cigarettes that are imported, the company should submit information. There decided to be a full disclosure of ingredients, additives and smoke constituents by brand or only for "Characterizing flavours ".

For this reason, tobacco additives should be seen as a public health issue and an appropriate regulatory framework would require the tobacco companies to disclose used additives in countries of Balkan region.

Additives and tobacco industry

Increased knowledge about cigarette additives makes it clear that modern cigarettes are very different from cigarettes of the past. Bates et al. (1999) report that there are 600 tobacco additives allowed in the cigarettes, most of them were included in the manufacturing of cigarettes after 1970.

Chitanondh (2000) suggests that the need for additives arose with the development of filters and "low-tar" products in response to consumer demand for a reduction in health risks. Filters, low tar and nicotine alter flavour – the smoke becomes drier, losing much of its body. Taste yield is reduced and the descriptive analytical profile of the smoke flavour changes. A repeatedly described taste deficiency is that of a "dry mouth feeling".

The rise of additives in tobacco products is linked with the strategy to reduce tar yields. When health concerns related to smoking were first raised in the 1950s, cigarette manufacturers responded by introducing filtered cigarettes. Manufacturers competed with one another to reassure health conscious smokers with new, "healthier" products (Pollay and Dewhirst, 2000). By the early 1960s in the USA, some health groups expressed concern that these health claims were not backed by objective scientific data. In 1967, the Federal Trade Commission (FTC) in the USA began a program to test cigarettes for tar and nicotine yields in cigarette smoke (Peeler, 1996).

"Light" cigarettes were introduced on the market in the 1970s. Typical for light cigarettes is their high grade of ventilation. Due to the delivery of less tar, the impact and taste of the "diluted" smoke is also decreased. It is therefore probable that the light cigarettes were "enriched" by adding more substances, and in higher amounts, to compensate for reduced taste and impact. At the same time an extremely lax regulatory regime for additives has emerged (SCENIHR, 2010).

In 1984, the US Department of Health and Human Services began requiring tobacco companies to submit annually a confidential, aggregated list of ingredients added to cigarettes manufactured in or imported into the United States.

There is evidence that the percentage of additives by weight may have increased in the 1990s, especially the use of sweeteners (Bates et al., 1999).

In 1994, National Public Radio reported on a number of these ingredients, which caused a public outcry. Subsequently, in that same year, the 6 major US tobacco companies made the list public. This was the only time the list was made public, and there is no current public list of tobacco additives (Rabinoff et al.,2007, Additive list, 1994).

Tobacco companies have devoted a significant amount of research and development to the use and inclusion of different additives in cigarettes (Bates et al., 2000; Lewis and Wackowski, 2006).

According to various tobacco company documents, many of these additives are used by manufacturers to influence the pharmacological effects of nicotine, make individual brands taste more appealing to smokers and mask the taste and immediate discomfort of smoke (Bates et al., 1999, Paschke et al., 2002, Pötschke-Langer M., 2012)

Other important reason for using additives is to give the cigarette a specific and standardised taste. A specific taste is important for the company to be competitive on the consumer market in view of the large variety of brands available. The specific taste of a certain product must be preserved (standardised) to compensate for the yearly variation of the natural tobacco, because consumers do not like to smoke a product that changes from year to year.

Additives and manufacturing process

Tobacco companies intentionally add additives to tobacco for several reasons including increasing the moisture-holding capacity, enhancing the taste of tobacco to make the product more desirable (e.g. sweeteners, flavourings, menthol), masking the smell and visibility of sidestream smoke, and decrease smoke irritability (Wigand, 2006, SCENIHR, 2010).

There are two distinct classes of additives, namely intentional and unintentional. Intentional additives encompass *all* chemicals, substances, complex botanical extracts, flavorings, ingredients, inks, gums, combustion modifiers, aesthetic, inorganic salts or functional chemicals, etc., that are deliberately added to a traditional tobacco product, that is either combusted or heated (Wigand, 2006).

Unintentional additives have not specific purpose but are by-products of the tobacco growing (fertilizers, pesticides, metals absorbed from the soil etc.), handling and manufacturing processes (conveyor belt fragments, oil from the machinery) (Wigand, 2006).

Also, additives may be natural or synthetic; they may include artificial tobacco substitutes, flavour extracts of tobacco and other plants, exogenous enzymes, powdered cocoa, and other synthetic flavouring substances.

Additives are defined as any ingredient or substance that is added, except water, during the course of manufacture of a tobacco product, including casings, humectants, flavours, and processing aids (Wigand, 2006).

Typical tobacco additives include:

Humectants are substances which increase the moisture-holding capacity of the tobacco. Preservatives include substances that protect the product from deterioration caused by microorganisms (Wayne and Connolly, 2002.).

Solvents are substances used to dissolve or dilute ingredients, without altering their function, in order to facilitate their handling and application.

Binders and strengtheners are substances that make it possible to maintain the physical state of the product. Fillers allow the product to keep the volume without contributing significantly to odor, taste or flavor.

In cigarettes, flavours may be added to tobacco, cigarette paper, the filter, in a plastic pellet placed in the filter or the foil wrapper, in an attempt to enhance the tobacco flavour, mask unpleasant odour, and deliver a pleasant cigarette-pack aroma. Internal industry documents reveal additional flavour technologies such as flavour micro encapsulation in the paper, carbon beads, and polymer-based flavour fibres inserted into the filter, flavoured tipping etc. (Nikolić, 2004, WHO, 2007, SCENIHR, 2010).

They may be used as casing ingredients or flavourings (sometimes referred to as top flavours). Casing ingredients are substances used to enhance the tobacco product sensory quality by balancing sensory attributes and developing certain required taste and flavour characteristics. Casing refers to the sauce composed of a variety of ingredients such as humectants, sugars, cocoa, liquorice and fruit extracts. They are usually applied to tobacco strips or leaf early in the primary processing scheme to tone down or mute the strength or harshness of tobacco smoke, improve processibility of tobacco and add deep flavour notes to the smoke. Being used early in the process, different casings can be applied to the various tobacco blends (e.g., burley, flue-cured, oriental, etc) (Brown, 1990, Fisher, 1999, Nikolić, 2004).

Flavorings (or top flavours) are substances used to impart a specific taste and flavor in a tobacco product. They are applied to the cut and processed tobacco prior to cigarette manufacture, usually in parts per million (ppm) quantities in a complex mixture in solution. Those tobacco blends that contain flavours and flavourings are usually held in a bin to allow for equilibration across the blend before it is passed to the making machine as the final blend. Flavorings are used to improve quality of smoke, impart a pleasant pack aroma and side stream aroma, and give the tobacco brand its unique sensory characteristics (Fisher, 1999, Nikolić, 2004).

The tobacco industry has pursued many non-conventional flavour technologies to address the goal of unique flavour delivery. For example, internal documents reveal that polymer pellet technology, using a flavoured filter pellet (polyethylene bead), was designed to provide controlled release of flavour for delivery to the smoker (Arzonico, 1989, Saintsing, 1992). Philip Morris also explored flavour release technology using carbon beads (Moore, 1994.) and various additives (i.e. cinnamaldehyde and gluco-vanillin) designed to flavour mainstream and sidestream smoke (Houminer, 1989) with a sweet, vanillin-type aroma. Additional flavour technologies described in tobacco industry documents include flavour microencapsulation in the paper, packaging technology, polymer-based flavour fibres inserted into the filter, and flavoured tipping (Saintsing, 1992, Douglas, 1989, Robinson, 1992.).

Menthol may be added at any of the following stages; spraying onto the final blend, through addition to the filter via a thread, or by application to the cigarette paper or the foil used to wrap the cigarettes. Due to the high level of volatility of menthol, different manufacturers have over the years developed a variety of methods for producing mentholated products that are as consistent as possible in terms of their finished product menthol levels (SCENIHR, 2010, Neck, 2010.).

“Flavoured tobacco product” means any tobacco product or any component part thereof that contains a constituent that imparts a characterizing flavour. “Characterizing flavour” means a distinguishable taste or aroma, other than the taste or aroma of tobacco, menthol, mint or wintergreen, imparted either prior to or during consumption of a tobacco product or component part thereof, including, but not limited to, tastes or aromas relating to any fruit, chocolate, vanilla, honey, candy, cocoa, dessert, alcoholic, beverage, herb or spice; provided, however, that no tobacco product shall be determined to have a characterizing flavour solely because of the use of additives or flavourings or the provision of ingredient information… Non-characterizing flavours are licorice and cocoa (The Tobacco Control Legal Consortium, 2011).

Processing aids facilitate the manufacture of cigarettes, such as by making cured tobacco less brittle. These include several ammonia compounds, carbon dioxide and ethyl alcohol.

Ammonia compounds are added to some brands in order to increase the level of unprotonated nicotine in the smoke (Bates et al., 1999a).

Regulation

There are important efforts to regulate tobacco products, such as the World Health Organization’s Framework Convention on Tobacco Control (FCTC) and the US Congress’ consideration of legislation to give the Food and Drug Administration (FDA) regulatory authority over tobacco (WHO, 2003).

The Framework Convention on Tobacco Control (FCTC), to which 176 countries are currently parties, contains a number of key demand-reducing strategies, such as tobacco taxation, education about health effects (including health warnings on packages), removal of misleading product descriptors, and support for cessation. As of August 30, 2012, 168 countries have signed and ratified or accepted the treaty. This is more than 85% of the world's population.

Nine countries have signed but not yet ratified the WHO FCTC: Argentina, Cuba, El Salvador, Ethiopia, Haiti, Morocco, Mozambique, Switzerland, and USA.

On the basis of public health protection, the Tobacco Products Directive (2001/37/EC) foresees measures on the manufacture, presentation and sale of tobacco products. It sets maximum limits for tar, nicotine and carbon monoxide yields of cigarettes and requires the industry to report on ingredients in tobacco products (Directive 2001/37/EC, 2001).

Siem (2000), has reported on the wide differences between the measures countries have taken in their efforts to regulate tobacco as a product. Until recently, the most relevant legal instruments to this issue - those to regulate consumer products and foodstuffs – have only exceptionally been used. He suggests that "the way is now open" for States to legislate their rights to verify the content of tobacco and smoke, to inspect production sites, limit the amount of certain ingredients, and approve additives. They can also request the declaration of certain additives, the purpose for their use and a toxicological evaluation.

Under Articles 9 and 10 of the FCTC, parties will be developing systems to regulate the contents, design and emissions of products and to require reporting on the same from manufacturers. To date, several countries currently have limits on tar and nicotine emissions (for example, European Union, Brazil, South Africa, Egypt) and others require regular reporting of tar and nicotine emissions (for example, Canada, United States, Hong Kong).

Article 6 of the Tobacco Products Directive 2001/37/EC requires that manufacturers and importers of tobacco products submit a list of all ingredients, and quantities thereof, used in the manufacture of those tobacco products by brand name and type. It specifies the content of this list and requires that the list be accompanied by the toxicological data available to the manufacturer and importer.

In 2001, the European Parliament adopted Directive 2001/37/EC concerning the harmonization of the regulation of tobacco products in the member states of the European Union (Directive 2001/37/EC of the European Parliament and of the Council of 5 June 2001 on the approximation of the laws, regulations and administrative provisions of the Member States concerning the manufacture, presentation and sale of tobacco products).

The production of tobacco and cigarettes and their consumption in European Union is influenced by the World Health Organization WHO through the EU health policy, based on the principles of Warszawa declaration and the EU strategy for tobacco control (ESTC-European Strategy for Tobacco Control). All this principles are harmonized with the Framework Convention for tobacco (FCTC), under guidance of the WHO, and signed by all European Union, and some countries in development.

Reporting requirements have been adopted in various jurisdictions including European Union, Canada, Brazil, Thailand and some state of the US. FDA has developed an electronic submission tool, eSubmitter, to streamline submission and receipt of the ingredient information required by sections 904(a)(1) and 904(c) of the act.

The European Commission's Practical Guide on "Reporting on tobacco product ingredients", issued on 31 May 2007, and specifies the reporting format, which is the basis for the EMTOC system. Additionally required information is collected solely for the purposes of data processing and communication (Practical Guide Reporting on tobacco product ingredients, 2007).

In Canada, for example, tobacco manufacturers must report the quantity of all ingredients used in the cigarette paper and filler, as well as the levels of specified chemical constituents in tobacco and mainstream and sidestream smoke emissions. On May 26 2009, Canada's Health Minister tabled Bill C-32, which would ban flavoured cigarillos, cigarettes and blunts (tobacco rolling papers), but exempt menthol as an additive (House of Commons of Canada, 2009).

In the United States, New Zealand, and Chile, tobacco manufacturers report a composite list of additives which may be present in any tobacco product. New Zealand additionally requires manufacturers to provide the maximum levels of each additive present in various types of tobacco products, including cigarettes (Thomson, 2005).

Brazil's National Health Surveillance Agency (ANVISA) passed new regulations which ban the use of additives and flavorings in all tobacco products. The new regulations will ban the use of menthol, clove and ammonia among a long list of additional banned additives.

Moreover, the bill that would the United States Food and Drug Administration (FDA) to regulate tobacco, would ban flavoured cigarettes, but exempts menthol (Waxman, 2009).

Australia has been negotiating with Australian cigarette manufacturers for the disclosure of ingredients (Health Government of Australia. 2009). The sale of most confectionary-flavoured, confectionary-scented and fruit-flavoured or fruit-scented cigarettes is banned in ACT, New South Wales and South Australia.

The first report on the application of the Tobacco Products Directive 2001/37 EC indicated that in the EU different reporting formats are used for the submission of tobacco products ingredient and emission information. The data sets delivered by manufacturers and importers to Member States are often incomplete. The first report therefore suggested that the Commission develops harmonised data collection methods that are based on a common EU format and improved definitions (Directive report, 2005).

In a Germany and the UK, the government publishes lists of ingredients that are permitted or prohibited for use in tobacco. The UK sets a maximum limit for each additive based on percentage added by weight of tobacco (Geiss and Kotzias, 2007).

Germany prohibits use of a number of additives. In both cases, disclosure of additives is not required from manufacturers, although the UK House of Commons Health Committee recently required the companies to submit additives by brand to them. These were then published in full, by brand, on the Health Committee's website, although flavourings are not broken down into their constituent parts (WHO Monograph, 2000).

The Tobacco Manufacturers Association of Denmark disclosed in July 2000 a list of 37 additives used in cigarettes on the Danish market which also included the additives' purpose of use (WHO Monograph, 2000).

Philip Morris and Imperial Tobacco have voluntarily submitted their own composite additive lists. However, a look at the websites of the larger cigarette manufacturers show, that, in a given brand, typically less than 10 different tobacco ingredients are used at concentrations higher than 0.1% (PMI, 2012).

In addition to the purposes listed above, this publication also included several further types of ingredients, including burn additives, plasticizers, preservatives, adhesives, dyes, and processing aids. Several of these types of ingredients are placed into or form part of the cigarette filter, where smoker exposure, if there is any, would be restricted to the ingredients in their natural form (i.e., not subject to changes incurred during the burning of tobacco) (Dempsey et al., 2011).

In the Republic of Macedonia, according to the Law on Tobacco and Tobacco Products "Manufacturers and importers of tobacco products at the request of the Ministry of Health are required to submit the data on the additives used in the manufacture of tobacco products and tobacco smoke onto the prescribed form. The form and content of the form referred to in paragraph 4 of this Article, its delivery and use of data by the Minister of Health (Law on Tobacco and Tobacco Products). Lists of permitted and non-permitted additives are given in the Official Gazette of the Republic of Macedonia No.56, 2007 (Lists of permitted and non-permitted additives, 2007).

The requirements under section 4 apply to each "tobacco product manufacturer or importer." We interpret this to mean that domestic manufacturers are to submit the required additive information for products they manufacture and, for tobacco products that are imported, the required ingredient information is to be submitted by either the foreign manufacturer or the importer of the product. This includes any regulated tobacco product, whether for sale to consumers or for further manufacturing. But this legal provision is not fully implemented.

DISSCUSION

Increased knowledge about cigarette additives makes it clear that modern cigarettes are very different from cigarettes of the past. Cigarettes are produced by an industrial process which, through a number of steps in handling the tobacco and by the use of additives, is tailoring the product to satisfy the user (Siem, 2000).

Tobacco companies have devoted a significant amount of research and development to the use and inclusion of additives in cigarettes and the industry has acknowledged using 600 different cigarette additives. According to various tobacco company documents, many of these additives are used to improve taste and decrease harshness (Rabinoff, 2007).

Tobacco companies claims that all of the additives used in the manufacture of cigarettes and other tobacco products are approved for use by either by the FDA GRAS list or the Flavor and Extract Manufacturers Association (FEMA). But, although a tobacco ingredients and additives may indeed appear on the GRAS list, this does not guarantee that this ingredient is safe. The main problem with the GRAS argument is that the ingredients on the GRAS list were never intended to be burned or inhaled.

Justification for the use of tobacco additives cannot be based solely on their approved use in food since, potentially, they could decompose into other substances during tobacco combustion in the smoking process. An assessment should include consideration of possible thermal decomposition of the ingredients, their effects on smoke chemistry and potential impact on smoke toxicity

The overall outcomes of investigations of manufacturers were the finding that tobacco ingredients - at levels typically used in tobacco products - have very limited influence on smoke chemistry. Three major reviews were published in 2002 that examined studies on the chemical and biological effects of tobacco ingredients on smoke properties conducted over the last 50 years (Paschke et al., 2002; Rodgman, 2002 a, b).

Rodgman conclusions were similar to those of Paschke et al., namely that ingredients added to tobacco during commercial cigarette manufacture in the U.S.A. do not increase the toxicity of cigarette smoke.

Also during 2002, Carmines and co-workers published a series of four papers that described a comprehensive study on the evaluation of 333 ingredients used in Philip Morris products. This study included smoke chemistry, in vitro genotoxicity and cytotoxicity, and animal sub-chronic inhalation toxicity (Carmines, 2002; Rustemeier et al., 2002; Roemer et al., 2002; Vanscheeuwijck et al., 2002). Taking into account all the smoke chemistry and biological data, they concluded that the addition of the ingredients to tobacco did not increase the toxicity of the smoke.

Some additives have been used to increase the apparent impact and addictiveness of cigarettes, it may also be possible to reduce the addictiveness of cigarettes without eliminating the nicotine by either prohibiting certain additives or requiring them (Henningfield et al., 1998). Increasing smoke delivery of nicotine has also been achieved using chemical additives such as nicotine maleate (Robertson, 2000). Chitanondh (2000) describes the several known carcinogenic or otherwise dangerous additives currently known to be present in cigarettes.

In 2002 Seeman et al. published a comprehensive review of the formation of acetaldehyde in mainstream cigarette smoke and its bioavailability in the smoker (Seeman et al., 2002).

Because of the importance of sugars as tobacco ingredients in American-blend cigarettes, many studies have been performed in order to understand any potential toxicological impact of sugars use (Baker, 2006; Talhout et al., 2006; Roemer et al., 2012). All of these studies used tobacco blends representative of American-blend cigarettes to produce research cigarettes that were cased with various levels of the respective sugar ingredient. It was found that sugars and polysaccharides materials were associated with increased formaldehyde yields. Cahours et al. (2012) supports the conclusion that structural material in the tobacco plant is the main source of acetaldehyde in mainstream smoke after combustion during cigarette smoking.

There is speculation that added ingredients which may be Generally Regarded as Safe (GRAS) from studies on usage in foods, may increase the toxicity of the smoke by forming ‘‘new’’ smoke constituents (or increase the concentrations of existing constituents) during pyrolysis and combustion (Thielmann and Potschke-Langer, 2005).

In the Scientific Committee on Emerging and Newly Identified Health Risks (SCENIHR) report published in 2010, attractiveness with regard to tobacco products is defined as the stimulation to use a tobacco product. The attractiveness of tobacco products may be increased by a number of additives. Specific additives can mask the bitter taste, improve the flavour and reduce the irritation of inhaled smoke. Examples of flavouring substances include sugars, benzaldehyde, maltol, menthol and vanillin. Altogether, these additives have the potential to enhance the attractiveness of cigarettes. The use of a combination of ingredients includes the investigation of any potential interactions among the ingredients, while the use of single ingredients allows greater exaggeration of the application levels of the particular ingredient beyond use levels found in commercial cigarettes (Connolly et al., 2000, SCENIHR 2010). Some additives, although not directly toxic in themselves, may nevertheless increase tobacco-related harm by making cigarettes more palatable, attractive, or addictive to consumers (Bates et al., 1999 a).

Tobacco industry has acknowledged that the addition of alkali such as ammonia to increase smoke pH increases the availability of "free" or "unbound" nicotine and thereby increases the nicotine addictiveness for given nicotine content (Henningfield et al., 2004).

The intended purpose of additives needs to be fully understood. If the purpose is to facilitate extra smoking or to increase the addictiveness of the product, it hardly matters whether the additive itself is toxic or benign.

European Commission Health and Consumer Protection Directorate (2010) recommended that ingredients have usually been evaluated as design principles rather than repeatedly for each product. For principle-based testing, ingredients may be applied to research cigarettes, either individually or in combination, in a manufacturing process which corresponds to that of commercial cigarettes.

The European Commission has announced that it is considering legislation concerning further restrictions on cigarette tar and nicotine yields, as well as new provisions to regulate additives and the labelling of the tobacco products.

The third meeting of the working group on economically sustainable alternatives to tobacco growing in relation to Articles 17 and 18 of the WHO FCTC was held in Geneva from 14 to 16 February 2012. The meeting was attended by Key Facilitators and Partners of the group. Participants also included representatives of WHO, the International Labour Organization, the Food and Agriculture Organization of the United Nations and the United Nations Environment Programme, as well as representatives of nongovernmental organizations accredited as observers to the Conference of the Parties (COP) and invited experts. The group discussed the draft policy options and recommendations presented by the Key Facilitators, including issues concerning standardization of terms and a methodological framework, for submission to COP5.

The fifth session of the Conference of the Parties (COP5) will be held from 12 to 17 November 2012 at COEX Convention Centre in Seoul, Republic of Korea.

The Parties to the Framework Convention on Tobacco Control, which also include Macedonia, Serbia and Bosnia and Herzegovina, have reached an agreement on the introduction of assistance programs for smoking cessation in national health systems and support campaigns to raise awareness of the population, according to a WHO statement.

Regulation of additives will be an important issue for research and potential future regulatory attention.

CONCLUSION

Cigarettes are manufactured by an industrial process which, through a number of steps in handling the tobacco, non-tobacco materials, and by the use of additives, is tailoring the product to satisfy the user. American style of blended tobacco cigarettes is increasing. For example, when worldwide cigarette demand was growing only 1% per year, the demand for American style products was projected to grow 3% annually (EUROMONITOR, 2012).

Comprehensive disclosure of smoke constituents and additives would improve consumer information, and removal of misleading branding and labelling of tobacco products.

The regulators should know which additives are in which brands and only permit them in brands where it could be proven that they would facilitate a public health gain. All additives should be covered by the regulations, including those added to non-tobacco materials (cigarette papers, filters, filter wrappers, and overwrappers). For particular cigarette, manufacturer should disclose the contents of all additives at regular intervals.

Proposed elements of tobacco product regulation

- Manufacturers should be required to disclose all additives (intentional and unintended) used in cigarettes, by brand, to a regulator.
- Some information should not be confidential, but made available to the public through publications, the Internet or on request from the regulator.
- There may be some additives that should be listed as ingredients on tobacco product packaging.
- Regulatory framework should permit additives necessary for the manufacture and storage of cigarettes providing these are safe, but should challenge all additives that may influence smoking behavior.

REFERENCES

1. Arzonico B.W., 1989. TF-4 accelerated aging study - TF products using various flavor and packaging systems to retard spearmint flavor migration, 11 October 1989. RJ Reynolds. Bates No. 508295999-508296015.
2. Australian Capital Territory Government, 2009. Tobacco (Prohibited Smoking Products) Declaration 2009 No.1. Available at: www.legislation.act.gov.au/ni/2009-53/current/pdf/2009-53.pdf. (accessed October 2012).
3. Baker R.R. The generation of formaldehyde in cigarettes—Overview and recent experiments. Food and Chemical Toxicology 44. 1799–1822
4. Bates C., Jarvis M., Connolly G., 1999. Tobacco additives, Cigarette engineering and nicotine addiction, UK ASH (Action of Smoking and Health). Available at: http://www. ash.org.uk/html/regulation/html/additives.html. Accessed October, 2012.
5. Bates C., McNeill A., Jarvis M., Gray N.J., 1999 a. The future of tobacco product regulation and labelling in Europe: implications for the forthcoming European Union directive. Tobacco Control.; 8: 225–235.
6. Bates M., Fowles J., Noiton D., 2000. The chemical constituents in cigarettes and cigarette smoke: priorities for harm reduction. Philip Morris. Bates no. 2505913406/3471. Available at:http://legacy.library.ucsf.edu/tid/nsn81c00. (accessed October 2012).
7. Browne C.L., 1990. The design of cigarettes, 3rd ed: Hoechst Celanese, 8-57.
8. Cahours X., Verron T., Purkis S., 2012. Effect of Sugar Content on Acetaldehyde Yield in Cigarette Smoke. Beiträge zur Tabakforschung International, Vol.25, No.2, 381-395.
9. Carmines, E.L., 2002. Evaluation of the potential effects of ingredients added to cigarettes. Part 1: Cigarette design, testing approach, and review of results. Food and Chem. Toxicol. 40, 77–91.
10. Chitanondh H., 2000. Tobacco industry response to ingredient disclosure law. Paper presented at the International Conference: Advancing Knowledge on Regulating Tobacco Products, Oslo, Norway, 9-11 February.
11. Cigarette additive list. http://en.wikipedia.org/wiki/List_of_additives_in_cigarettes. (accessed October 2012).
12. Conference of the Parties to the WHO FCTC, 15 -20 October 2010, Uruguay Available at: http://www.who.int/fctc/cop/sessions/fourth_session_cop/en/index.html. Accessed October, 2012.
13. Connolly G.N., Wayne G.D., Lymperis D., Doherty M.C., 2000. How cigarette additives are used to mask environmental tobacco smoke. Tob Control, 9:283–291

14. Dempsey R., Coggins C. R. E., Roemer E., 2011. Toxicological assessment of cigarette ingredients. Regulatory Toxicology and Pharmacology 61 119–128.
15. Directive 2001/37/EC of the European Parliament and of the Council, 2001. Available at: http://eur lex.europa.eu/LexUriServ/LexUriServ.do?uri=OJ:L:2001:311:0067:0128:en:PDF. (accessed October, 2012).
16. Douglas S.H., 1989. New technology exploration process. Project FAT qualitative research. (Atlanta, GA). RJ Reynolds. Bates No.507126759-507126764.
17. European Commission Health and Consumer Protection Directorate General Scientific Committee on Emerging and Newly Identified Health Risks. (2010).
18. FDA Tobacco Products Guidance Compliance Regulatory Information Availableat:http://www.fda.gov/ / ucm191982.htm. (accessed October, 2012).
19. Fisher P., 1999. Cigarette manufacture, in Davis D. L, Tobacco Production, Chemistry and Technology, 346-352.
20. Geiss O., Kotzias D., 2007. Tobacco, Cigarettes and Cigarette Smoke An Overview. Joint Research Centre. European Commission. Available at: http://publications.jrc.ec.europa.eu/repository/bitstream/111111111/8885/1/7472%20-%20EUR22783EN_Geiss_Kotzias.pdf. (accessed October 2012).
21. Георгиев С., 2002. Технология на тютюневите изделия, второ издание, Антоан Георгиев, Пловдив, 122-155.
22. Health Government of Australia. 2009. Available at: http://www.health.gov.au/internet/main/publishing.nsf/Content/3672CD0332CA36A1CA257575000F51AB/$File/b5.pdf. (accessed October 2012).
23. Henningfield J., Pankow J., Garrett B., 2004. Ammonia and other chemical base tobacco additives and cigarette nicotine delivery: issues and research needs. Nicotine Tob Res., 6:199–205.
24. Henningfield J.E, Benowitz N. L, Connolly G. N, Davis R. M, Gray N, Myers M. L, Zeller M, Reducing tobacco addiction through tobacco product regulation, Tob Control 2004, 13:132-135.
25. Henningfield J.E., Djordjevic M., 2004. Menthol cigarettes, research needs and challenges, Nicotine and Tobacco Research, 6 (Supplement 1): S11-S16.
26. Houminer Y., 1989. Potential flavoring of sidestream by using flavor-release technology. Philip Morris. Bates No. 2022206963-2022206964.
27. House of Commons of Canada, 2009. BILL C-32 : An Act to amend the Tobacco Act, May 26, 2009. Available at: www.parl.gc.ca/content/hoc/Bills/402/Government/C-32/C-32_1/C-32_1.PDF. (accessed October 2012).
28. Law on Tobacco and Tobacco Products (“Official Gazette of the Republic of Macedonia “no. 24/0, 88/08, 31/10, 36/11 and 53/11).
29. Lewis M.J., Wackowski O., 2006. Dealing with an innovative industry: a look at flavored cigarettes promoted by mainstream brands,” Am J Public Health, Feb; 96(2): 244-51. Available at: http://ajph.aphapublications.org/cgi/reprint/96/2/244.pdf. (accessed October 2012).
30. List of permitted additives (“Official Gazette of the Republic Macedonia “no. 56/07);
31. List to modify the list of prohibited additives (“Official Gazette of the Republic of Macedonia” no. 61/07).
32. Manus P., 1989. Flavorings: The right combination. Tobacco Reporter; April: 2.
33. Moore H.E., 1994. Calgon Carbon Corporation (untitled), Bates No. 2024583505-2024583506.
34. Neck J. D., 2010. A review and assessment of menthol employed as a cigarette flavoring ingredient, Food and Chemical toxicology, 48, S1-S38.
35. Nikolić M., 2004. Tehnologija prerade duvana, Univerzitet u Beogradu, Poljoprivredni fakultet, 1-33; 114-117.

36. Norman A., 1999. Cigarette design and materials, in Davis D. L, Tobacco Production, Chemistry and Technology, 353-387.
37. Paschke T., Scherer G., Heller W., 2002. Effects of Ingredients on Cigarette Smoke Composition and Biological Activity: A Literature Overview. Beiträge zur Tabakforschung International, Vol.20, No.3, 107-247.
38. Peeler C.E., 1996. Cigarette testing and the Federal Trade Commission: a historical overview. The FTC cigarette test method for determining tar, nicotine, and carbon monoxide yields of US cigarettes: report of the NCI Expert Committee. Bethesda, Maryland: National Institutes of Health, 1996. (NIH Publication No 96-4028.)
39. Philip Morris International. Available at: http://www.pmi.com/eng/our_products/ingredients/pages/technical_products_information.aspx. (accessed October 2012).
40. Pollay R.W., Dewhirst T., 2000. Successful images and failed fact: the dark side of marketing seemingly Light cigarettes. History of Advertising Archives, Working Paper #99.4.1:52.
41. Pötschke-Langer M., 2012. Additives in Tobacco Products. German Cancer Research Center (DKFZ), Heidelberg,1-31.
42. Rabinoff M., 2007. Pharmacological and Chemical Effects of Cigarette Additives. American Journal of Public Health 97: 1981-1991.
43. Rabinoff M., Caskey N., Rissling A., Park C., 2007. Pharmacological and chemical effects of cigarette additives. Am J Public Health. 97:1981–91.
44. Robinson A.L., 1992. Advanced technology products. Atp92 303. Review of qualitative and quantitative research: Chelsea and Horizon. 0113 - Project Xb", RJ Reynolds. Bates No. 513502565-513502596.
45. Rodgman, A., 2002a. Some studies of the effects of additives on cigarette mainstream smoke properties. I. Flavorants. Beitr. Tabak. Int. 20, 83–103.
46. Rodgman, A., 2002 b. Some studies of the effects of additives on cigarette mainstream smoke properties. II. Casing materials. Beitr. Tabak. Int. 20, 279–299.
47. Roemer E., Schorp M.K., Piade J., Seeman J.I., Leyden D.E., Haussmann H., 2012. Scientific assessment of the use of sugars as cigarette tobacco ingredients: A review of published and other publicly available studies. Critical Review in Toxicology. 42 (3). 244-278.
48. Roemer, E., Tewes, F.J., Mesigen, T.J., Veltel, D.J., Carmines, E.L., 2002. Evaluation of the potential effects of ingredients added to cigarettes. Part 3: In vitro genotoxicity and cytotoxicity. Food and Chem. Toxicol. 40, 105–111..
49. Rustemeier, K., Stabbert, R., Haussmann, H.-J., Roemer, E., Carmines, E.L., 2002. Evaluation of the potential effects of ingredients added to cigarettes. Part 2: Chemical composition of mainstream smoke. Food and Chem. Toxicol. 40, 93–104.
50. Saintsing B., 1992. Key objectives (Barry Saintsing), RJ Reynolds. Bates No. 510784522-510784527.Available at: (http://tobaccodocuments.org/preps/510784522-4527.html., (accessed October, 2012).
51. Scientific Committee on Emerging and Newly Identified HealthRisks (SCENIHR) 2010. Addictiveness and attractiveness of tobacco additives, Available at: http://ec.europa.eu/health/scientific_committees/emerging/docs/scenihr_o_031.pdf (accessed October 2012).
52. Seeman, J.I., Dixon, M., Haussmann, H.-J., 2002. Acetaldehyde in mainstream tobacco smoke: formation and occurrence in smoke and bioavailability in the smoker. Chem. Res. Toxic 15, 1331–1350
53. Siem H., 2000. Comparative regulatory frameworks: The legal framework for regulation of the product tobacco. Paper presented at the International Conference: Advancing Knowledge on Regulating. Tobacco Products, 9-11 Feb. Oslo, Norway.

54. Talhout R., Opperhuizen A., van Amsterdam J.G.C., 2006.Sugars as tobacco ingredient: Effects on mainstream smoke composition, Food and Chemical Toxicology 44, 1789–1798.
55. The European Commission's Practical Guide. 2007. Reporting on tobacco product ingredients", Available at: http://www.zigarettenverband.de/pos-data/page_img/file/EU_ingredients_practical_guidance_en.pdf. (accessed October 2012).
56. The first report on the application of the Directive 2001/37/EC.2005. Available at: http://www.ms.gov.ro/documente/283_576_Primul%20raport%20al%20Comisiei%20Europene%20despre%20Directiva%20200137EC.pdf. (accessed October 2012).
57. Thielmann H.W., Potschke-Langer M., 2005. Increased health hazards due to additives of tobacco products–consequences for product regulation. Heidelberg: Deutsches Krebsforschungszentrum.
58. Thomson G., 2005. Implementation failures in the use of two New Zealand laws to control the tobacco industry: 1989–2005. Australia and New Zealand Health Policy.
59. Vanscheeuwijck, P.M., Teredesai, A., Terpstra, P.M., Verbeeck, J., Kuhl, P., Gerstenberg, B., Gebel, S., Carmines, E.L., 2002. Evaluation of the potential effects of ingredients added to cigarettes. Part 4: Subchronic inhalation toxicity. Food Chem. Toxicol. 40, 113–131.
60. Waxman H.A., 2009. United States. House of Representatives 1256: the Family Smoking Prevention and Tobacco Control Act. (April 2nd version). Available at: http://energycommerce.house.gov/Press_111/20090304/hr1256.pdf. (accessed October 2012).
61. Wayne G.F, Connolly G.N., 2002. How cigarette design can affect youth initiation into smoking: Camel cigarettes 1983–93. Tobacco Control. 11 (Suppl 1): i32–i39.
62. WHO Monograph. 2000. Advancing knowledge on regulating tobacco products Available at: http://www.who.int/tobacco/media/en/OsloMonograph.pdf. (accessed October 2012).
63. Wigand J.S., 2006. Additives, Cigarette Design and Tobacco Regulation, Third WHO Study Group on Tobacco Regulation (TobReg), Kobe, Japan, 1-43.
64. World Health Organization (WHO). 2007. The scientific basis of tobacco product regulation: Report of a WHO Study Group. WHO Technical Report Series 945. Geneva, Switzerland: World Health Organization Press; 25-40.
65. World Health Organization (WHO). 2003. Framework Convention on Tobacco Control. Geneva: WHO, Available at: http://www.who.int/tobacco/fctc/text/en/fctc_en.pdf2003. (accessed October 2012).

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